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NEGATIVES IN SYMMETRIC INPUT-OUTPUT TABLES: THE IMPOSSIBLE QUEST FOR THE HOLY GRAIL

Louis de MESNARD

Université de Bourgogne & CNRS
UMR 5118 Laboratoire d'Economie et de Gestion
Pôle d'Economie et de Gestion, 2 boulevard Gabriel, 21000 Dijon, France

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Laboratoire d'Economie et de Gestion
Université de Bourgogne & CNRS UMR 5118

Pôle d'Economie et de Gestion - 2 boulevard Gabriel BP 26611 - F21066 DIJON cedex
Tel. +33 (0)3 80 39 54 30 Fax +33 (0)3 80 39 54 43
secretariat.leg@u-bourgogne.fr - www.u-bourgogne.fr/leg

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Louis de Mesnard

Résumé

Dans le modèle input-output « Supply-Use » (ou « Make-Use »), les hypothèses de « technologies basée sur les biens » (TB) ou de « structure fixe de vente des industries » (SFVI) sont plus largement adoptées (SCN, Eurostat) pour dériver des tables input-output symétriques (TIOS) que les hypothèses de « technologies basée sur les industries » ou de « structure fixe de vente de des biens », mais engendrent des termes négatifs dans la TIOS. Une TIOS déduite du modèle Supply-Use est considérée comme satisfaisante dès qu'elle ne contient pas plus négatifs : les chercheurs ont focalisé sur les termes négatifs dans les TIOS et sur la façon de les supprimer. Toutefois, comme les TIOS peuvent ne pas comprendre de termes négatifs, même s'il ya quelques termes négatifs dans l'inverse de la matrice Supply, nous avons complètement inversé le raisonnement. Un contre-exemple montre en effet que le calcul de l'inverse de la matrice Supply, comme l'imposent les hypothèses TB ou SFVI, est mathématiquement une opération interdite, même lorsque la TIOS ne comprend pas de termes négatifs ; ce résultat est nouveau. Ainsi, le calcul d'une TIOS sous les hypothèses TB ou SFVI doit être rejeté. Trois applications sont fournies : Autriche en 2000 et 2005 et USA en 2007.

Mots-clés

Input-Output, Supply-Use, Make-Use, Product technology, SIOT, SNA, Eurostat.

Abstract

In the Supply-Use (or Make-Use) input-output model, "product-technology" (PT) or "fixed-industry-sales-structure" (FISS) assumptions are more widely adopted (SNA, Eurostat) for deriving symmetric input-output tables (SIOT) than "industry-technology" or "fixed-product-sales-structure" assumptions, but generate negatives in the SIOT. A SIOT deduced from the Supply-Use model is considered as satisfactory as soon as it contains no more negatives: scholars have focused on the negatives in the SIOT and on how to remove them. However, as a SIOT may include no negatives even if there are some negatives in the inverse Supply matrix, we have completely reversed the reasoning. A counter-example demonstrates that computing the inverse Supply matrix, as imposed by PT or FISS assumptions, is mathematically a nonsense operation *even when the SIOT does not include any negative*; this result is new. Hence, deriving a SIOT under PT or FISS assumptions must be rejected. Three applications are provided: Austria 2000 and 2005 and USA 2007.

Keywords

Input-Output, Supply-Use, Make-Use, Product technology, SIOT, SNA, Eurostat.

Classification JEL :

C67, D57

1 Introduction

The two-matrix input-output model—the Supply-Use model or Make-Use model—is the basis for most national accounting schemes, such as the *System of National Accounts* (SNA) (United Nations 1968, 1993, 1999, 2001; Lawson 1997; Guo et al. 2002; Eurostat 2008; Horowitz and Planting 2009), but it is also considered very useful and more realistic than the traditional input-output model for regional, multiregional, interregional or international modeling because it forms the basis of many approaches to interregional and regional SAM models (Oosterhaven 1984; Shao and Miller 1990; Siddiqi and Salem 1995; Dietzenbacher and van der Linden 1997; Israilevich et al. 1997; Susiluoto 1997; Harris and Aying 1998; Jackson 1998; Madsen and Jensen-Butler 1998; Eding et al. 1999; Nijmeijer, de Vet and Eding 1999; Schaffer 1999; Comer and Jackson 2003; Sayapova and Slobodyanik 2008; Perez, Dones and Llano 2009): the model is considered more realistic than the simpler single-matrix Leontief model. It is also useful for sectoral analyses such as fisheries (e.g. Garcia-Negro 2004), transport, forestry (e.g. Forestry Department 1998), water (e.g. Anderson and Manning 1983), tourism (e.g., Manrique-de-Lara-Peñate et al. 2008; van de Steeg and Steenge 2008), etc., or for sustainable development analysis (e.g. Ravetz 2004; Wood 2008). This model is based on two matrices because the Leontief one-to-one correspondence sector/product is abandoned and replaced by the distinction—due to Stone—between industries and products, with one and the same product being able to be produced by many industries, and vice-versa. One finds:

- The *Use* matrix which is analogous to the Leontief matrix and which describes a linear production function with complimentary inputs.
- The *Supply* or *Make* matrix which describes which industry produces which product and reciprocally.

Currently Eurostat (2008) considers two types of tables: the product-by-product input-output tables by making an assumption about technology, and the industry-by-industry input-output tables by assuming “fixed sales structure”; see also Rueda-cantuche et al. (2009). In this paper, we focus on the product-by-product input-output tables, which are theoretically the better ones but the results can be transposed *mutatis mutandis* to the fixed sales structure assumption.

In order to make symmetric product-by-product input-output tables two hypotheses are set by the SNA 1993 for transferring outputs and associated inputs:

- The *product-technology assumption*, also called *commodity-technology*, which corresponds to Eurostat Model A: “Each product is produced in its own specific way, irrespective of the industry where it is produced” (Eurostat 2008, p. 297). This hypothesis is recommended by the new SNA even if it generates negatives, because it fulfils all four desirable axioms: material balance, financial balance, price invariance, scale invariance (Kop Jansen and Thijs Ten Raa 1990; ten Raa and Rueda-Cantuche 2003; ten Raa 2005) and (United Nations 1999, pp. 100–103). For the SNA, “Economically, the product technology assumption makes more sense than the industry technology assumption” (United Nations 1999, p. 87).

- Alternately, the *industry-technology assumption* corresponding to Eurostat Model B: “Each industry has its own specific way of production, irrespective of its product mix” (Eurostat 2008, p. 297). This hypothesis was recommended by the former SNA 1968 (United Nations 1968) but the present SNA 1993 considers that it is incoherent because it leads to incoherent “cooking recipes” (United Nations 1999, p. 99; Almon 2000). Unlike the product-technology assumption, it violates the last three of the four desirable axioms: financial balance, price invariance, scale invariance. Following ten Raa (2005), this is an obvious reason for abandoning the hypothesis based on industries.

For making symmetric industry-by-industry input-output tables, two hypotheses can also be set:

- The *fixed-industry-sales-structure* assumption is posited, corresponding to Eurostat’s model C: “Each industry has its own specific sales structure, irrespective of its product mix” (Eurostat 2008, p. 297). It generates negatives.

- The *fixed-product-sales-structure* assumption, which corresponds to Eurostat’s model D: “Each product has its own specific sales structure, irrespective of the industry where it is produced” (Eurostat 2008, p. 297). It is not affected by negatives, that is, no more than the industry-assumption in Eurostat’s model B.

It is clear that the SNA, Eurostat, and most scholars in Regional Science consider the product-technology and the fixed-industry-sales-structure assumptions to be the best ones. The choice seems clear: the industry-technology and the fixed-product-sales-structure assumptions should be discarded. It is known that the product-technology or the fixed-industry-sales-structure assumptions may generate negatives in the symmetric input-output table: see Table 1. The negatives are generally considered as caused by errors in the data or by the presence of different technologies or by heterogeneous classifications. As they are obviously annoying and cannot receive any satisfactory interpretation, various approaches have been proposed to eliminate them when the product-technology or the fixed industry sales structure assumptions are chosen (Almon 1970, 2000; Armstrong 1975; Rainer 1989; Steenge 1990; Rainer and Richter 1992; Mattey 1993; Mattey and ten Raa 1997; Braibant 2002); for a review, see United Nations (1999), ten Raa and Rueda-Cantuche (2003), Hoekstra (2005) or Eurostat (2008).

The result of the application of these remedies is that a symmetric input-output table is considered as satisfactory by most scholars as soon as it contains no more negatives.[1] While scholars have focused on the negatives in the symmetric input-output table, we will show in this paper that these negatives in the symmetric input-output table are only the symptom of the model's disease but are not the disease itself: the disease is much serious. We will completely displace the reasoning and demonstrate with the help of a counter-example that computing the inverse Supply matrix, as imposed by the product-technology or the fixed-industry-sales-structure assumptions, is mathematically a nonsense operation *even when the symmetric input-output table does not include any negative*. This has never been demonstrated before. For helping readers, we will use Eurostat's classroom example (2008), plus three real applications: Austria for the year 2000 and 2005 and the USA for the year 2007.

The paper is organized as follows. Section 2 reminds how symmetric input-output tables are derived; Section 3 exposes the classical approach of the problem of the negatives; Section 4 explains why computing the inverse Supply matrix should not be done even if no negatives are present in the symmetric input-output table; Section 5 concludes; Section 6 contains the bibliographical references and Section 1 is this introduction.

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[1] There are two notable exceptions. Ten Raa (1988) has understood that negative terms are not due to errors in the data but to the model. De Mesnard (2004) has shown that, if the model is interpreted in terms of circuit, it cannot be demand-driven as usual: the product-technology assumption corresponds to the so-called supply-driven Ghosh model, largely criticized (Dietzenbacher 1997; de Mesnard 2009).

Category of assumption	Technology		Fixed sales structure	
Input-output table	Product by Product		Industry by Industry	
Model	A	B	C	D
Assumption	Product-technology	Industry-technology	Fixed-industry-sales-structure	Fixed-product-sales-structure
Comment	Each product is produced in its own specific way, irrespective of the industry where it is produced	Each industry has its own specific way of production, irrespective of its product mix	Each industry has its own specific sales structure, irrespective of its product mix	Each product has its own specific its sales structure, irrespective of the industry where it is produced
Negatives	With	Without	With	Without

Table 1 The four models for deriving symmetric input-output tables from supply and use tables (extracted from Eurostat 2008, p. 295–296). The row “Negatives” will be discussed later.

2 Remind: Derivation of symmetric IO tables with the four models

The derivation of the symmetric input-output table will follow the standard presentation of Miller & Blair (1985 pp. 159 -…; 2009 pp. 230-…) but it is compatible with Eurostat’s equation (Eurostat 2008, pp. 348-349).[2] In the rectangular models such as the SNA, one considers that two rectangular homogeneous matrices are given, as compiled by the system of national accounts of each country.

First, the Use matrix, noted **U**, indicates which quantity of each product each industry buys in order to produce: $u_{ij} \geq 0$ is the quantity of input *i* used by industry *j*. For example, for two industries and three products:

[2] See also Aidenoff (1970), United Nations (1999, pp. 86-103), Gilchrist et al. (2000), Guo and Planting (2007), Rueda-Cantuche et al. (2009). Shao and Miller (1990) have focused on the multiregional case; there is a remarkable survey in Guo et al. (2002).

$$(1)\qquad \begin{matrix} \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix} & \begin{matrix} e_1 & q_1 \\ e_2 & q_2 \\ e_3 & q_3 \end{matrix} & \text{Commodities} \\ \begin{matrix} w_1 & w_2 \\ x_1 & x_2 \end{matrix} & & \\ \text{Industries} & & \end{matrix}$$

where x_i is the output of industry i ($x_i > 0$ for any i), w_j is the value added of industry j ($w_j > 0$ for any j), q_i is the total production of product i ($q_i > 0$ for any i), e_i is the amount of product i sold to final demand ($e_i > 0$ for any i).

Second, the Supply (or Make) matrix, noted **V**, indicates which quantity of each product each industry produces, where $v_{ij} \geq 0$ is the quantity of good j produced by industry i. For example:

$$(2)\qquad \begin{matrix} \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} & \begin{matrix} x_1 \\ x_2 \end{matrix} & \text{Industries} \\ \begin{matrix} q_1 & q_2 & q_3 \end{matrix} & & \\ \text{Commodities} & & \end{matrix}$$

Four accounting identities are given, **s** being the sum or identity vector, i.e., $\mathbf{s}' = (1...1)$, prime denoting the transposition:

(3) $\mathbf{x} = \mathbf{V}\,\mathbf{s}$

(4) $\mathbf{x} = \mathbf{U}'\,\mathbf{s} + \mathbf{w}$

(5) $\mathbf{q} = \mathbf{U}\,\mathbf{s} + \mathbf{e}$

(6) $\mathbf{q} = \mathbf{V}'\,\mathbf{s}$

Technical coefficients are defined by:

(7) $\mathbf{B} = \mathbf{U}\,\hat{\mathbf{x}}^{-1}$

By combining (5) and (7), one obtains:

(8) $\mathbf{q} = \mathbf{B}\,\mathbf{x} + \mathbf{e}$

2.1 Product-by-product tables

2.1.1 The product-technology assumption (Eurostat Model A)

In the product-technology assumption, as Miller & Blair (1985, p. 165) say "…the total output [x_i] of any industry [i] is composed of goods [j] in fixed proportions", and the input structure of a product does not depend on the industry that actually produces this product; that is, the matrix **C** is fixed:

(9) $c_{ij} = \frac{v_{ij}}{x_i}$ or $\mathbf{C} = \mathbf{V}'\,\hat{\mathbf{x}}^{-1}$

For Miller & Blair (1985), this assumption is applicable to secondary products but for Rainer (1989), it is unsuitable for some secondary products such as the mineral oil industry. The 1993 System of National Accounts prescribes the product-technology hypothesis (United Nations 1999, p. 98–99), mainly because it fulfills the four desirable axioms cited above (material balance, financial balance, price invariance, and scale invariance).

It ensues from (6) and (9):

(10) $\mathbf{x} = \mathbf{C}^{-1}\,\mathbf{q}$

which indicates how the goods are produced by industries but involves calculating the inverse of **C**. Remember that **C** is invertible because it is the product of **V** and of an invertible matrix from (9)), **V** being invertible from ten Raa's theorem 7.1 (ten Raa and van der Ploeg 1989, p. 89). Combining (10) with (8) gives:

(11) $\mathbf{q} = \mathbf{B}\,\mathbf{C}^{-1}\,\mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^{C}(\mathbf{U},\mathbf{V})\,\mathbf{q} + \mathbf{e}$

by denoting

(12) $\mathbf{A}_P^C(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{C}^{-1}$

the matrix of direct intermediate consumption of products in the product-technology hypothesis when symmetric product-by-product tables are derived; this matrix is a matrix of constants as are **B** and **C**; Almon calls it a "recipe matrix" because it indicates how products are produced regardless of the industries where they are made. Note that

(13) $\mathbf{A}_P^C(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{V}'^{-1}$

by using (7) and (9).

The model's solution is $\mathbf{q} = \left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{e}$. Finally, the product-by-product symmetric input-output matrix is equal to

(14) $\mathbf{S}_P^C(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{C}^{-1}\,\hat{\mathbf{q}} = \mathbf{U}\,\mathbf{V}'^{-1}\,\hat{\mathbf{q}} = \mathbf{U}\,\mathbf{V}'^{-1}\left\langle \mathbf{V}'\,\mathbf{s}\right\rangle$

Note that in any case the number of goods must be equal to the number of industries so that the inverse of matrix **C** can be calculated as in (9). Hence, Supply and Use matrices must be square, which is highly restrictive (Eurostat 2008, p. 295). It is known that the product-technology assumption may generate negative terms in the symmetric input-output table.

2.1.2 The industry-technology assumption (Eurostat Model B)

Under the industry-technology assumption "…we assume that the total output [q_j] of a commodity [*j*] is provided by industries [*i*] in fixed proportions", as said by Miller & Blair (1985, p. 165). The input structure of an industry does not depend on the goods that it produces; that is, the matrix **D** is fixed:

(15) $$d_{ij} = \frac{v_{ij}}{q_j} \text{ or } \mathbf{D} = \mathbf{V}\,\hat{\mathbf{q}}^{-1}$$

This assumption corresponds to a fixed market share of all industries (realistic in the short run and for the by-products). It must be recalled that the industry-technology assumption violates three of the four axioms cited above: financial balance, price invariance and scale invariance and respects only the axiom of material balance. Combining (3) and (15) gives

(16) $$\mathbf{x} = \mathbf{D}\,\mathbf{q}$$

which, when plugged into (8), gives the model:

(17) $$\mathbf{q} = \mathbf{B}\,\mathbf{D}\,\mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^{I}(\mathbf{U},\mathbf{V})\,\mathbf{q} + \mathbf{e}$$

by denoting

(18) $$\mathbf{A}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{D}$$

the product-by-product symmetric coefficient table when a symmetric product-by-product input-output table is derived, that is, the matrix of direct consumption of products in the industry-based hypothesis; this matrix is fixed as are **B** and **D**; it is also a recipe matrix but deduced from a different assumption. Note that

(19) $$\mathbf{A}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\langle \mathbf{V}\,\mathbf{s}\rangle^{-1}\,\mathbf{V}\,\langle \mathbf{V}'\,\mathbf{s}\rangle^{-1} \equiv \mathbf{U}\,\hat{\mathbf{x}}^{-1}\,\mathbf{V}\,\hat{\mathbf{q}}^{-1}$$

by using (7), (15), (3) and (6).

The model's solution is $\mathbf{q} = (\mathbf{I} - \mathbf{B\,D})^{-1}\mathbf{e}$. The product-by-product symmetric input-output table deduced by post-multiplying the technological matrix by $\hat{\mathbf{q}}$ is equal to

(20) $\mathbf{S}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{B\,D}\,\hat{\mathbf{q}} = \mathbf{U}\,\hat{\mathbf{x}}^{-1}\,\mathbf{V} = \mathbf{U}\,\langle \mathbf{V\,s}\rangle^{-1}\,\mathbf{V}$

The industry-technology assumption may lead to absurd recipes of production (chocolate into cheese as Almon says (2000)…).

2.2 *Industry-by-industry tables*

Even if Eurostat separates the *technology* assumptions (Eurostat Models A and B) from the *fixed sales structures* assumptions (Eurostat Models C and D), the derivation of the industry-by-industry symmetric tables from the above equations is very simple if we follow Miller and Blair (1985, 2009), providing results identical to those of Eurostat (2008). However, we will be shorter than above.

2.2.1 The fixed-industry-sales-structure assumption (Eurostat Model C)

For deriving in the Eurostat model C the industry-by-industry symmetric input-output table, it follows from (10) that $\mathbf{q} = \mathbf{C x}$, which carried into (11) gives $\mathbf{q} = \mathbf{B C}^{-1}\,\mathbf{q} + \mathbf{e}$ $\Leftrightarrow \mathbf{Cx} = \mathbf{B\,x} + \mathbf{e}$ $\Leftrightarrow \mathbf{x} = \mathbf{C}^{-1}\,\mathbf{B\,x} + \mathbf{C}^{-1}\,\mathbf{e}$. As the final demand addressed to industries is $\mathbf{e} = \mathbf{C\,f} \Leftrightarrow \mathbf{C}^{-1}\,\mathbf{e} = \mathbf{f}$, this equation gives $\mathbf{x} = \mathbf{C}^{-1}\,\mathbf{B\,x} + \mathbf{f}$, the model's solution being $\mathbf{x} = (\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B})^{-1}\,\mathbf{f}$, where $\mathbf{A}_F^I(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\,\mathbf{B}$ is the industry-by-industry symmetric coefficient table and $\mathbf{S}_F^I(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\,\mathbf{B}\,\hat{\mathbf{x}}$ the industry-by-industry symmetric input-output table. The model may generate negatives in the symmetric input-output table.

2.2.2 The fixed-product-sales-structure assumption (Eurostat Model D)

For deriving Eurostat model D, the industry-by-industry symmetric input-output table, we know from (16) and (17): $\mathbf{D\,q} = \mathbf{D\,B\,D\,q} + \mathbf{D\,e} \Leftrightarrow \mathbf{x} = \mathbf{D\,B\,x} + \mathbf{f}$ as the final demand of products addressed to the industries writes as $\mathbf{f} = \mathbf{D\,e}$, where $\mathbf{A}_F^C(\mathbf{U},\mathbf{V}) = \mathbf{D\,B}$ is the industry-by-industry symmetric coefficient table, the model solving as $\mathbf{x} = (\mathbf{I} - \mathbf{D\,B})^{-1}\mathbf{f}$. $\mathbf{S}_F^C(\mathbf{U},\mathbf{V}) = \mathbf{D\,B}\,\hat{\mathbf{x}}$ is the industry-by-industry symmetric input-output table. The model generates no negatives in the symmetric input-output table.

3 The classical approach of the problem of the negatives

3.1 The problem of negatives

Under the industry-technology assumption, $\mathbf{A}_P^I(\mathbf{U},\mathbf{V})$ is nonnegative as it is the product of **U** and **V**, $\hat{\mathbf{x}}^{-1}$ and $\hat{\mathbf{q}}^{-1}$, which are all nonnegative. Consider the example of Eurostat (2008, p. 318) scenario B); we assume that the unit of money is the “million”, denoted M (Tables 2, 3). For the Model B, the industry technology, we find no negatives $\mathbf{A}_P^I(\mathbf{U},\mathbf{V})$:

$$\mathbf{A}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{D} = \begin{bmatrix} .05392 & .17142 & .07057 \\ .18010 & .21612 & .14142 \\ .12104 & .14594 & .21162 \end{bmatrix} \text{ Products}$$

Products

In the symmetric product-by-product input-output table, no cells are negative:

$$\mathbf{S}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{A}_P^I(\mathbf{U},\mathbf{V})\hat{\mathbf{q}} = \begin{bmatrix} 5.931 & 51.426 & 17.643 \\ 19.811 & 64.835 & 35.354 \\ 13.315 & 43.781 & 52.904 \end{bmatrix} \text{ Products}$$

Products

Supply table (in M)	Product A	Product B	Product C	Total **x**
Industry A	90	70	50	210
Industry B	5	180	45	230
Industry C	15	50	155	220
Total **q’**	110	300	250	660

Table 2. Eurostat scenario B, Supply table: example

Use table (in M)	Industry A	Industry B	Industry C	Final demand **e**	Total **q**
Product A	10	60	5	35	110
Product B	40	60	201	180	300
Product C	20	30	60	140	250
Value added **w’**	140	80	135	0	355
Total **x’**	210	230	220	355	1015

Table 3. Eurostat scenario B, Use table: example

The fixed-product-sales-structure assumption (Eurostat Model D) also generates no negatives in the symmetric input-output table.

On the contrary, under the product-technology assumption, negatives may appear in the symmetric product-by-product input-output table. The matrix $\mathbf{A}_P^C(\mathbf{U},\mathbf{V})$ given by (13) as well as $\mathbf{S}_P^C(\mathbf{U},\mathbf{V})$ given by (14) may eventually include negative terms while this is never the case for $\mathbf{A}_P^I(\mathbf{U},\mathbf{V})$ given by (19) or $\mathbf{S}_P^I(\mathbf{U},\mathbf{V})$ given by (20).

The negative may be very numerous in the symmetric product-by-product input-output table under the product-technology assumption (Eurostat 2009, pp. 325-326). By using a 2x2 example, Ten Raa and van der Ploeg (1989, p. 89) explain that negatives occur in $\mathbf{A}_P^C(\mathbf{U},\mathbf{V})$ when the diagonal terms of **U** are large. They have not explored larger matrices. Are the negatives frequent in symmetric product-by-product input-output tables? We will verify it in a classroom example and in three real matrices.

Example. Eurostat scenario B, Model A (2008, p. 318).

$$\text{(21)} \quad \mathbf{B} = \mathbf{U}\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .04762 & .26087 & .02273 \\ .19048 & .26087 & .09091 \\ .09524 & .13043 & .27273 \end{bmatrix}$$

$$\text{(22)} \quad \mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1} = \begin{bmatrix} .81818 & .23333 & .2 \\ .04546 & .6 & .18 \\ .13636 & .16667 & .62 \end{bmatrix} \text{ Industries}$$

1 1 1

Products

$$\text{(23)} \quad \mathbf{C} = \mathbf{V}'\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .42857 & .02174 & .06818 \\ .33333 & .78261 & .22727 \\ .2381 & .19565 & .70455 \end{bmatrix} \text{ Products}$$

1 1 1

Industries

In this example, with the product-technology assumption (Eurostat Model A), three cells are negative:

$$\mathbf{A}_P^C(\mathbf{U},\mathbf{V}) = \mathbf{B}\mathbf{C}^{-1} = \begin{bmatrix} -.12563 & .35429 & -.06987 \\ .18724 & .32676 & .00551 \\ -.04062 & .07618 & .36645 \end{bmatrix} \text{ Products}$$

Products

$$\mathbf{S}_P^C(\mathbf{U},\mathbf{V}) = \mathbf{A}_P^C(\mathbf{U},\mathbf{V})\hat{\mathbf{q}} = \begin{bmatrix} -13.819 & 106.290 & -17.468 \\ 20.597 & 98.027 & 1.3768 \\ -.4.4676 & 22.855 & 91.613 \end{bmatrix} \text{ Products}$$

Products

Example. Austria 2000, 6x6. This example comes from Eurostat (2000, p. 70 and 122): it is a real table but reduced to 6x6 for Austria, year 2000.

$$\mathbf{x}' = (6467 \quad 115925 \quad 29161 \quad 84164 \quad 70961 \quad 56112)$$

$$\mathbf{q}' = (7663 \quad 117344 \quad 28957 \quad 84686 \quad 67486 \quad 57013)$$

$$\mathbf{U} = \begin{bmatrix} 1705 & 4104 & 30 & 482 & 11 & 95 \\ 1678 & 55020 & 9212 & 14043 & 3701 & 7730 \\ 99 & 542 & 1993 & 950 & 3695 & 1445 \\ 83 & 4420 & 401 & 11129 & 1321 & 1493 \\ 171 & 7400 & 1732 & 10490 & 21810 & 4618 \\ 102 & 1328 & 77 & 813 & 1682 & 3052 \end{bmatrix} \text{ Products}$$

Industries

$$\mathbf{V}' = \begin{bmatrix} 6467 & 0 & 0 & 0 & 0 & 0 \\ 889 & 111350 & 626 & 2749 & 62 & 248 \\ 140 & 1132 & 27356 & 429 & 36 & 67 \\ 150 & 3375 & 399 & 79355 & 447 & 439 \\ 13 & 1428 & 211 & 1953 & 66939 & 416 \\ 4 & 58 & 5 & 200 & 2 & 55843 \end{bmatrix} \text{ Products}$$

Industries

$$\mathbf{S}_P^C(\mathbf{U},\mathbf{V}) = \begin{bmatrix} 2020.3 & 4066.8 & -50.8 & 347.7 & -50.7 & 93.0 \\ 1988.3 & 57050.0 & 9049.4 & 12978.0 & 2602.1 & 7829.0 \\ 117.3 & 467.1 & 2093.7 & 928.2 & 3671.4 & 1472.2 \\ 98.3 & 4230.0 & 314.1 & 11721.0 & 993.9 & 1493.7 \\ 202.6 & 7325.2 & 1679.0 & 10750.0 & 21602 & 4684.1 \\ 120.9 & 1340.7 & 56.5 & 786.0 & 1637.5 & 3113.3 \end{bmatrix} \text{ Products} \tag{24}$$

Products

In this 6x6 example, only two nondiagonal terms ($\{1,3\}$ and $\{1,5\}$) of the symmetric product-by-product input-output table are negative, and then only slightly so: they could be neglected in first approximation. This is why the negatives seem acceptable and insignificant for many economists. These negatives are economic flows for which subtle explanation is complex and unconfirmed and must be rejected.

If we compare with the 15x15 table of the United States for year 2007 (BEA 2008) for the product-technology assumption—remember that the USA prefer using the industry-based assumption—, the number of negatives is very low: only six over $15 \times 15 = 225$ cells, that is 2.67%; if we remove the fifteen diagonal terms never negative, it remains 210 nondiagonal terms, that is, 2.86% of negatives; see Table 4. Moreover, these negatives are very small and could be probably neglected.

Commodities	**Rows**		**Columns**	
	Number of negatives	%	Number of negatives	%
Agriculture, forestry, fishing, and hunting	3	25.00	0	0
Mining	2	16.67	1	7.14
Utilities	0	0	0	0
Construction	0	0	0	0
Manufacturing	0	0	0	0
Wholesale trade	0	0	0	0
Retail trade	1	8.33	0	0
Transportation and warehousing	0	0	0	0
Information	0	0	2	14.29
Finance, insurance, real estate, rental, and leasing	0	0	0	0
Professional and business services	0	0	0	0
Educational services, health care, and social assistance	0	0	0	0
Arts, entertainment, recreation, accommodation, and food services	0	0	0	0
Other services, except government	0	0	0	0
Government	0	0	3	21.43
Total	**6**	**2.67**	**6**	**2.67**

Table 4. Number of negatives per industry and commodity in symmetric product-by-product input-output table for the USA, 2007
(the percentages are given by respect to 15 nondiagonal terms)

This seems reassuring but if we examine a much larger table, those of Austria for year 2005, a 57x57 table (Statistics Austria 2009), the number of negatives is much larger. The commodities are denoted C01 to C93 and the industries are denoted Y01 to Y93; C12 “Uranium and Thorium Ores” and C13 “Metal ores” as well as Y12 “Mining of Uranium and Thorium Ores” and Y13 “Mining of Metal Ores” are excluded as not provided in the Austrian tables (Y95 is also excluded as full of zeros). There are 609 negatives, over $56 \times 56 = 3136$ terms, that is, 19.41%. If we remove the diagonal terms that are never negative, it is 19.77% of the nondiagonal terms, as shown by Table 5 and Figure 1. There are obviously the same number of negatives for rows and columns. However, the dispersion of percentages is larger for rows than for columns; many rows with low percentages are associated to columns with a larger percentage or a low percentage; by it is the inverse for large percentages of rows; finally, the majority of points are over the first diagonal in Figure 1.

We deduce of the above examples that the number of negatives in the symmetric product-by-product input-output tables does not follow a clear rule when the size of the table increases, even if the remark of ten Raa and van der Ploeg (1989, p. 89) probably may hold for larger matrices.

Notice that the fixed-industry-sales-structure assumption (Eurostat Model C) may also generate negatives in the symmetric input-output table even if we focus on the product-technology assumption in this paper.

Commodities	Rows		Columns	
	Number of negatives	%	Number of negatives	%
C01	22	39.29	15	26.79
C02	22	39.29	6	10.71
C05	29	51.79	0	0.00
C10	17	30.36	9	16.07
C11	24	42.86	18	32.14
C14	27	48.21	15	26.79
C15	11	19.64	4	7.14
C16	0	0.00	20	35.71
C17	10	17.86	12	21.43
C18	6	10.71	12	21.43
C19	14	25.00	10	17.86
C20	14	25.00	12	21.43
C21	3	5.36	7	12.50
C22	6	10.71	9	16.07
C23	3	5.36	34	60.71
C24	3	5.36	8	14.29
C25	0		10	17.86
C26	19	33.93	7	12.50
C27	22	39.29	6	10.71
C28	2	3.57	15	26.79
C29	6	10.71	6	10.71
C30	17	30.36	20	35.71
C31	3	5.36	6	10.71
C32	28	50.00	16	28.57
C33	15	26.79	10	17.86
C34	19	33.93	7	12.50
C35	22	39.29	7	12.50
C36	5	8.93	10	17.86
C37	30	53.57	15	26.79
C40	1	1.79	11	19.64
C41	4	7.14	12	21.43
C45	2	3.57	10	17.86
C50	1	1.79	5	8.93
C51	18	32.14	5	8.93
C52	36	64.29	9	16.07
C55	2	3.57	9	16.07
C60	8	14.29	9	16.07
C61	31	55.36	18	32.14
C62	4	7.14	13	23.21
C63	4	7.14	16	28.57
C64	2	3.57	8	14.29
C65	0	0.00	7	12.50
C66	1	1.79	14	25.00
C67	24	42.86	12	21.43
C70	2	3.57	11	19.64
C71	2	3.57	14	25.00
C72	7	12.50	17	30.36
C73	6	10.71	24	42.86
C74	0	0.00	11	19.64
C75	2	3.57	3	5.36
C80	1	1.79	3	5.36
C85	11	19.64	5	8.93
C90	3	5.36	13	23.21
C91	2	3.57	10	17.86
C92	30	53.57	6	10.71
C93	6	10.71	8	14.29
Total	**609**	**19.77**	**609**	**19.77**

Table 5. Number of negatives per rows and columns
in symmetric product-by-product input-output table for Austria, 2005
(the percentages are given by respect to 56 nondiagonal terms)

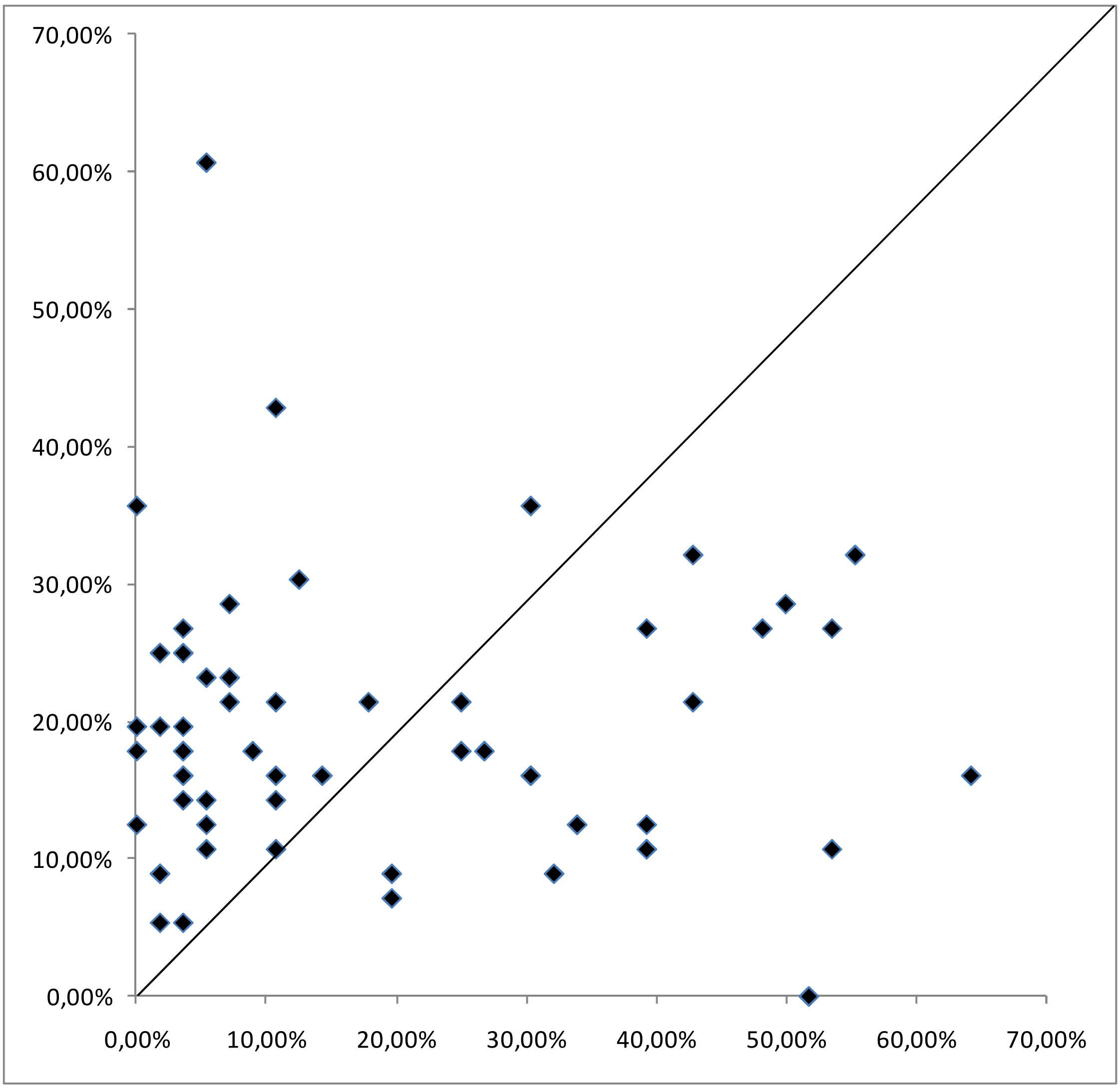

Figure 1. Percentage of negatives in the symmetric product-by-product input-output table for Austria, 2005, under the product-technology assumption (rows in the X-axis, columns in the Y-axis)

3.2 Discussing the patches proposed to remove the negatives

As the negative terms may be very small in absolute value in national accounting matrices, most authors or scholars tend to neglect them or try to remove them by some process however theoretically unsatisfactory. A complete review can be found in United Nations (1999), ten Raa and Rueda-Cantuche (2003), Eurostat (2008) and particularly Hoekstra (2005). Most authors have thought that the negatives are caused by nonhomogeneities (Rainer 1989) or by measurement errors (Steenge 1990). They have tried to eliminate them by various methods that are completely correct in themselves. For example one may quote the method of the SNA (United Nations 1999) or Eurostat approach (2008), Almon's sophisticated iterative method (Almon 2000) or ten Raa and van der Ploeg's statistical adjustment (1989) (even if they reject the product-technology hypothesis) or the non-negativity constraints (Ten Raa 2005, p. 96).[3] Alternatively, a transition matrix between **B** and **C** has been proposed (Steenge 1990), which is a matter for another category of methods. Rainer (1989) lists three methods to alleviate negatives: set the negatives to zero, set the negatives to zero iteratively as done by Almon (1970), or set the negatives to zero by replacing some by a positive value as done by Armstrong (1975).

Most of these methods take us away from the input-output model. The SNA 1993 thinks that over-specification, misclassifications, differences between secondary products and products, and above all, errors in data, are the cause of the negative terms; see also United Nations (1999, p. 96–97) or Eurostat (2009, pp. 325-326). What Hoekstra explains very clearly (2005, pp. 31–37) is absolutely true: the negatives can be eliminated by manipulations on the data that are attributed to the correction of measurement errors, heterogeneous production process, aggregation (by disaggregating), or non-uniform prices. Eurostat proposes to merge industries, change the primary producer, apply industry technology within the product technology framework, make by-products, introduce new products, correct errors in the supply and use table, and make manual corrections to symmetric input-output tables Eurostat (2009, pp. 323–325). Then, Eurostat proposes to "continue until the value and number of negative elements becomes acceptable". This is the case when it can be considered that these negatives are the normal 'noise' in the compilation process" (Eurostat 2008, p. 325). The criterion of acceptability is obviously fuzzy and one may wonder what a "normal noise" is, even if it is true that small negative elements (as well as positive ones) might be neglected.

Almon's method (2000) is different but can be criticized in the same way. This iterative method that consists in progressively removing the negatives by modifying the terms of $\mathbf{S}_P^C$ and hence of **U** and **V** progressively such that the final $\mathbf{S}_P^C$ contains no negative but fulfils the accounting equation $\mathbf{S}_P^C\mathbf{s} = \mathbf{U}\mathbf{s}$. Again, the original data are changed to make the model acceptable. We may cite also Bohlin and Widell's method (2006) which minimizes the variance of coefficients that depend on **U** and **V**.

---

[3] This method is absolutely correct in itself but the terms that should be negative will tend to accumulate on the border of the set delimited by the non-negativity constraints, that is, are replaced by zeros, which might be somewhat unrealistic.

The fact remains that the symmetric product-by-product input-output table is formed with the product-technology assumption by the product of matrix **U—**nonnegative—and of the inverse of matrix **V**—the terms of which are systematically negative (whenever the **V** matrix is not strictly diagonal). However, the SNA, Eurostat and most commentators have failed to understand that the negatives come from the inversion of matrix **V**. Thus, correcting the negatives in the symmetric product-by-product input-output tables by affecting both **U** and **V** as done in authors' method described above is a mistake because the negatives are in $\mathbf{V}'^{-1}$ only and not in **U** (even if the premultiplication of $\mathbf{V}'^{-1}$ by **U** generates more or less negatives in the symmetric input-output table), even if one cannot say that authors' point-of-views are false: the remedies that they indicate really remove the negatives. However, we consider that "a table is a table": an input-output table, as **U** and **V** are, cannot be changed. Modifying the data because some negatives appear in the symmetric input-output table is a very serious epistemological deviation: it is like, in the sixteenth century, trying to make the Sun orbit the Earth because we have a theory that says that the Earth is the center of the universe and we reject the theory of Copernicus and Galileo that it is the Sun that is at the center. All told, authors' recommendations amount, for an econometrician, to changing the data because the model is not significant! Even if they use to remove non-significant variables or to add new ones because they improve the correlation coefficient, never econometricians modify the data. Nonetheless, our point-of-view does not prevent to say that over-specification, problems of definition of secondary products, heterogeneities, errors in data, etc. could be encountered in the Supply and Use tables**—**as the SNA or Eurostat believe—: they should be corrected if they exist and this must be done for deriving all types of symmetric input-output tables, with the product-technology assumption as well as with the industry-technology assumption.

4 The real issue: the impossibility of computing the inverse Supply matrix

4.1 The systematic character of the negatives in the inverse Supply matrix

The existence of negative terms in the Supply-Use model under the product-technology hypothesis has been poorly understood in the past. Most Researchers and practitioners have not seen that (i) the negatives come from $\mathbf{V}'^{-1}$ (or $\mathbf{C}^{-1}$): this is a systematic phenomenon and (ii) what matters is not to detect when the negatives *are* present in

$\mathbf{A}_P^C(\mathbf{U},\mathbf{V})$ but whether they *might be* present even if they are not actually depending on the available data.

De Mesnard (2004) has tried to demonstrate that the negatives in $\mathbf{C}^{-1}$ are systematically unavoidable. Let us verify it by returning to the classroom example and three real matrices examined above. With the example of Eurostat scenario B (Table 2), we find:

$$\text{(25)}\qquad \mathbf{V}'^{-1} = \underbrace{\begin{bmatrix} .01177 & -.00005 & -.00112 \\ -.00383 & .00606 & -.00158 \\ -.00268 & -.00174 & .00727 \end{bmatrix}}_{\text{Products}} \text{Industries}$$

In this example, all nondiagonal terms of matrix $\mathbf{V}'^{-1}$ are negative (and of $\mathbf{C}^{-1}$ or $\mathbf{D}^{-1}$). In order to verify the result, we may first return to the 6x6 example of Austria 2000 for $\mathbf{C}^{-1}$:[4]

$$\text{(26)}\qquad \mathbf{C}^{-1} = \underbrace{\begin{bmatrix} 1 & -.00792 & -.0477 & -.00153 & -.00004 & -.00006 \\ 0 & 1.0424 & -.04242 & -.044 & -.02082 & -.00092 \\ 0 & -.00587 & 1.0663 & -.00509 & -.00309 & -.00007 \\ 0 & -.02611 & -.01553 & 1.062 & -.03036 & -.00377 \\ 0 & -.00044 & -.00128 & -.00595 & 1.0603 & -.00002 \\ 0 & -.00209 & -.00228 & -.00543 & -.00603 & 1.0048 \end{bmatrix}}_{\text{Industries}} \text{Products}$$

In the matrix $\mathbf{C}^{-1}$, virtually all the non-diagonal elements are negative except for cell $\{1,5\}$, which is slightly positive. The negative character of $\mathbf{C}^{-1}$ cannot be neglected, unlike in the corresponding symmetric input-output table (24).

If we examine now a larger table, those of the USA for year 2007 (BEA 2008), the percentage of negatives is of 43.59%, that is, rather large. One understands that many negatives in $\mathbf{C}^{-1}$ do not imply many negatives in the symmetric product-by-product input-output table: it depends also on the structure of the Use matrix.

[4] Obviously, there is the same number of negatives, in the same cells, in $\mathbf{C}^{-1}$ as in $\mathbf{V}'^{-1}$ since both are linked by (9): using (13) instead of (12) changes nothing even if the model is more difficult to understand. That is why we are able to use $\mathbf{C}^{-1}$.

	Industries		Commodities	
	Number of negatives	%	Number of negatives	%
Agriculture, forestry, fishing, and hunting	1	8.33	9	64.29
Mining	5	41.67	7	50.00
Utilities	1	8.33	3	21.43
Construction	0		0	
Manufacturing	8	66.67	3	21.43
Wholesale trade	5	41.67	0	
Retail trade	5	41.67	0	
Transportation and warehousing	6	50.00	4	28.57
Information	5	41.67	6	42.86
Finance, insurance, real estate, rental, and leasing	6	50.00	4	28.57
Professional and business services	7	58.33	7	50.00
Educational services, health care, and social assistance	6	50.00	6	42.86
Arts, entertainment, recreation, accommodation, and food services	6	50.00	3	21.43
Other services, except government	6	50.00	4	28.57
Government	1	8.33	12	85.71
Total	**68**	**43.59**	**68**	**43.59**

Table 6. Number of negatives per industry and commodity in $\mathbf{C}^{-1}$ for the USA, 2007

However, if we compare with the much larger table of Austria for year 2005 (the 57x57 table), the percentage of negatives in $\mathbf{C}^{-1}$ is similar to the USA. We exclude industry Y05 and commodities C01, C05, C10, C16, C62, C75 and C91 that are full of zeros in matrix $\mathbf{C}$ (except the diagonal term); we will see that they are not able to generate negatives in $\mathbf{C}^{-1}$. There are 1152 negatives over $(56-1)\times(56-7)=55\times 49=2695$ terms, that is, 42.75%. If we remove the 49 diagonal terms, which are always nonnegative, 43.54% of nondiagonal terms are negative. In this larger example, the negatives in $\mathbf{C}^{-1}$ are not 100% of the nondiagonal terms but they remain very numerous: about three-seventh, much more than in the symmetric product-by-product input-output table. Many of the negatives are very small; but many are large! Table 7 and Figure 2 indicate the number of negatives per industry (by respect to 55 nondiagonal terms) and per commodity (by respect to 49 nondiagonal terms). For commodities as well as for industries, the percentages of negatives are much larger than for the symmetric input-output table examined in Table 5 and Figure 1. The dispersion of percentages is a little larger for commodities than for industries.

Unfortunately, the demonstration of the present author (de Mesnard 2004) has two flaws. First, the demonstration was partially incomplete because it has not considered the case of a quasi-diagonal matrix **C**, that is, a matrix decomposable by blocks, especially when one block is itself diagonal:

(27) $$\mathbf{C} = \begin{bmatrix} \begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix}$$

Second, the cases where all the terms of a row or a column are equal to zero, except the diagonal term, should be handled. For example:

(28) $$\mathbf{C} = \begin{bmatrix} .7 & .1 & .1 \\ .3 & .9 & .4 \\ 0 & 0 & .5 \end{bmatrix}$$

(29) $$\mathbf{C} = \begin{bmatrix} .6 & .1 & 0 \\ .3 & .7 & 0 \\ .1 & .2 & 1 \end{bmatrix}$$

It is necessary to add these particular cases because the corresponding Supply matrices are more often encountered in real life than full matrices: real life matrices are often large, with a few terms off the diagonal: the more disaggregated the data are, the fewer off-diagonal terms are encountered. Moreover, the demonstration should be more general, in order to handle the cases of $\mathbf{V}'^{-1}$ and $\mathbf{D}^{-1}$: the demonstration was provided for matrix $\mathbf{C}^{-1}$ while it should have been given for **D** and above all for **V**. Hence, the theorem will be given below in more generality for any matrix **Z** such that $z_{ij} \geq 0$ for any *i*, *j* and $z_{ii} > 0$ for any *i*.

Industries			Commodities		
	Number of negatives	**%**		**Number of negatives**	**%**
Y01	27	55.10	C01		
Y02	31	63.27	C02	1	1.82
Y05			C05		
Y10	19	38.78	C10		
Y11	11	22.45	C11	28	50.91
Y14	17	34.69	C14	22	40.00
Y15	20	40.82	C15	12	21.82
Y16	26	53.06	C16		
Y17	20	40.82	C17	16	29.09
Y18	14	28.57	C18	15	27.27
Y19	15	30.61	C19	18	32.73
Y20	17	34.69	C20	17	30.91
Y21	18	36.73	C21	13	23.64
Y22	20	40.82	C22	22	40.00
Y23	14	28.57	C23	18	32.73
Y24	22	44.90	C24	26	47.27
Y25	26	53.06	C25	31	56.36
Y26	22	44.90	C26	21	38.18
Y27	21	42.86	C27	27	49.09
Y28	26	53.06	C28	21	38.18
Y29	25	51.02	C29	34	61.82
Y30	16	32.65	C30	25	45.45
Y31	18	36.73	C31	18	32.73
Y32	18	36.73	C32	18	32.73
Y33	23	46.94	C33	15	27.27
Y34	20	40.82	C34	21	38.18
Y35	17	34.69	C35	23	41.82
Y36	22	44.90	C36	22	40.00
Y37	17	34.69	C37	23	41.82
Y40	22	44.90	C40	27	49.09
Y41	19	38.78	C41	26	47.27
Y45	27	55.10	C45	36	65.45
Y50	19	38.78	C50	24	43.64
Y51	32	65.31	C51	38	69.09
Y52	26	53.06	C52	36	65.45
Y55	19	38.78	C55	25	45.45
Y60	24	48.98	C60	19	34.55
Y61	16	32.65	C61	26	47.27
Y62	22	44.90	C62		
Y63	23	46.94	C63	29	52.73
Y64	23	46.94	C64	22	40.00
Y65	27	55.10	C65	27	49.09
Y66	26	53.06	C66	24	43.64
Y67	23	46.94	C67	24	43.64
Y70	14	28.57	C70	36	65.45
Y71	23	46.94	C71	26	47.27
Y72	17	34.69	C72	28	50.91
Y73	17	34.69	C73	16	29.09
Y74	21	42.86	C74	44	80.00
Y75	24	48.98	C75		
Y80	29	59.18	C80	14	25.45
Y85	19	38.78	C85	42	76.36
Y90	19	38.78	C90	24	43.64
Y91	23	46.94	C91		
Y92	19	38.78	C92	14	25.45
Y93	17	34.69	C93	18	32.73
Total	**1152**	**43.54**	**Total**	**1152**	**43.54**

Table 7. Number of negatives per industry and commodity in $\mathbf{C}^{-1}$ for Austria, 2005

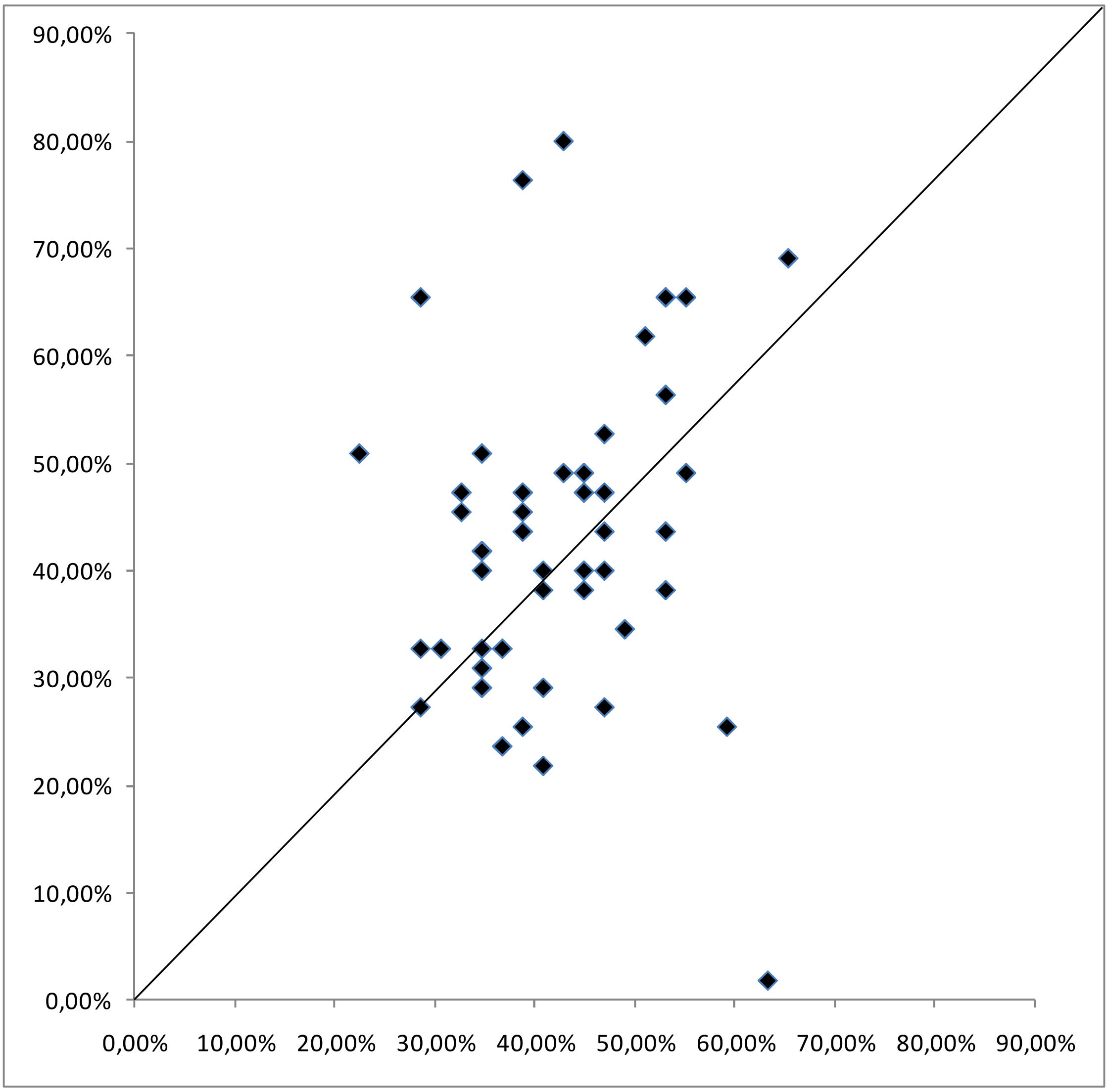

Figure 2. Percentage of negatives in $\mathbf{C}^{-1}$ for Austria, 2005,
under the product-technology assumption,
in each couple industry/commodity
(industries in the X-axis, commodities in the Y-axis)

Theorem 1. In a nonnegative matrix **Z,** which has one or more nondiagonal blocks, consider the k^{th} of these blocks, denoted $\mathbf{Z}^k$. To each row (respectively column) of $\mathbf{Z}^k$ where at least one term is strictly positive (in addition to the diagonal term) corresponds a row (respectively a column) in matrix $\left(\mathbf{Z}^k\right)^{-1} \equiv \mathbf{X}^k$ where at least one term is negative.

Theorem 1 covers the cases described in the examples (27), (28) and (29) of a bloc-diagonal matrix and of a matrix where some rows or columns are full of zeros, except the diagonal term. Following this theorem, except in trivial cases, each industry and product has at least one negative term in $\mathbf{Z}^{-1}$: the negatives are thus systematic in $\mathbf{Z}^{-1}$. This holds if **Z** is

V, **C** or **D**, that is, for $\mathbf{V}'^{-1}$, $\mathbf{C}^{-1}$ or $\mathbf{D}^{-1}$. For example, in the inverse of matrix (27), all the nondiagonal terms of the nondiagonal block are negative in the inverse matrix:

$$(30)\qquad \mathbf{C}^{-1} = \begin{bmatrix} \begin{bmatrix} 3/2 & -1/3 \\ -1/2 & 4/3 \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix}$$

In the inverse of matrices (28) and (29), respectively the last row and the column are full of zeros (except the diagonal term) but all the other nondiagonal terms are negative:

$$(31)\qquad \mathbf{C}^{-1} = \begin{bmatrix} 1.5 & -.1667 & -.1667 \\ -.5 & 1.1667 & -.8333 \\ 0 & 0 & 2 \end{bmatrix}$$

$$(32)\qquad \mathbf{C}^{-1} = \begin{bmatrix} 1.7949 & -.2564 & 0 \\ -.7692 & .15385 & 0 \\ -.0256 & -.2821 & 1 \end{bmatrix}$$

Proof.

The inverse of a block-diagonal matrix **Z** (that is, of a decomposable matrix) is the block-diagonal matrix formed by the inverse of each block:

$$(33)\qquad \begin{bmatrix} [\mathbf{Z}^1] & & & & \mathbf{0} \\ & \cdots & & & \\ & & [\mathbf{Z}^k] & & \\ & & & \cdots & \\ \mathbf{0} & & & & [\mathbf{Z}^n] \end{bmatrix}^{-1} = \begin{bmatrix} [(\mathbf{Z}^1)^{-1}] & & & & \mathbf{0} \\ & \cdots & & & \\ & & [(\mathbf{Z}^k)^{-1}] & & \\ & & & \cdots & \\ \mathbf{0} & & & & [(\mathbf{Z}^n)^{-1}] \end{bmatrix}$$

$$\equiv \begin{bmatrix} [\mathbf{X}^1] & & & & \mathbf{0} \\ & \cdots & & & \\ & & [\mathbf{X}^k] & & \\ & & & \cdots & \\ \mathbf{0} & & & & [\mathbf{X}^n] \end{bmatrix}$$

Hence, we can treat each block $\mathbf{Z}^k$ separately. As $\mathbf{Z}^k$ is nonnegative by assumption and $\mathbf{Z}^k\,\mathbf{X}^k = \mathbf{I}$, the nondiagonal terms of the unit matrix **I** being equal to zero, one can posit the following formula:

(34) $\sum_{p} z_{ip}^{k}\, x_{pj}^{k} = 0$ for any i and j, $i \neq j$

where x_{pj}^{k} is the term $\{p, j\}$ of $\mathbf{X}^k$. We have two cases for the couple $\{i, j\}$. Either $z_{ip}^{k} = 0$ for any $p \neq i$ in row i of $\mathbf{Z}^k$; then (34) turns out to be $z_{ii}^{k}\, x_{ij}^{k} = 0$ for any j, what implies $x_{ij}^{k} = 0$ for any j: all nondiagonal terms of the row i of $\mathbf{X}^k$ are equal to zero. Or, it exists a p such that $z_{ip}^{k} \neq 0$ in row i of $\mathbf{Z}^k$; therefore, in (34), for the couple $\{i, j\}$, at the very least, there is one p such that $x_{pj}^{k} < 0$ in column j of $\mathbf{X}^k$; thus, there is one negative term per column in $\left(\mathbf{Z}^k\right)^{-1}$, that is, per product.

However, one could have written $\mathbf{X}^k\, \mathbf{Z}^k = \mathbf{I}$ in an equivalent way and hence posit the following formula:

(35) $\sum_{p} x_{ip}^{k}\, z_{pj}^{k} = 0$ for any i and j, $i \neq j$

We have again two cases for the couple $\{i, j\}$. Either $z_{pj}^{k} = 0$ for any $p \neq j$ in column j of $\mathbf{Z}^k$; then (35) turns out to be $x_{ij}^{k}\, z_{jj}^{k} = 0$ for any i, what implies $x_{ij}^{k} = 0$ for any i: all nondiagonal terms of the column j of $\mathbf{X}^k$ are equal to zero. Or, it exists a p such that $z_{pj}^{k} \neq 0$ in column j of $\mathbf{Z}^k$; therefore, in (35), for the couple $\{i, j\}$, at the very least, there is one p such that $x_{ip}^{k} < 0$ in row i of $\mathbf{X}^k$; thus, there is one negative term per row in $\left(\mathbf{Z}^k\right)^{-1}$, that is, per industry. •

Even if the inverse Supply matrix is systematically negative, what cannot be denied, the phenomenon seems inconsequential when the corresponding symmetric input-output table is virtually nonnegative. However, we will now use this result, $\mathbf{V}'^{-1}$ (and matrices that derive of it, $\mathbf{C}^{-1}$ or $\mathbf{D}^{-1}$) is systematically largely negative, to show why the model flaws.

4.2 The mistake in the product-technology assumption

Here, we conduct the discussion for the technology assumption (Eurostat models A and B) but it can be transposed, *mutatis mutandis*, to the fixed-sales-structure assumption. The way in which the SNA and Eurostat present both assumptions of product technology and industry technology hides the underlying problem of the negatives in $\mathbf{V}'^{-1}$ and $\mathbf{C}^{-1}$ (or $\mathbf{D}^{-1}$) and its consequences. It is really a pity to see that most people accept the idea of computing the inverse matrix **V** only because the negatives in real-world symmetric product-by-product input-output tables are finally rather rare and small, and all the more so since these matrices are large (hundreds of row and columns): even if the whole matrix symmetric product-by-product input-output table could be nonnegative—or as the negatives would be small—the model would be unacceptable.

Even if it has not yet been recognized, it is particularly easy to explain why $\mathbf{A}^C(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{C}^{-1}$ of formula (12) poses a problem, that is, $\mathbf{A}^C(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{C}^{-1}$ is nonsense even when it is nonnegative, unlike $\mathbf{A}^I(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{D}$ of formula (18). We will use the exact theoretical definition of matrices **B**, **C** and **D**. In order to be as clearer as possible, we will need a classroom example with *no negatives* in the symmetric product-by-product input-output table: even if this makes the tables always acceptable for both assumptions, we will show that the product-technology assumption must be rejected. We deliberately choose the 3x3 example which includes no negatives in $\mathbf{A}^C(\mathbf{U},\mathbf{V})$, that is, Eurostat's scenario A (2008, p. 318), to prove that the problem is not with the negatives in the symmetric input-output tables.

We take scenario A of Eurostat's example (2008, p. 318). We assume that the unit of money is the "million", denoted M.

Supply table (in M)	Product A	Product B	Product C	Total **x**
Industry A	100	20	10	130
Industry B	5	270	20	295
Industry C	5	10	220	235
Total **q'**	110	300	250	660

Table 8. Eurostat scenario A, Supply table

Use table (in M)	Industry A	Industry B	Industry C	Final demand **e**	Total **q**
Product A	6	64	5	35	110
Product B	24	75	21	180	300
Product C	12	34	64	140	250
Value added **w'**	88	122	145	0	355
Total **x'**	130	295	235	355	1015

Table 9. Eurostat scenario A, Use table

$$\mathbf{B}=\mathbf{U}\hat{\mathbf{x}}^{-1}=\begin{bmatrix} .04615 & .21695 & .02128 \\ .18462 & .25424 & .08936 \\ .09231 & .11525 & .27234 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{V}\,\hat{\mathbf{q}}^{-1} = \begin{bmatrix} .90909 & .66667 & .04 \\ .04546 & .9 & .08 \\ .04546 & .03333 & .88 \end{bmatrix} \text{ Industries}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Products

$$\mathbf{C} = \mathbf{V}'\,\hat{\mathbf{x}}^{-1} = \begin{bmatrix} .76923 & .01695 & .02128 \\ .15385 & .91525 & .04255 \\ .07692 & .06780 & .93617 \end{bmatrix} \text{ Products}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Industries

$$\mathbf{C}^{-1} = \begin{bmatrix} .76923 & .01695 & .02128 \\ .15385 & .91525 & .04255 \\ .07692 & .06780 & .93617 \end{bmatrix} \text{ Products}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Industries

$$\mathbf{C}^{-1} = \begin{bmatrix} 1.3073 & -.02208 & -.02871 \\ -.21547 & 1.0999 & -.04510 \\ -.09181 & -.07784 & 1.0738 \end{bmatrix} \text{ Industries}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Products

Obviously, the industry-technology assumption (Model B) never generates negatives:

$$\mathbf{A}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{D} = \begin{bmatrix} .05279 & .19904 & .03793 \\ .18345 & .24410 & .10636 \\ .10153 & .11896 & .25257 \end{bmatrix} \text{ Products}$$

Products

$$\mathbf{S}_P^I(\mathbf{U},\mathbf{V}) = \mathbf{A}_P^I(\mathbf{U},\mathbf{V})\,\hat{\mathbf{q}} = \begin{bmatrix} 5.8065 & 59.712 & 9.4814 \\ 20.180 & 73.23 & 26.59 \\ 11.169 & 35.688 & 63.143 \end{bmatrix} \text{ Products}$$

Products

However, there are also no negatives in the symmetric product-by-product input-output table with the product-technology assumption (Model A):

$$\mathbf{A}_P^C(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{C}^{-1}=\begin{bmatrix} .01164 & .23595 & .01174 \\ .17836 & .26861 & .07919 \\ .07083 & .10353 & .28459 \end{bmatrix}\ \text{Products}$$

Products

The symmetric product-by-product input-output table includes no negatives:

$$\mathbf{S}_P^C(\mathbf{U},\mathbf{V})=\mathbf{A}_P^C(\mathbf{U},\mathbf{V})\hat{\mathbf{q}}=\begin{bmatrix} 1.2799 & 70.786 & 2.9344 \\ 19.62 & 80.583 & 19.798 \\ 7.7917 & 31.06 & 71.148 \end{bmatrix}\ \text{Products}$$

Products

Most scholars would say that the story must stop there! We will now show that it is false: the symmetric product-by-product input-output table should be derived. We begin by the symmetric table that poses no problem, that is, by the industry-technology assumption.

With the industry-technology assumption (model B), suppose that one has to increase the production of product 1 by 10M. We want to know how much of product 3 is required. If we read the matrix $\mathbf{A}^I(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{D}$ in its cell a_{31}^I, we see that it is $.10153\times 10\text{M}=1.015\text{M}$ but how is this deduced? Actually, $a_{31}^I=b_{31}\,d_{11}+b_{32}\,d_{21}+b_{33}\,d_{31}$ following the rules for matrix computing. Hence, there are three parallel ways to obtain product 3: through industry 1, industry 2 and industry 3, all of which are able to provide product 3 simultaneously (this is the essence of the Supply-Use model!). If we go through industry 1, product 1 is produced by industry 1 for 90.909% (as read from cell d_{11}) and industry 1 needs 9.231% of its output in product 3 (as read from cell b_{31}): producing product 1 via industry 1 requires $90.909\%\times 9.231\%=8.3916\%$ of 10M, that is, .839M of product 3. When going through industry 2, product 1 is produced by industry 2 for 4.546% and industry 2 needs 11.525% of its output in product 3: producing product 1 via industry 2 requires $4.546\%\times 11.525\%=.524\%$, that is, .0525M. Finally, with industry 3, product 1 is produced by industry 3 for 4.546% and industry 3 needs 27.234% of its output in product 3: producing product 1 via industry 3 requires $4.546\%\times 27.234\%=1.238\%$, that is, .1238M. In all, it is .839M + .0524M + .1238M = 1.015M. The BEA (Horowitz and Planting 2009, pp. 12-13) similarly explains the product $\mathbf{A}^I(\mathbf{U},\mathbf{V})=\mathbf{B}\,\mathbf{D}$.

Now, if we do the same thing with the product-technology assumption, suppose that one has to increase the production of product 1 by 10M and again we want to know how much of product 3 is required, that is, what the value of a_{31}^{C} is. We see that it is $.07083\times 10\text{M} = .708\text{M}$ if we read the matrix $\mathbf{A}^{C}(\mathbf{U},\mathbf{V}) = \mathbf{B}\mathbf{C}^{-1}$ in its cell a_{31}^{C}. However, one may wonder how and why. Actually, $a_{31}^{C} = b_{31}\sigma_{11} + b_{32}\sigma_{21} + b_{33}\sigma_{31}$ following the rules of matrix computing, σ_{ij} being the cell $\{i, j\}$ of $\mathbf{C}^{-1}$. Hence, again we have three simultaneous possibilities: going through industries 1, 2 and 3. When we go through industry 1, product 1 is produced by industry 1 for 130.73% (as read from cell σ_{11}): one understands that this number is nonsense as it exceeds 100%. We should stop there but we continue the reasoning because we wish to explain the whole matrix product $\mathbf{B}\mathbf{C}^{-1}$. Industry 1 needs 9.231% of its output in product 3 (as read from cell b_{31}); hence, producing product 1 via industry 1 requires $130.73\%\times 9.231\% = 12.067\%$ of 10M, that is, 1.207M of product 3. If we go through industry 2, product 1 is produced by industry 2 for -21.547%: this is impossible as the quantities are negative, but we continue! Industry 2 needs 11.525% of its output in product 3: producing product 1 via industry 2 requires $-21.547\%\times 11.525\% = -2.483\%$ of inputs, that is, $-.248\text{M}$, a negative amount! Finally, going through industry 3, product 1 is produced by industry 3 at -9.181% and industry 3 needs 27.234% of its output for product 3: producing product 1 via industry 3 requires $-9.181\%\times 27.234\% = -2.5\%$, that is -.25M, again negative. However, all told, we have 1.207M - .248M - .25M = .708M, which is nonnegative. Readers will have understood that, even if the result in cell a_{31}^{C} is here nonnegative, it is nonsense. Similar reasoning holds for the fixed-industry-sales-structure assumptions.

A similar demonstration could be conducted with the 6×6 example of Austria or those of the USA for 2007 where the symmetric table is practically nonnegative; but also with any table where the number of negatives in the symmetric table is large and much lower than the number of negatives in the inverse Supply matrix as in the example of Austria for 2005. This demonstration is significant because of Table 7. Without it, the negatives could be exceptional in the inverse Supply matrix; anybody could say that zero negatives in the symmetric table generally mean zero negatives in the inverse Supply matrix: each of the above counter-examples would be a very particular case. However, Theorem 1 demonstrates that the negatives are systematic in the inverse Supply matrix: the difficulty exposed in the counter-example will be encountered systematically and not exceptionally.

5 Conclusion

What practitioners wish to do with the Supply-Use model when they derive a symmetric input-output table—a problem which is central in all applications recalled in introduction—is discovering the universal "recipe", that is, the universal production function, used to produce any product, irrespective of the industry where it is used. This is the Holy Grail of national or regional accounting! There are two possibilities: either the product-technology assumption (Eurostat Model A) and the fixed-industry-sales-structure assumption (Eurostat Model C) or the industry-technology assumption (Eurostat Model B) and the fixed-product-sales-structure assumption (Eurostat Model D).

If the product-technology assumption is adopted (Eurostat Model A), the derivation of a product-by-product symmetric input-output table must be rejected, even when it is completely nonnegative. Indeed, we have demonstrated by the mean of counter-examples that the problem is not in the negatives eventually present in the symmetric input-output table but in the negatives systematically present (at least one per row and one per column in each of its nondiagonal blocks) in the inverse Supply matrix. Consequently, computing the inverse Supply matrix is nonsense, *even if no negatives are present in the symmetric input-output table*. One deduces that SNA's and Eurostat's approach to fixing the problem of negatives is wrong: the difficulty cannot be solved by arranging the data or by creating a mixed hypothesis. The simple possibility of negatives is sufficient to reject this model. Obviously, the same conclusions can be transposed, *mutatis mutandis*, to the industry-by-industry input-output tables: when a symmetric industry-by-industry input-output table is derived and the fixed-industry-sales-structure assumption is chosen, computing the inverse Supply matrix is again a forbidden operation, even when the resulting symmetric table is completely nonnegative.[5]

However, deriving a symmetric input-output table is perhaps an impossible quest because the alternative hypotheses, the industry-technology and the fixed-product-sales-structure assumptions, violate three axioms. The truth is that there are as many production functions as there are industries, that is, as many symmetric product-by-product input-output

[5] Nonetheless, the symmetric industry-by-industry input-output tables are less questionable because the idea of a universal table irrespective of the product considered is more acceptable: this table can be considered as the mean of all the industry-by-industry tables for each product.

tables as industries. Ideally, collecting these tables directly is desirable but not realistic. The supply-use model purports to recreate them from the Use table and the inverse of the Supply table: this is probably an inaccessible dream. What is funny is that the devil lies in the Use table: each time an industry is questioned about its intermediary consumptions of inputs for forming the Use table, it answers globally for all the products it produces. From the outset, the information about the production function used by an industry for a specific product is lost, assuming that the industry is able to know the production function for each of its outputs. The Supply matrix does not pose such a problem as it collects all the information necessary about what the industry produces (beyond the impossibility of computing the inverse of the Supply table…). Notice that the hybrid assumptions must also be rejected for the same reason: they include the inverse Supply matrix.

Globally, the result presented in the paper seems particularly frustrating, especially when one considers the vast amount of labor and intelligence expended by the people who have elaborated the SNA and Eurostat and when one observes the very large number of national, international, regional, multiregional and interregional applications that utilize the Supply-Use model. However, what is questioned is neither the work of the "national accountants" (they obviously do a great job) nor the Supply and Use tables themselves (a table is a table)! It is only what the mathematical model is able to do with these tables.

6 Bibliography

Aidenoff, Abraham, 1970. "Input-Output Data in the United Nations System of National Accounts," in Anne P. Carter and Andrew Brody, 1970, Applications of Input-Output Analysis, North Holland; reprinted in Ira Sohn, Ed., 1986, *Readings in Input-Output Analysis. Theory and Applications*. Oxford University Press, New-York: 130-150.

Almon, Clopper. 1970. "Investment in input-output models and the treatment of secondary products," in Carter Anne P. and Andrew Brody (Eds.), *Applications of input-output analysis*, Amsterdam-London: North-Holland: 103-116.

Almon, Clopper. 2000. "Product-to-product tables via product technology with no negative flows," *Economic Systems Research*, 12, 1: 27-43.

Anderson A.W. and T.W. Manning. 1983. "The Use of Input-Output Analysis in Evaluating Water Resource Development," *Canadian Journal of Agricultural Economics*, 31, 1: 15-26.

Armstrong, A. G. 1975. "Technology assumptions in the construction of U.K. input-output tables," in Allen R.I.G. and W.F. Gossling. 1975. *Estimating and projecting input-output coefficients*, London: Input-Output Publishing: 68-93.

Bohlin, Lars and Lars M. Widell. 2006. "Estimation of Commodity-by-Commodity Input-Output Matrices," *Economic Systems Research*, 18, 2: 205-215.

Braibant, Michel. 2002. “Transformation of Supply and Use Tables to Symmetric Input-Output Tables”, 14th Conference on Input-Output Techniques, Montreal, October 10-15, 2002.

BEA. 2008. *Interactive Access to Input-Output Accounts Data*, Bureau of Economic Analysis, Last updated December, 15, 2008, http://www.bea.gov/industry/iotables/table_list.cfm?anon=97514

Comer, Jonathan C. and Randall W. Jackson. 2003. “A note on adjusting national input-output data for regional table construction,” *Journal of Regional Science*, 37, 1: 145-153.

de Mesnard, Louis. 2004. “Understanding the shortcomings of commodity-based technology in input-output models: an economic-circuit approach,” *Journal of Regional Science,* 44, 1: 125-141.

de Mesnard, Louis. 2009. “Is the Ghosh model interesting?”, *Journal of Regional Science*, 2009, 49, 2: 361-372.

Dietzenbacher, Erik. 1997. “In Vindication of the Ghosh Model: A Reinterpretation as a Price Model,” *Journal of Regional Science*, 37,4: 629–651.

Dietzenbacher, Erik and Jan A. van der Linden. 1997. “Sectoral and Spatial Linkages in the EC Production Structure,” *Journal of Regional Science*, 37, 2: 235-257.

Eding, Gerard, Jan Oosterhaven, Bas de Vet and Henk Nijmeijer. 1999. “Constructing regional supply and use tables: Dutch experiences,” in Hewings Geoffrey J.D, Michael Sonis, Moss Madden and Yoshio Kimura, Eds. *Understanding and interpreting economic structure*, Berlin, New York, London, Tokyo: Springer Verlag.

Eurostat. 2008. Eurostat Manual of Supply, Use and Input-Output Tables.

Forestry Department. 1998. *Economic and Environmental Accounting for Forestry: Status and Current Efforts*, Planning and Statistics Branch, Policy and Planning Division.

Garcia-Negro, M.C., X.R. Doldán-Garcia, M.L. Chas-Amil, Y. Zotes-Tarrío, A. Carballo-Penela, E., Nogueira-Moure, S. Villasante-Larramendi. 2004. “Application of Input-Output methods for the study of the Galician fishing in 1999,” *Input -Output and Global Equilibrium: Data, Modeling, and Policy Analysis*, Brussels, September 2-4, 2004.

Gilchrist, Donald A., Larry V. StLouis and Debra J. Harper. 2000. “An examination of the Leontief and Ghoshian price and quantity models: a rectangular approach,” roneo, January 2000.

Guo, Jiemin, Ann M. Lawson, and Mark A. Planting. 2002. “From Make-Use to Symmetric I-O Tables: An Assessment of Alternative Technology Assumption,” *Bureau of Economic Analysis*, WP2002-03.

Guo, Jiemin and Mark A. Planting. 2007. “Integrating U.S. Input-Output Tables with SNA: Valuations and Extensions,” *16th International Input-Output Conference in Istanbul*, Turkey, July 2-7, 2007.

Harris, Richard I. D. and Liu Aying. 1998. “Input-Output Modelling of the Urban and Regional Economy: The Importance of External Trade,” *Regional Studies*, Vol. 32, No. 9, 851-862.

Hoekstra, Rutger. 2005. Economic growth, material flows and the environment: new applications of structural decomposition analysis and physical input-output tables, Glos, UK: Edward Edgar publishing.

Horowitz Karen J. and Mark A. Planting. 2009. *Concepts and Methods of the U.S. Input-Output Accounts*, Bureau of Economic Analysis, US Department of Commerce, September 2006, updated April 2009.

Israilevich, Philip R., Geoffrey J. D. Hewings, Michael Sonis and Graham R. Schindler. 1997. “Forecasting structural change with a regional econometric input-output model,” *Journal of Regional Science*, 37, 4: 565-590.

Jackson, Randall W. 1998. “Regionalizing National Commodity-by-Industry Accounts,” *Economic Systems Research*, Vol. 10, No. 3, 223-38.

Kop Jansen, Pieter and Thijs ten Raa. 1990. “The choice of model in the construction of input-output matrices,” *International Economic Review*, 31: 213-227.

Lawson, Ann M. 1997. “Benchmark input-output accounts for the U.S. economy, 1992,” *Survey of Current Business*, Bureau of Economic Analysis, U.S. Department of Commerce, Washington, November 1997: 36-82.

Madsen, Bjarne and Chris Jensen-Butler. 1998. “Commodity Balance and Interregional Trade: Make and Use Approaches to Interregional Modelling,” Paper presented to the *12th International Conference on Input-Output Techniques*, New York 18-22 May 1998.

Manrique-de-Lara-Peñate, Casiano, Maria del Carmen Langa-Seva and Dolores Santos-Peñate. 2008. “Application of IO models to Use and Destination Tourism Accounts”, *The 2008 International Input-Output Meeting, Input - Output & Environment*, Seville, July 9 - 11, 2008.

Mattey, J.P. 1993. “Evidence on input-output technology assumptions from the Longitudinal Research Database,” discussion paper 93-8, *Center for Economic Studies*, US Bureau of Economic Census, Washington DC.

Mattey, J.P. and Th. ten Raa. 1997. “Primary versus secondary production techniques in US manufacturing,” *Review of Income and Wealth*, 43: 449-464.

Miller, Ronald E. and Peter D. Blair. 1985. *Input-output analysis: foundations and extensions*, Prentice-Hall, Englewood Cliffs (New-Jersey).

Miller, Ronald E. and Peter D. Blair. 2009. *Input-output analysis: Foundations and Extensions. Second edition*, Cambridge (UK): Cambridge University Press.

Nijmeijer, Henk, Bas de Vet and Gerard Eding. 1999. *Screening the Dutch Regional Accounts: Experiences from the Regional Input-Output Project*. Draft Report, Statistics Netherlands.

Oosterhaven, Jan. 1984. “A Family of Square and Rectangular Interregional Input-output Tables and Models,” *Regional Science and Urban Economics*, 14: 565-582.

Perez, Julián, Milagros Dones and Carlos Llano. 2009. “An interregional impact analysis of the EU structural funds in Spain (1995–1999),” *Papers in Regional Science*.

Rainer, Norbert. 1989. “Descriptive versus analytical Make-Use systems: some Austrian experiences,” in Miller, Ronald E., Karen R. Polenske and Adam R. Rose (Eds). 1989. *Frontiers of input-output analysis*, Oxford University Press, New-York: 50-64.

Rainer, Norbert and Joseph Richter. 1992. “Some aspects of the analytical use of descriptive make and absorption tables,” *Economic Systems Research*, 4: 159-172.

Ravetz Joe. 2004. *Measuring regional sustainable development. A review of modeling tools in the UK*. Eco-region NW, A BiffaWard project on sustainable resource use.

Rueda-Cantuche Jose-M. Joerg Beutel, Frederik Neuwahl, Ignazio Mongelli and Andreas Loeschel. 2009. “A symmetric input-output table for EU27: latest progress” *Economic Systems Research*, 21, 1: 59-79.

Sayapova Alsu and Svetlana Slobodyanik. 2008. “Compilation of Regional Input-Output Tables in Russia,” *The 2008 International Input-Output Meeting, Input - Output & Environment*, Seville, July 9 - 11, 2008

Schaffer William. 1999. *Regional Impact Models*, Regional Research Institute, West Virginia University.

Shao, Gang and Ronald E. Miller. 1990. “Demand-side and supply-side commodity-industry multiregional input-output models and spatial linkages in the US regional economy,” *Economic Systems Research*, 2: 385-405.

Siddiqi, Y. M. and M. Salem. 1995. *Regionalization of commodity-by-industry input-output accounts: the Canadian Case*. Statistics Canada.

Statistics Austria. 2009. *National Accounts*. Compiled on 12 February 2009. - NACE/CPA 11 incl. 13 (http://www.statistik.at/web_en/statistics/national_accounts/input_output_statistics/index.html).

Steenge, Albert E. 1990. "The commodity technology revisited: theoretical basis and an application to error location in the make-use framework," *Economic Modelling*, 7: 376-387.

Stone, Richard. 1961. *Input-output and national accounts*. Paris: OECD.

Susiluoto, Ilkka. 1997. An Input-Output Study of Regional Economic Growth: The Helsinki Region and the Rest of Finland. Studies Published by City of Helsinki Urban Facts, 1997:2.

ten Raa, Thijs. 1988. "An alternative treatment of secondary products in input-output analysis: frustration," *The Review of Economics and Statistics*, 70, 3: 535-538.

ten Raa, Thijs. 2005. *The Economics of Input-Output Analysis*, Cambridge: Cambridge University Press.

ten Raa, Thijs and Rick van der Ploeg. 1989. "A statistical approach to the problem of negatives in input-output analysis," *Economic Modelling*, 6: 2-19.

ten Raa, Thijs and Jose-M. Rueda-Cantuche. 2003. "The construction of Input-Output Coefficients Matrices in an Axiomatic Context: Some Further Considerations," *Economic Systems Research*, 15: 439-455.

ten Raa, Thijs and Jose-M. Rueda-Cantuche. 2007. "A generalized expression for the commodity and the industry technology models in input-output analysis," *Economic Systems Research*, 19, 1: 99-104.

United Nations, Department Of Economic and Social Affairs. 1968. *A System of National Accounts (SNA)*, series F, No. 2, Rev. 3, United Nations Studies in method. United Nations, New York.

United Nations, Department Of Economic and Social Affairs. 1999. *Handbook of National Accounting: Input/Output Tables - Compilation and Analysis*. United Nations Studies in method, United Nations, New York.

United Nations, Statistics Division – National Accounts. 2001. *System of National Accounts 1993*. (http://unstats.un.org/unsd/sna1993/tocLev8.asp?L1=15&L2=4).

van de Steeg, Annemieke M and Albert E. Steenge. 2008. "Integrating the TSA and Input-Output Methodology in Tourism Impact Studies," *The 2008 International Input-Output Meeting, Input - Output & Environment*, Seville, July 9 - 11, 2008.

Wood, Richard. 2008. "Development of a Time Series of Australian Input-Output Tables and Greenhouse Emissions**",** *The 2008 International Input-Output Meeting, Input - Output & Environment*, Seville, July 9 - 11, 2008.