

On the fallacy of forward linkages:

A note in the light of recent results

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Abstract. Following on from de Mesnard's (2009) radical criticism of the Ghosh supply-driven model, this paper draws the dramatic consequences for the widespread use of forward linkages in input-output analysis applied to regional science: the practice must be abandoned. The arguments are based on three points: (i) it is impossible simultaneously to choose the Leontief model for the backward effects and the Ghosh model for the forward effects; (ii) it is impossible simultaneously to consider a production function of complementary inputs (Leontief) and a production function of perfectly substitutable inputs (Ghosh); and most importantly (iii) price effects and output effects remain inextricably mixed in the Ghosh model, so precluding its use for analyzing forward effects on quantities or prices.

Running head. Fallacy of forward linkages.

1. Introduction

Backward and forward linkages has been very popular in Regional Science for evaluating the impact of one industry or region on the remainder of the economy ever since Miller's pioneering works (Miller 1966, 1969, 1986; Polenske and Hewings 2004, pp. 275–276). They have now become one of the main tools in input-output analysis, particularly in Regional Science. While backward effects are based on multipliers deduced from the Leontief model, forward effects are based on the Ghosh model. The Ghosh model assumes that the row coefficients (the allocation coefficients) of an interindustry and/or interregional matrix of economic flows are stable. However, in a very recent paper, de Mesnard (2009) probably closes a long debate¹ about the question of the fallacy of the Ghosh model. He demonstrates that this model is uninteresting and, at best, contributes nothing more than the Leontief model. As de Mesnard's paper can be considered as putting to rest the Ghosh model as a theoretical model, the logical consequence that must be deduced from the vacuity of the Ghosh model is that the measures of the forward effects, i.e., the forward multipliers, must be abandoned. A French proverb says: *quand le vin est tiré, il faut le boire* (when the wine is out of the barrel, it must be drunk); we must follow the argument, wherever it leads.

After recalling what forward linkages are, the discussion will turn around three points; although familiar enough, the first two must be recalled here because they reinforce the third one: (i) it is impossible, at on and the same time, to choose the Leontief model for the backward effects and the Ghosh model for the forward effects; (ii) it is impossible, at one and the same time, to consider a production function of complementary inputs (Leontief) and a production function of perfectly substitutable inputs (Ghosh); and (iii) the vacuity of the Ghosh model has now been clearly demonstrated.

2. Reminder about forward multipliers

The typical situation is where an industry must increase or reduce its activity. What are the effects, backward and forward? We expound what they are as is done generally, with data given in terms of value. Both approaches, backward and forward, are perfectly exposed in Miller and Blair (1985, pp. 323-325) but also in Dietzenbacher, van der Linden and Steenge (1993, pp. 191–193) or in Miller and Lahr (2001, pp. 409–411).²

Backward, the Leontief model gives the answer with the idea of standard multipliers which are equal to the column sum of the Leontief matrix, that is,

$$(1) \quad \mathbf{x} = \mathbf{A} \mathbf{x} + \mathbf{f}$$

where

$$(2) \quad \mathbf{A} = \mathbf{Z} \hat{\mathbf{x}}^{-1}$$

is the matrix of technical coefficients when the following accounting matrix is considered: $\mathbf{Z} \mathbf{s} + \mathbf{f} = \mathbf{x}$, where \mathbf{Z} denotes the matrix of flows, \mathbf{x} the output vector, and \mathbf{f} the final demand vector (both defined when data are given in value); the hat over a vector denotes the matrix formed by diagonalizing this vector. Model (1) solving as $\mathbf{x} = \mathbf{L} \mathbf{f}$, where \mathbf{L} is the Leontief inverse matrix, $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$, the vector of backward multipliers is given by summing \mathbf{L} by rows: $\mathbf{m}^B = \mathbf{i}' \mathbf{L}$ (Rasmussen 1956). It is also possible to derive the vector of direct backward effects from $\mathbf{i}' \mathbf{A}$ which sums \mathbf{A} by columns (Chenery and Watanabe, 1958).

Forward, the Ghosh model (Ghosh, 1958) gives the answer:

$$(3) \quad \mathbf{x}' \mathbf{B} + \mathbf{v}' = \mathbf{x}'$$

where

$$(4) \quad \mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{Z}$$

is the matrix of allocation coefficients and \mathbf{v} is the value added vector (the prime denoting the transposition operation). Model (3) solving as $\mathbf{x}' = \mathbf{v}' \mathbf{G}$, where $\mathbf{G} \equiv (\mathbf{I} - \mathbf{B})^{-1}$, the vector of total forward linkages is given by

$$(5) \quad \mathbf{m}^F = \mathbf{G} \mathbf{i}.$$

The vector of direct forward effects can be derived by computing $\mathbf{B} \mathbf{i}$ which sums \mathbf{B} by rows.³

The theory has hesitated in the past about which definition must be chosen for the forward linkages. First, it can be deduced from (2) and (4) that

$$(6) \quad \mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{A} \hat{\mathbf{x}}$$

This is why Miller and Lahr (2001, pp. 410–411) emphasize that this formula allows us to deduce the forward linkages by avoiding the Ghosh model: $\mathbf{G} \mathbf{i} = \hat{\mathbf{x}}^{-1} \mathbf{L} \hat{\mathbf{x}} \mathbf{i} = \hat{\mathbf{x}}^{-1} \mathbf{L} \mathbf{x}$. However, they also say that the cross ratio x_j/x_i “do[es] not seem to have an appealing economic

interpretation". Second, the total linkage—or economic importance⁴—of a given sector or region is deduced by calculating the difference between the total output $\mathbf{i}'\mathbf{x} = \mathbf{i}'\mathbf{L}\mathbf{f}$ and the same quantity, $\mathbf{i}'\tilde{\mathbf{x}} = \mathbf{i}'\tilde{\mathbf{L}}\mathbf{f}$, when the sector or region is removed from the economy (the superscript $\tilde{\mathbf{x}}$ denotes the output vector when the sector has been removed from the economy, etc.).⁵ The idea of key sectors created by Hewings (1982) follows from this. Such “hypothetical extraction” of a sector can be best conducted by partitioning the matrix, as in Cella (1984),⁶ which allows us to deduce the total linkage of a group of sectors or regions to the rest of the economy. It is important to note that for some researchers, including Cella (1984), the total linkages can be decomposed into backward and forward linkages, which clearly means that forward linkages can be deduced by summing the Leontief inverse matrix by rows. However, Guccione (1986), followed by Dietzenbacher, van der Linden and Steenge (1993, p. 192), criticized this idea: the terms viewed by Cella as describing the forward linkage of a sector i over the rest of the economy (terms a_{ij} of row i , with $j \neq i$) actually describe the backward linkage of the rest of the economy on sector i (see Miller and Lahr, 2001). This observation is particularly true because the production function remains that of Leontief: this point will be discussed later.

3. Discussion

Firstly, it is known that both analyses of backward and forward effects cannot be conducted simultaneously; such an idea was exposed by Cella (1984). Even if for Miller and Lahr (2001, p. 422–423) the problem is “more grey than black or white”, we defend the idea that it is not possible to use both models at the same time. The analysis of backward effects is based on the Leontief model, which requires the technical coefficients of matrix \mathbf{A} to be stable; the analysis of forward effects is based on the Ghosh model, which requires the allocation coefficients to be stable. However, it has been well known since Chen and Rose (1986, 1991) and Rose and Allison (1989) that both types of coefficients cannot be stable simultaneously in the general case and it is only exceptionally that they are simultaneously stable. The demonstration is simple. Consider two intervals of time, t_1 and t_2 where $\mathbf{x}_1 \neq \mathbf{x}_2$ and return to formula (6) which implies that $\mathbf{B}_2 = \hat{\mathbf{x}}_2^{-1} \mathbf{A}_2 \hat{\mathbf{x}}_2$. When the technical coefficients are assumed to be stable, that is $\mathbf{A}_1 = \mathbf{A}_2$, the allocation coefficients cannot be stable: $\mathbf{B}_2 = \hat{\mathbf{x}}_2^{-1} \mathbf{A}_1 \hat{\mathbf{x}}_2 \neq \mathbf{B}_1$; and conversely, except in a very special case, *absolute joint stability* (Chen and Rose, 1986 and 1991), that is, the homothetic variation of the gross output of all

sectors: $\mathbf{x}_2 = k \mathbf{x}_1$, which implies that $\mathbf{B}_2 = \hat{\mathbf{x}}_2^{-1} \mathbf{A}_1 \hat{\mathbf{x}}_2 = \mathbf{B}_1$ when $\mathbf{A}_1 = \mathbf{A}_2$. Consequently, if the backward effects are examined, the forward effects are nonsense and reciprocally: both studies cannot be conducted simultaneously, particularly for detecting which sector generates the largest backward *and* forward effects, or for knowing whether the ordering of industries remains stable.⁷ Even if Miller and Lahr (2001, p. 423-425) rightly demonstrate that “four of the seven possible extraction structures will generate identical results—for the Leontief quantity model and for the Ghosh price model—it may be unnecessary to quibble about the relative plausibilities of the various extractions”, the fact remains that the two models are based on two completely contradictory hypotheses (except in the trivial case of absolute joint stability).

Moreover, there is, as a corollary, a second justification of the impossibility of examining backward and forward effects simultaneously: the Leontief model corresponds to a particular production function (a production function with complementary inputs) (El-Hodiri and Nourzad 1988) while it has been advocated (Gruver, 1989, p. 449; Dietzenbacher 1997) that the Ghosh model corresponds to a production function with perfectly substitutable inputs.⁸ Hence, passing from backward effects to forward effects is equivalent to dramatically changing the production function. This can be explained with a simple example. Consider two industries: j builds airplanes while i produces jet engines; a 10% reduction in the deliveries of industry i compels industry j to reduce its use of i 's intermediate product; hence, j must reduce its own output because the Leontief production function is of complementary inputs; airplanes production will be cut by 10% because of the shortage of jet engines: $\frac{\Delta x_j}{x_j} = \frac{\Delta x_i}{x_i}$; and the

value added of the airplane maker j will also be reduced. There is no Ghoshian effect here, only the hard law of the production function with complementary inputs. If the production function with complementary inputs is abandoned,⁹ to the benefit of a substitutable inputs function (Cobb-Douglas, CES, etc.), this effect disappears: the production of the forward industry is only marginally affected by the fall in deliveries, which is completely unrealistic in most industries, and particularly for airplanes, at least in the short and medium term because it is impossible to sell airplanes without all their engines.¹⁰ Nonetheless, it is true that in the long run, if the reduction in the deliveries of jet engines continues, the airplane maker will design lighter or slower airplanes requiring fewer engines. However, in the long run, it is known that the central hypothesis of input-output analysis—the stability of technical coefficients—cannot be considered valid. This could suggest that the Ghosh model is perhaps

a long-run model; unfortunately, the Ghosh model is not an acceptable model at all as will be shown now.

The third and final argument against forward linkages—and certainly the most important one—is that the Ghosh model is largely problematical and uninteresting, as demonstrated by de Mesnard (2009), a paper that followed on from many others but that introduced a definitive new theoretical result:

(i) When data are available in value—remember that it is the operational case¹¹—the true equation of the Ghosh model (in the dual, the primal providing a mathematical identity) is

$$(7) \quad \tilde{\mathbf{x}}' \mathbf{B} + \tilde{\mathbf{v}}' = \tilde{\mathbf{x}}'$$

where the tilde over vectors denotes the “shifted values”, $\tilde{\mathbf{x}} = \hat{\boldsymbol{\pi}} \mathbf{x}$ (and $\tilde{\mathbf{v}} = \hat{\boldsymbol{\pi}}^V \mathbf{v}$) the product of a quantity vector \mathbf{x} (or \mathbf{v}) at the base year by an index price, $\boldsymbol{\pi} = \hat{\mathbf{p}}_1^{-1} \hat{\mathbf{p}}_2$ and \mathbf{p}_1 and \mathbf{p}_2 the prices for the base year and the current prices respectively.¹² The model solves as $\tilde{\mathbf{x}}' = \tilde{\mathbf{v}}' (\mathbf{I} - \mathbf{B})^{-1} \equiv \tilde{\mathbf{v}}' \mathbf{G}$. Hence, when data are available in value, the Ghosh model finds $\tilde{\mathbf{x}} = \hat{\boldsymbol{\pi}} \mathbf{x}$ and is incapable of separating outputs in value \mathbf{x} and index prices $\boldsymbol{\pi}$.¹³ Dietzenbacher (1997) takes the outputs in value as fixed, providing an escape from this problem.¹⁴ However, the index prices could also be considered as fixed, without any rule to decide between which set of data must be fixed: as the dual of the Ghosh model provides “shifted values”, $\tilde{x}_i = x_i \pi_i$, both outputs in value x_i or index prices π_i can be chosen as fixed at an arbitrary level.¹⁵ As outputs in value (respectively index prices) are completely arbitrary in the Ghosh model, the index prices that are deduced (respectively the outputs in value) are also completely arbitrary: even if the forward multipliers remain indicated, the row sum of \mathbf{G} , that is, by $\mathbf{m}^F = \mathbf{G} \mathbf{i}$ or (5), deducing any clear forward effect on outputs in value or index prices is impossible, both remain indissolubly linked in the as shifted-valued outputs when the forward effects are measured.¹⁶

(ii) When data are available in physical terms, the outcome is similar. The true equation of the Ghosh model (in the dual, the primal providing again a mathematical identity) is

$$(8) \quad \mathbf{x}' \overline{\mathbf{B}} + \mathbf{v}' = \mathbf{x}'$$

which solves as $\mathbf{x}' = \mathbf{v}' (\mathbf{I} - \overline{\mathbf{B}})^{-1} \equiv \mathbf{v}' \overline{\mathbf{G}}$, the bar over a matrix or a vector denoting physical quantities. This is a big issue because when the Ghosh model finds $\mathbf{x} = \hat{\boldsymbol{\pi}} \mathbf{p}$, it is incapable of

separating physical outputs $\bar{\mathbf{x}}$ from prices \mathbf{p} . Moreover, as $\mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{Z} = \langle \hat{\mathbf{p}} \bar{\mathbf{x}} \rangle^{-1} \hat{\mathbf{p}} \bar{\mathbf{Z}} = \hat{\mathbf{x}}^{-1} \bar{\mathbf{Z}} = \bar{\mathbf{B}}$ implying $\bar{\mathbf{G}} = \mathbf{G}$ (de Mesnard 2009, Property 2) where \mathbf{p} is the vector of current prices, equation (8) turns out to be $\mathbf{x}' \mathbf{B} + \mathbf{v}' = \mathbf{x}'$, which is identical to what is generally considered as the equation of the Ghosh model when data are available in value, that is, the so-called Ghosh-model equation indicated by (3): many authors are mistaken... Finally, even if the forward multipliers remain indicated by the row sum of $\bar{\mathbf{G}} = \mathbf{G}$, that is, by $\bar{\mathbf{m}}^F = \mathbf{G} \mathbf{i}$,¹⁷ any measure of forward impact on physical outputs (respectively forward impact on prices) is impossible, unless it is assumed arbitrarily that prices (respectively physical outputs) are fixed at an arbitrary level: physical outputs and prices are indissolubly linked in outputs in value when forward effects are measured.

4. Conclusion

In a nutshell, the Ghosh model by itself is unable to evaluate forward effects on the output of one industry over the output of other industries which are its customers. As the backward linkages refer to the Leontief model and the forward linkages refer to the Ghosh model, (i) one may not assume that both technical coefficients and allocation coefficients are fixed simultaneously; (ii) as the backward linkages correspond to a production function with complementary inputs while the forward linkages correspond to a production function with perfectly substitutable inputs, studying both effects means that two incompatible production functions are used at the same time; and (iii) price effects and output effects are not separately determined: they always remain inextricably mixed with the Ghosh model, precluding its use for analyzing forward effects on quantities or on prices; fixing prices (or index prices) or physical outputs (or outputs in value) is completely arbitrary.

It would remain possible to explore the consequences of forward linkages on econometric tests: that is another story. However, de Mesnard's demonstration does not imply that the row coefficients of an exchange matrix cannot be empirically rather stable over time. Some studies have found that the Ghosh allocation (row) coefficients are as stable as the Leontief technical (column) coefficients, by measuring the structural change in the production matrix, without predetermining whether the model is demand-driven or supply-driven. This was done by de Mesnard (1988, 1997, 2002) by using a biproportional projector (RAS) or later, and slightly differently, by van der Linden and Dietzenbacher (1995, 2000), Dietzenbacher and Hoekstra (2003) and Walmsley and McDougall (2007). Previous approaches, such as the founding

work of Leontief (1936) or that of Carter (1970), supposed that the technical coefficients alone must be stable.

Many scholars and practitioners in Regional Science will think that the price to pay for the problems posed by the Ghosh model is enormous: abandoning the study of forward linkages in interindustry and/or regional analysis. However, it is the condition for having a “clean” use of input-output analysis from an epistemological point of view and hence for having significant results. One cannot give up the rigor of scientific analysis only because analyzing forward linkages was interesting for regional scientists in operational terms. Until scholars invent a new way of measuring forward effects, they will invariably be equal to 100%—as shown in the above example of the airplanes and their engines—when the production function is of complementary inputs, that is, in the context of the traditional input output analysis.¹⁸

5. References

- CAI, Junning and PingSun LEUNG. 2005. “An alternative interpretation of the ‘pure’ linkage measures,” *The Annals of Regional Science*, 39: 49-54.
- CARTER, Anne P. 1970. *Structural Change in the American Economy*.” Cambridge Mass: Harvard University Press.
- CELLA, Guido. 1984. “The input-output measurement of interindustry linkages,” *Oxford Bulletin of Economics and Statistics*, 46, 1: 73-84.
- CHEN, Chia-Yon and Adam ROSE. 1986. “The joint stability of input-output production and allocation coefficients,” *Modeling and Simulation*, 17: 251-5.
- CHEN, Chia-Yon and Adam ROSE. 1991. “The absolute and relative joint stability of input-output production and allocation coefficients,” in A.W.A. Peterson (Ed.) *Advances in Input-Output Analysis*. Oxford University Press, New-York, pp. 25-36.
- CHENERY, H. and WATANABE, T. 1958. “An international comparison of the structure of production,” *Econometrica*, 26: 487-521.
- CRONIN, Francis J. 1984. “Analytical assumptions and causal ordering in interindustry modeling,” *Southern Economic Journal*, 51, 2: 521–529.
- de MESNARD L. 1988. *Analyse de la dynamique de la structure interindustrielle française par filtrage biproportionnel. Méthode et application à la période 1970-1985*, PhD doctoral thesis in Economic Science, Paris: University Paris I Panthéon-Sorbonne.
- de MESNARD, Louis. 1997. “A biproportional filter to compare technical and allocation coefficient variations,” *Journal of Regional Science*, 37, 4: 541-564.
- de MESNARD, Louis. 2002. “Forecast output coincidence and biproportion: Two criteria to determine the orientation of an economy. Comparison for France (1980-1997),” *Applied Economics*, 34, 16: 2085-2091.
- de MESNARD, Louis. 2002. “Normalizing biproportional methods”, *The Annals of Regional Science*, 2002, 36, 1: 139-144.
- de MESNARD, Louis. 2004. “On the idea of ex ante and ex post normalization of biproportional methods”, *The Annals of Regional Science*, 2004, 38, 4: 741-9.

- de MESNARD, Louis. 2009. "Is the Ghosh model interesting?" *Journal of Regional Science*, 49, 2: 361-372.
- DIETZENBACHER, Erik. 1989. "On the relationship between the supply-driven and the demand-driven Input-Output Model," *Environment and Planning A*, 21, 11: 1533-1539.
- DIETZENBACHER, Erik. 1997. "In vindication of the Ghosh model: a reinterpretation as a price model," *Journal of Regional Science*, 37, 4: 629-651.
- DIETZENBACHER, Erik and Rutger HOEKSTRA. 2003. "The RAS structural decomposition approach," in HEWINGS G.J.D., SONIS M. and D. BOYCE, Eds., *Trade, Networks and Hierarchies. Modelling Regional and Interregional Economies*. Berlin: Springer: 179-199.
- DIETZENBACHER, Erik and Bart LOS. 2002. "Externalities of R&D Expenditures," *Economic Systems Research*, 14(4): 407-425.
- DIETZENBACHER, Erik, Isidoro ROMERO LUNA and Niels S. BOSMA. 2005. "Using average propagation lengths to identify production chains in the Andalusian economy," *Estudios de Economía Aplicada*, 23, 2: 405-422.
- DIETZENBACHER Erik and Jan A. van der LINDEN. 1997. "Sectoral and spatial linkages in the EC production structure," *Journal of Regional Science*, 37: 235-257.
- DIETZENBACHER Erik, Jan A. van der LINDEN and Albert E. STEENGE. 1993. "The Regional Extraction Method: EC Input-Output Comparisons," *Economic Systems Research*, 5, 2: 185-206.
- GHOSH Ambica. 1958. "Input-output approach to an allocative system," *Economica*, 25, 1: 58-64.
- GRUVER, Gene W. 1989. "A comment on the plausibility of supply-driven input-output models," *Journal of Regional Science*, 29, 3: 441-50.
- GUCCIONE, Antonio. 1986. "The input-output measurement of interindustry linkages: A comment," *Oxford Bulletin of Economics and Statistics*, 48: 373-377.
- EL-HODIRI Mohamed A. and Farrokh NOURZAD. 1988. "A note on Leontief technology and input substitution," *Journal of Regional Science*, 28, 1, 119-120.
- HELMSTADTER, Ernst and Jürgen RICHTERING. 1982. "Input coefficients versus output coefficients types of models and empirical findings," in *Proceedings of the Third Hungarian Conference on Input-Output Techniques*, Budapest, Statistical Publishing House, pp. 213-224.
- HEWINGS, Geoffrey J.D. 1982. "The empirical identification of key sectors in an economy: a regional perspective," *The Developing Economies*, 20, 2: 173-195.
- HIRSCHMAN, A.O. 1958. "Interdependence and industrialization" in *The Strategy of Economic Development*, New Haven: Yale University Press.
- LANTNER, Roland. 1974. *Théorie de la dominance économique*. Paris: Dunod.
- LANTNER, Roland and Frédéric CARLUER. 2004. "Spatial dominance: a new approach to the estimation of interconnectedness in regional input-output tables", *The Annals of Regional Science*, 38, 3: 451-467.
- LEONTIEF, Wassily. 1936. "Quantitative input-output relations in the economic system of the United States," *Review of Economics and Statistics*, 18, 3: 105-125.
- LOS Bart. 2004. "Identification of strategic industries: a dynamic perspective," *Papers in Regional Science*, 83, 4: 669-698.

- MILLER, Ronald E. 1966. "Interregional feedbacks in input-output models: Some preliminary results," *Papers of the Regional Science Association*, 17: 105-125.
- MILLER, Ronald E. 1969. "Interregional feedbacks in input-output models: Some experimental results," *Western Economic Journal*, 7: 57-70.
- MILLER, Ronald E. 1986. "Upper bounds on the sizes of interregional feedbacks in multiregional input-output models," *Journal of Regional Science*, 26: 285-306.
- MILLER, Ronald E. 1989. "Stability of supply coefficients and consistency of supply-driven and demand-driven input-output models: a comment," *Environment and Planning A*, 21, 8: 1113-1120.
- MILLER, Ronald E. and Peter D. BLAIR. 1985. *Input-output analysis: foundations and extensions*. Prentice-Hall, Englewood Cliffs.
- MILLER, Ronald E. and Michael L. LAHR. 2001. "A taxonomy of extractions", in *Regional science perspectives in economic analysis: a festschrift in memory of Benjamin H. Stevens* (Contributions to economic Analysis), Michael L. LAHR and Ronald E. MILLER, Eds, Amsterdam: North-Holland, pp. 407-441.
- OOSTERHAVEN, Jan. 1988. "On the Plausibility of the Supply-Driven Input-Output Model," *Journal of Regional Science*, 28, 2: 203-217.
- OOSTERHAVEN, Jan. 1989. "The supply-driven input-output model: A new interpretation but still implausible," *Journal of Regional Science*, 29, 3: 459-465.
- OOSTERHAVEN, Jan. 1996. "Leontief versus Ghoshian price and quantity models," *Southern Economic Journal*, 62, 3: 750-759.
- PARK, Ji Young. 2007. "The supply-driven IO model: a reinterpretation and extension," paper presented at the *Western Regional Science Association 46th Annual Meeting*, Newport Beach.
- POLENSKE Karen R. and Geoffrey J.D. HEWINGS. 2004. "Trade and spatial economic interdependence," *Papers in Regional Science*, 83: 269-289.
- PONSARD Claude. 1972. "Les éléments fondamentaux de la théorie des graphes", in *Graphes de Transfert et Analyse Economique*, Claude PONSARD Ed, Publication hors-série de la *Revue Economique*, Institut de Mathématiques Economiques de Dijon, Paris : Sirey, pp. 1-25.
- RASMUSSEN, P. Nørregaard. 1956. *Studies in Intersectoral Relations*, North-Holland, Amsterdam
- ROSE, Adam and Tim ALLISON. 1989. "On the plausibility of the supply-driven input-output model: empirical evidence on joint stability," *Journal of Regional Science*, 29: 451-458.
- SONIS, Michael and Geoffrey J.D. HEWINGS. 1992. "Coefficient change in input-output models: theory and applications," *Economic Systems Research*, 4, 2: 143-157.
- SONIS Michael, Joaquim GUILHOTO, Geoffrey J.D. HEWINGS, Eduardo MARTINS. 1995. "Linkages, key sectors and structural change: some new perspectives," *Developing Economies*, 33: 233-270.
- ten RAA, Thijs and Pierre MOHNEN. 1994. "Neoclassical input-output analysis," *Regional Science and Urban Economics*, 24, 1: 135-158.
- van der LINDEN, Jan A. and Erik DIETZENBACHER. 1995. "The nature of changes in the EU cost structure of production 1965-85: an RAS approach," in H.W. ARMSTRONG and R.W. VICKERMAN, Eds., *Convergence and divergence among European Regions*, Pion Limited, London, 124-139.

- van der LINDEN, Jan A. and Erik DIETZENBACHER. 2000. "The determinants of structural change in the European Union: A new application of RAS," *Environment and Planning (A)*, 32: 2205-29.
- WALMSLEY, Terrie L and Robert A. McDOUGALL. 2007. *Using Entropy to Compare IO tables*, GTAP Research Memorandum No. 09.

Endnotes

¹ See Helmstadter and Richtering (1982), Cronin (1984), Chen and Rose (1986, 1991), Oosterhaven (1988, 1989, 1996), Dietzenbacher (1989, 1997), Gruver (1989), Miller (1989), Rose and Allison (1989), Sonis and Hewings (1992), de Mesnard (1988, 1997, 2002), Park (2007).

² See also Dietzenbacher and van der Linden (1997), Los (2004), Dietzenbacher et al. (2005) or Cai and Leung (2005).

³ The Rasmussen index is derived from the row sum of \mathbf{L} actually.

⁴ Hirschman (1958) may be considered the father of the idea of total linkages.

⁵ See in Dietzenbacher, van der Linden and Steenge (1993) how this is done for a matrix mixing both sectors and regions.

⁶ Miller and Lahr (2001, p. 412) attribute the idea of partitioning for measuring linkages to Cella (1984). However, such an idea can be found in a slightly different way in the pioneering works of the “Dijon school” (Ponsard 1972; Lantner 1974): the analysis is mainly based on graph theory (but there is a strict equivalence between graphs and matrices) and the idea of determinant of matrices which leads to comparing the determinant of the whole matrix and the determinant of a submatrix. These works are little known because they were published in French and never translated into English, except in Lantner and Carluer (2004).

⁷ This remark is not incompatible with Dietzenbacher and Los’s demonstration (2002, p. 412) for the R&D effects (or any similar question): the matrices of backward and forward effects can be equal after introducing the coefficients of R&D intensity (but the forward effects become the weighted row sums of the \mathbf{G} matrix): since $\mathbf{H}_B = \hat{\mathbf{r}} \hat{\mathbf{x}}^{-1} \mathbf{L}$ for the backward effects and $\mathbf{H}_F = \hat{\mathbf{r}} \mathbf{G} \hat{\mathbf{x}}^{-1}$ for the forward effects, where \mathbf{r} is the vector of R&D expenditure and $\hat{\mathbf{r}} \hat{\mathbf{x}}^{-1}$ is the vector of intensities of R&D, we have $\mathbf{H}_B = \mathbf{H}_F$.

⁸ Formula (6) does not mean that the production function in the Ghosh model is of complementary inputs! On the contrary, as emphasized above, (6) means that when the Ghosh model is chosen, the allocation coefficients must be fixed, compelling the technical coefficients to be variable.

⁹ Obviously, backward effects cannot be analyzed if the production function with complementary inputs is abandoned.

¹⁰ There is some contradiction between perfectly substitutable inputs on the one hand and considering that the Ghosh model is a model where quantities are fixed in the short run on the other hand.

¹¹ It must be recalled that in usual input-output analysis (national accounting, regional analysis) the data are always available in value for homogeneity reasons (e.g. it becomes possible to aggregate different type of cars). The case of physical data remains theoretical as in the models of production prices (Ricardo, Marx and Sraffa).

¹² Index prices are introduced because multiplying a value (a number in \$) by a price (a number in \$) is nonsense, while an index price (a “dimensionless” number) can be validly multiplied by a value (a number in \$) to give a number in \$.

¹³ Mathematically, this is because when data are available in quantities, the primal of the Ghosh model only provides a mathematical identity—that is, nothing—while the primal of the Leontief model provides quantities and its dual provides prices; in the Ghosh model, only the dual provides something, namely, values. Formally, it is a question of the quantity of information supplied by each model: each model is allowed to provide only two categories of information (quantities and prices), with two sets of equations (the primal and the dual). If a model provides nothing with one set of equations, the two categories of information must be mixed in the other set: this is what happens in the Ghosh model.

¹⁴ Dietzenbacher (1997) has rightly conducted cost-push exercises, that is, propagation of cost variations upon prices, by assuming that quantities (actually, outputs \mathbf{x} and labor \mathbf{v} in value are fixed). Nevertheless, de Mesnard (2009) has shown that the cost-push exercises are equivalently performed with the dual Leontief model in a strictly equivalent way, but, and the big difference lies there, without being obliged to assume, even implicitly, that the production function has changed from complementary inputs to substitutable inputs.

¹⁵ It should be noticed that the fixity of index prices is a hypothesis added to the Ghosh model, while the Leontief model finds quantities in values in its primal and index prices in its dual *without* additional hypotheses (even if the prices in its primal and the quantities in its dual are undetermined). It should be noted also that it is only when the index prices are

assumed fixed in the dual Ghosh model—that is, when the prices are assumed constant—that equation (3) is retrieved when data are available in value.

¹⁶ It is interesting to note that a rather similar impossibility exists elsewhere in regional science. When the RAS method is used to deduce absorption-substitution (**R**) and fabrication-transformation effects (**S**), it has been shown that the diagonal matrices **R** and **S** of RAS are hyperbolically homogenous: if the terms of **R** are multiplied by λ , the terms of **S** must be divided by λ : both effects are indissolubly linked. In a word, **R** and **S** should be normalized but this has been proven to be impossible (de Mesnard 2002, 2004).

¹⁷ Notice that $\bar{\mathbf{m}}^F = \mathbf{m}^F$. However, in the Leontief model when data are given in quantities, which is written as $\bar{\mathbf{x}} = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\mathbf{f}}$ and solved as $\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} \bar{\mathbf{f}} \equiv \bar{\mathbf{L}}\bar{\mathbf{f}}$, the backward multipliers are given by $\bar{\mathbf{m}}^B = \mathbf{i}'\bar{\mathbf{L}} \neq \mathbf{m}^B$; remember also that it is not a matter of choice of physical units.

¹⁸ However, it is always possible to consider the neoclassical approach of input-output analysis as developed by ten Raa and Mohnen (1994). But that is another story...