More firms, more competition: is it certain?

The case of the fourth operator

in France's mobile telephony[1]

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Abstract. The French government plans to authorize a fourth operator to enter the country's mobile phone market alongside Orange, SFR and Bouygues Telecom. While the French government sees this as a way to foster competition, this paper predicts the move will prove a disappointment. Three points are examined. 1) If the operators are in four-way Cournot competition, minimizing the total profit fails to maximize the consumer surplus and the total surplus; the most realistic price fall is only of 1.11% compared to three-way Cournot competition. 2) The overall incentives for forming a monopoly are positive; when the fourth operator's costs are high, there will be no move from a three-way Cournot competition to a monopolistic cartel of four because Orange experiences negative incentives; there will be no move from a monopolistic cartel of three to a monopolistic cartel of four. 3) Moving from four-way Cournot competition to a partial cartel formed by Orange, SFR and Bouygues Telecom is unlikely; when the fourth operator enters a market dominated by the monopolistic cartel of Orange, SFR and Bouygues Telecom, these three operators will not continue forming a cartel; excluding the fourth operator from the monopolistic cartel of four is also losing; the cartel formed by SFR, Bouygues Telecom and the fourth operator is never credible either.

[1] This paper is a completely independent academic study. It has not been granted by France Telecom - Orange, SFR or Bouygues Telecom or any other company.

1. Introduction

To increase competition in France's mobile phone market, the French GSM, the government plans to authorize the entry of a fourth operator alongside the existing three—Orange (formerly the state-owned concern France Telecom, the oldest-established operator), SFR and Bouygues Telecom.[2] Some commentators suggest the new entrant could be the Internet provider Free, but nothing is sure. The point of introducing a fourth operator is debated in the industry but it seems no economic studies have been published on the topic. It is worthwhile, then, examining what will happen to the market with the entry of an additional firm. This is done here using the theory and tools of industrial economics. We shall ignore the recent phenomenon of the Mobile Virtual Network Operator (MVNO), where an operator leases its network and sells minutes of communication (at a price close to the marginal cost) to third-party operators who do not have their own networks.

The paper is organized as follows. The second section is very short one but absolutely necessary. It explains how the operators' costs are derived from their present market shares (which have been stylized to one-half, one-third and one-sixth for Orange, SFR and Bouygues Telecom, respectively) by assuming that these market shares have been formed in Cournot competition —a standard model for describing competition in an oligopoly. The third section examines the economic effects of the entry of a fourth operator when Cournot competition still prevails—the price fall is very moderate. The fourth section examines those effects when a monopolistic cartel still prevails—Orange would have negative incentives to form a monopolistic cartel of four. Finally, the fifth section considers the formation of partial cartels among Orange, SFR and Bouygues Telecom or among SFR, Bouygues Telecom and the fourth operator.[3]

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[2] Actually, it is a permit for the high speed "3G" (third generation) network but we may equate this to becoming a new mobile phone operator by considering that the product is homogenous (which is reasonable because the fourth operator will also provide a conventional telephone service). Bouygues Telecom is a 3G provider since April 2007; before it was only an EDGE (also named 2.75G) provider, an extension of the GPRS standard (also known as 2.5G).

[3] The partial cartel model will be based on the Cournot-competition-with-competitive-fringe model of oligopoly, where a group of firms, the partial cartel, is the natural leader and the other firms are followers.

Section 1 is this introduction while section 6 is the conclusion. An annex reminds readers of the theory behind the models of Cournot competition, monopolistic cartels and partial cartels.

2. Deriving operators' costs

We assume that (i) the non-uniform market shares, namely, one-half, one-third and one-sixth for Orange, SFR and Bouygues Telecom respectively, are those the firms had in Cournot competition and that (ii) the non-uniform market shares are used as quotas if the three operators decide to form a monopolistic cartel—which the *Conseil de la Concurrence* ruled they did. Following formula (5) the non-uniform market shares or quotas are entirely determined by the structure of the unit costs of production in Cournot competition, which means we can determine the unit costs of production from the market shares. Writing formula (5) for each of the three operators yields the following system of equations:

$$\left\{\frac{a-4c_1+3c(3)}{3a-c(3)}=\frac{1}{2};\frac{a-4c_2+3c(3)}{3a-c(3)}=\frac{1}{3};\frac{a-4c_3+3c(3)}{3a-c(3)}=\frac{1}{6}\right\}$$

As this system is under-determined, we choose to solve the system with respect to c_1, the cost of Orange, the "oldest-established operator", which leads to:

(1) $c_2=\frac{1}{9}(a+8c_1)$ and $c_3=\frac{1}{9}(2a+7c_1)$

and the order of unit costs is: $c_1<c_2<c_3$.

3. Cournot competition with four players

We assume that the current three operators, Orange, SFR and Bouygues Telecom, maintain the same costs that they had in three-way Cournot competition—indicated by (1), with the market shares $\lambda_1(3)=\frac{1}{2}$, $\lambda_2(3)=\frac{1}{3}$ and $\lambda_3(3)=\frac{1}{6}$. We assume also that the fourth operator's unit costs are given with respect to those of Orange, in the same form as for the other operators: $c_4=\alpha a+(1-\alpha)c_1$ (this is a linear combination of a and c_1, uniformly increasing with α because $a\geq c_1$). A particular case is $\alpha=0$, that is, $c_4=c_1$: the fourth operator has the same costs as Orange and c_4 is minimum.

We are able to recalculate what Cournot competition with four players implies in terms of output and profit. We have: $\overline{q}_1(4)=\frac{4+3\alpha}{15}\mu$, where μ denotes the quantity $\frac{(a-c_1)}{b}$, $\overline{q}_2(4)=\frac{7+9\alpha}{45}\mu$, $\overline{q}_3(4)=\frac{2+9\alpha}{45}\mu$ and $\overline{q}_4(4)=\frac{4-12\alpha}{15}\mu$, which implies $\alpha\leq\frac{1}{3}$ to avoid a negative output. The market size is $\overline{q}(4)=\frac{11-3\alpha}{15}\mu$ which is decreasing with α. From this, we are able to deduce the quotas: $\lambda_1(4)=\frac{4+3\alpha}{11-3\alpha}$, $\lambda_2(4)=\frac{1}{3}\frac{7+9\alpha}{11-3\alpha}$, $\lambda_3(4)=\frac{1}{3}\frac{2+9\alpha}{11-3\alpha}$ and $\lambda_4(4)=\frac{1-3\alpha}{11-3\alpha}$. Similarly, the price is $\overline{p}(4)=\frac{4a+11c_1+3\alpha(a-c_1)}{15}$; this function is uniformly linearly increasing with α. If φ denotes the quantity $\frac{(a-c_1)^2}{b}>0$, the profits are: $\overline{\Pi}_1(4)=\frac{(4+3\alpha)^2}{15^2}\varphi$, $\overline{\Pi}_2(4)=\frac{(7+9\alpha)^2}{9\times 15^2}\varphi$, $\overline{\Pi}_3(4)=\frac{(2+9\alpha)^2}{9\times 15^2}\varphi$: all are positive and increasing with α but the profit of the fourth operator, $\overline{\Pi}_4(4)=\frac{(4-12\alpha)^2}{15^2}\varphi$ is decreasing for $\alpha<\frac{1}{3}$ and equal to zero for $\alpha=\frac{1}{3}$. The total profit, which is equal to $\overline{\Pi}(4)=\frac{(1539\alpha^2-486\alpha+341)}{9\times 15^2}\varphi$, is decreasing for $\alpha<\frac{3}{19}$ and increasing thereafter; its maximum is reached for $\alpha=\frac{1}{3}$. This is depicted in Figure 1.

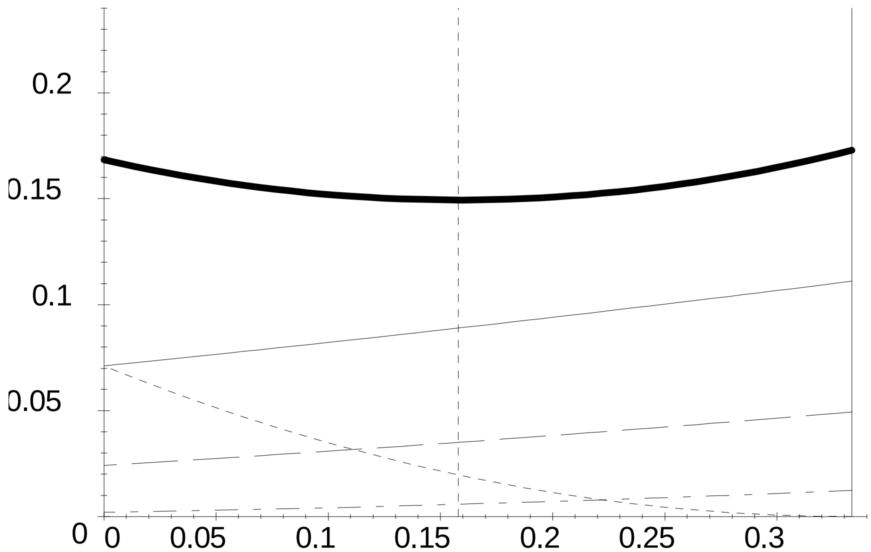

Figure 1. Profits for four operators with respect to α (with $\varphi = 1$)
(Orange: fine line; SFR: dashed line; Bouygues Telecom: dotted-dashed line;
fourth operator: dotted line;
total profit: bold line; vertical dots: minimum of the total profit)

One might think that it is clever to minimize the total profit with respect to α, that is to choose $\alpha = \frac{3}{19}$ so as to maximize the consumer surplus. However, the following logical result can be established:

Result 1. In Cournot competition with four operators, minimizing the total profit does not maximize the consumer surplus and the total surplus. In Cournot competition, total profit and consumer surplus are uniformly decreasing with α. Consumer surplus is minimum when the fourth operator is not in the market; the total surplus is minimum for $\alpha = \frac{1}{4}$. Both are maximum when the unit costs of the fourth operator are equal to those of Orange, which is difficult to achieve for a new operator.

Proof. The consumer surplus can be written:

$$\overline{S}_c(4) = \int_0^{\frac{11-3\alpha}{15}\mu} \left(a - b\,q - \frac{4\,a + 11\,c_1 + 3\alpha(a - c_1)}{15} \right) dq = \frac{(11-3\alpha)^2}{450}\varphi$$

which is minimum for $\frac{d\bar{S}_c(4)}{d\alpha} = -\frac{1}{75}(11-3\alpha)\varphi = 0 \Leftrightarrow \alpha = \frac{11}{3} > 1$. Hence, the practical minimum is reached for $\alpha = \frac{1}{3}$, where the fourth operator's output is zero; this comes about because the price is uniformly increasing with α. For $\alpha < \frac{1}{3}$, $\frac{d\bar{S}_c(4)}{d\alpha} < 0$: $\bar{S}_c(4)$ is decreasing and its maximum is for $\alpha = 0$. The total surplus $\bar{S}(4)$ is equal to $\bar{\Pi}(4) + \bar{S}_c(4)$. Its derivative is equal to zero for $\alpha = \frac{1}{4}$. •

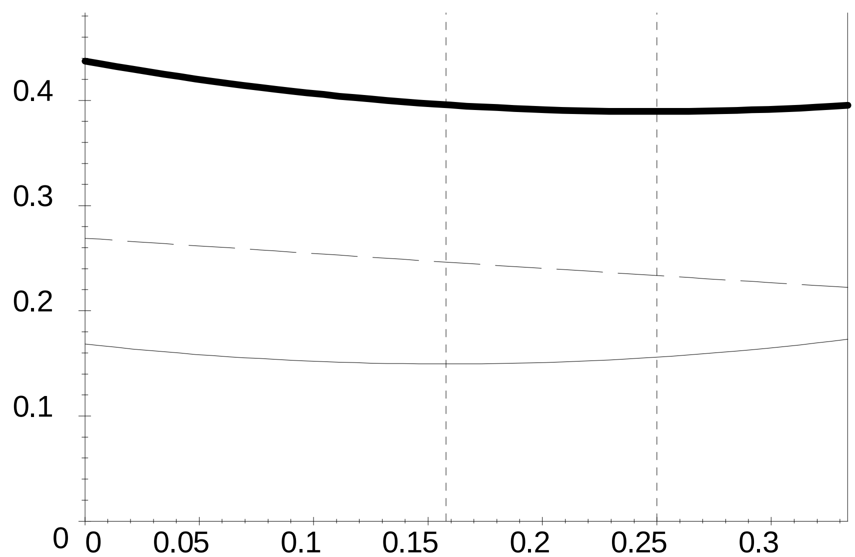

Figure 2. Profit (fine line), consumer surplus (dashed line) and total surplus (bold line) for four operators with respect to α
(with $\varphi = 1$; vertical dots: minimum of total profit and of total surplus)

To illustrate what happens with concrete examples, we now consider two cases (in addition to $\alpha = 0$ where $\bar{S}(4)$ is decreasing and the total profit is decreasing), which both correspond to relatively low values of α, all less than $\frac{1}{3}$ where $\bar{\Pi}_4(4)$ is positive and decreasing and less than $\frac{1}{4}$: (i) $\alpha = \frac{1}{9} = .111$, that is, $c_4 = c_2$: the fourth operator has the same costs as SFR (and the total profit is decreasing); and (ii) $\alpha = \frac{2}{9} = .222$, that is, $c_4 = c_3$: the fourth operator has the same costs as Bouygues Telecom (and the total profit is increasing). This leads to the market shares shown in Table 1.

Fourth operator's costs equal those of:	Market shares $\lambda_i(4)$ in Cournot competition: Orange	SFR	Bouygues Telecom	Fourth operator	Total output	Price $\overline{p}(4)$
Orange: $\alpha = 0$, $c_4 = c_1$	$\frac{12}{33}$	$\frac{7}{33}$	$\frac{2}{33}$	$\frac{12}{33}$	$\frac{33}{45}\mu$	$\frac{12a+33\,c_1}{45}$
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	$\frac{13}{32}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{1}{4}$	$\frac{32}{45}\mu$	$\frac{13a+32\,c_1}{45}$
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	$\frac{14}{31}$	$\frac{9}{31}$	$\frac{4}{31}$	$\frac{4}{31}$	$\frac{31}{45}\mu$	$\frac{14a+31\,c_1}{45}$

Table 1. Market shares, total output and price under Cournot competition with four operators

Orange's market share is growing with α while the price is increasing ($\overline{p}(4)\big|_{\alpha=\frac{2}{9}} > \overline{p}(4)\big|_{\alpha=\frac{1}{9}} > \overline{p}(4)\big|_{\alpha=0}$) and the total output is decreasing: unsurprisingly, the higher the unit cost of the fourth operator, the less competition there is. Hence, the French government must find an efficient operator. Table 10 in Annex indicates what the profits are under Cournot competition for each of the four operators: the total profit is decreasing with α then growing. It is more advantageous for Orange and SFR to compete against an inefficient fourth operator, but the opposite is true for Bouygues Telecom.

We may compare future four-way Cournot competition to present-day three-way Cournot competition. The market shares being $\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$, we follow equations (3) to (6) in the annex. As $c_m(3) = \frac{1}{9}(a + 8c_1)$, the price is $\overline{p}(3) = \frac{1}{3}(a + 2c_1)$. If we compare this price to the three prices of Table 1, the order is: $\overline{p}(3) > \overline{p}(4)\big|_{\alpha=\frac{2}{9}} > \overline{p}(4)\big|_{\alpha=\frac{1}{9}} > \overline{p}(4)\big|_{\alpha=0}$; hence, introducing a fourth operator clearly lowers the price. However,

Result 2. When a fourth operator is introduced in Cournot competition, the higher Orange's unit cost, the lower the price fall when passing from three-way to four-way Cournot competition. The *maximum maximorum* price fall is of 20%: the price fall is maximum when $c_1 = 0$ and then it is of 20% if $\alpha = 0$, 13.33% if $\alpha = \frac{1}{9}$, and of 6.67% if $\alpha = \frac{2}{9}$; it tends toward zero if c_1 tends toward *a*. However, the most realistic price fall is of 1.11% only; a rather disappointing result.

Proof. We have: $\frac{\bar{p}(4)\big|_{\alpha=0}-\bar{p}(3)}{\bar{p}(3)}=-\frac{3}{15}\frac{1-\gamma}{1+2\gamma}$, $\frac{\bar{p}(4)\big|_{\alpha=\frac{1}{9}}-\bar{p}(3)}{\bar{p}(3)}=-\frac{2}{15}\frac{1-\gamma}{1+2\gamma}$ and $\frac{\bar{p}(4)\big|_{\alpha=\frac{2}{9}}-\bar{p}(3)}{\bar{p}(3)}=-\frac{1}{15}\frac{1-\gamma}{1+2\gamma}$ where γ denotes the quantity $\frac{c_1}{a}$; they are maximum for $\gamma=0$, that is, $c_1=0$. One must understand that, even if the *maximum maximorum* price fall of 20% might seem attractive, it is achieved under two conditions: (i) the fourth operator is as efficient as Orange ($\alpha=0$), which is quite unlikely because it takes time for a new operator to become efficient (Bouygues Telecom is less efficient than SFR, which in turn is less efficient than Orange, which complies with the order of the entry in the market) and (ii) Orange is itself unrealistically efficient, with unit costs equal to zero! More realistically, the price fall could be obtained when the fourth operator is as efficient as Bouygues Telecom ($\alpha=\frac{2}{9}$) and Orange is averagely efficient ($c_1=\frac{1}{2}a\Leftrightarrow\gamma=\frac{1}{2}$), that is a price fall of $\frac{\bar{p}(4)\big|_{\alpha=\frac{2}{9}}-\bar{p}(3)}{\bar{p}(3)}=-1.11\%$ only. Figure 3 plots these variations and shows that the maximum price fall is reached when $\gamma=0$. •

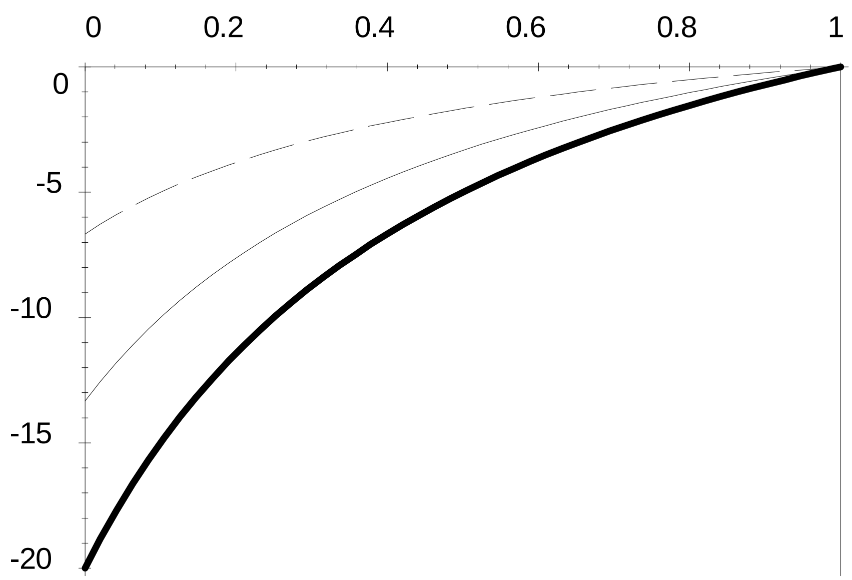

Figure 3. Prices variation in %
between three-way and four-way Cournot competition,
with respect to $\gamma=\frac{c_1}{a}$ ($\alpha=0$: bold line; $\alpha=\frac{1}{9}$: fine line; $\alpha=\frac{2}{9}$: dashed line)

Table 2 examines what happens when we move from three-way to four-way Cournot competition, knowing that with three players, the profits in Cournot competition are: $\overline{\Pi}_1(3) = .1111\,\varphi$, $\overline{\Pi}_2(3) = .04938\,\varphi$, $\overline{\Pi}_3(3) = .01235\,\varphi$ and the total profit is $\overline{\Pi}(3) = .1728\,\varphi$. The total profit decreases even if a fourth operator enters the market: the higher the fourth operator's costs, the greater the fall.[4]

	Relative variation of profit (%)			
Fourth operator's unit costs equal those of:	Orange	SFR	Bouygues Telecom	Total profit
Orange: $\alpha = 0$, $c_4 = c_1$	**-36**	**-51**	**-84**	**-2.57**
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	**-24.89**	**-36**	**-64**	**-12.57**
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	**-12.89**	**-19**	**-36**	**-11.71**

Table 2. Three-way and four-way Cournot competition compared

4. A monopolistic cartel with four players

The operators may also contemplate forming a monopolistic cartel. We assume that the operators that form the monopolistic cartel choose the quotas in proportion to the market shares they would have in four-way Cournot competition.

Result 3. The incentives for forming a four-way monopolistic cartel are greater overall when the fourth operator has the same costs as SFR ($\alpha = \frac{1}{9}$), as Orange ($\alpha = 0$) and finally as Bouygues Telecom ($\alpha = \frac{2}{9}$).

Proof. Indeed, we have $\Pi_i(4) = \lambda_i(4)\dfrac{(a - c(4))^2}{4\,b}$ following (12). The total profit in a monopolistic cartel of four $\Pi(4)$ is shown in Table 11 in Annex; this profit must be shared as per

[4] The contrasting pattern of total profit that decreases more and each operators' profit that decreases less when α grows is obviously caused by the profit of the fourth operator, which is much higher when $\alpha = 0$.

the quotas shown in Table 1. Obviously, the profits are ordered following $c_4 : \Pi(4)\big|_{c_4=c_1} > \Pi(4)\big|_{c_4=c_2} > \Pi(4)\big|_{c_4=c_3}$. Table 11 in Annex indicates also the global incentives to form a monopolistic cartel measured by the ratio $\frac{\Pi(4)-\overline{\Pi}(4)}{\overline{\Pi}(4)}$. •

Result 4. When the fourth operator's unit costs are equal to those of Bouygues Telecom (that is, $\alpha = \frac{2}{9}$, rather higher), switching from four-way Cournot competition to a monopolistic cartel of four is impossible because the incentives are negative for Orange.

Proof. Table 12 in Annex indicates what operator's profit are under monopolistic cartel of four. For each of the four operators, the incentives for forming a monopolistic cartel are measured by the ratio $\frac{\Pi_i(4)-\overline{\Pi}_i(4)}{\overline{\Pi}_i(4)}$ as a percentage, where $\Pi_i(4)=\lambda_i(4)\Pi(4)$, are given by Table 3. To prove the result, it is sufficient to see that $\frac{\Pi_1(4)-\overline{\Pi}_1(4)}{\overline{\Pi}_1(4)}\Big|_{c_4=c_1} = -3.32\%$ (in cell $\{3,1\}$ of Table 3): unanimity is obviously required. Nonetheless, it is possible to compute for what value of α the ratio $\frac{\Pi_1(4)-\overline{\Pi}_1(4)}{\overline{\Pi}_1(4)} = \frac{25\left(143-108\alpha+162\alpha^2\right)^2}{81(11-3\alpha)^3\,(4+3\alpha)}-1$ is negative: it is approximately for $\alpha \in \left].156, .405\right[$ (this solution has been graphically determined): $\frac{2}{9}$ falls within this interval. •

	Global incentives to switch from four-way Cournot competition to a monopolistic cartel of four (%)			
Fourth operator's unit costs equal those of:	Orange	SFR	Bouygues Telecom	Fourth Operator
Orange: $\alpha = 0$, $c_4 = c_1$	18.55	103.22	611.28	18.55
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	3.81	68.70	349.85	68.70
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	**-3.32**	50.39	238.38	238.38

Table 3. Incentives to switch from
four-way Cournot competition to a monopolistic cartel of four

As it is the weakest of the existing operators, Bouygues Telecom has the greatest incentives, measured by $\frac{\Pi_i(4)-\overline{\Pi}_i(4)}{\overline{\Pi}_i(4)}$, for forming a monopolistic cartel of four operators.

If we admit that the three operators (Orange, SFR and Bouygues Telecom) are in a monopolistic cartel, as held by the *Conseil de la Concurrence*, we can imagine that they could immediately include the new operator in their cartel. Indeed, it is shrewder to consider that the market moves from a monopolistic cartel of three to a monopolistic cartel of four rather than from three-way Cournot competition to four-way Cournot competition. Hence, we compare below the monopolistic cartel of four to the monopolistic cartel of three.

Result 5. Moving from a monopolistic cartel of three to a monopolistic cartel of four operators is unlikely.

When they move from a monopolistic cartel of three to a monopolistic cartel of four operators, the incentives for Orange, SFR and Bouygues Telecom are all negative. The operators logically experience the greatest losses when the costs of the fourth operator are the lowest, that is, when its costs are equivalent to those of Orange or $\alpha = 0$. However, Bouygues Telecom loses the most, Orange the least.

Proof. Following equations (11) to (13), the data for the monopolistic cartel of three are: $c(3) = \frac{1}{2}c_1 + \frac{1}{3}c_2 + \frac{1}{6}c_3 = \frac{1}{27}(2a + 25c_1)$, $q(3) = .15432\mu$, $\Pi(3) = .21433\varphi$, both to be shared in the proportions $\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$. Table 4 compares both monopolistic cartels: the total gains of profits when one moves from a three-operator monopolistic cartel to a four-operator monopolistic cartel are measured by $\frac{\Pi(4) - \Pi(3)}{\Pi(3)}$; the table also indicates by how much the total outputs increase. Table 4 further indicates the individual percentage gains in profits $\frac{\Pi_i(4) - \Pi_i(3)}{\Pi_i(3)}$: Bouygues Telecom could be in an uncomfortable position if the fourth operator is too efficient in terms of costs. •

Fourth operator's unit costs equal those of	Incentives for moving from a monopolistic cartel of three to a monopolistic cartel of four (%)			
	Orange	SFR	Bouygues Telecom	Global incentives[5]
Orange: $\alpha = 0$, $c_4 = c_1$	**-21.33**	**-31.17**	**-60.67**	+8.16
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	**-19.56**	**-25.38**	**-44.03**	**-.499**
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	**-12.68**	**-15.80**	**-25.16**	**-3.327**

Table 4. Incentives to move from a monopolistic cartel of three to a monopolistic cartel of four

5. Partial cartels

As before, we assume that the three operators that form a partial cartel choose the quotas proportionally to the market shares they had or would have in four-way Cournot competition.

5.1. Partial cartel Orange, SFR and Bouygues Telecom

First, one may wonder whether four-way Cournot competition is more beneficial than the partial cartel of three formed by Orange, SFR and Bouygues Telecom (the fourth operator remaining alone). By respect to the annex, $C(3)$ is the mean weighted unit costs of Orange, SFR and Bouygues Telecom, that is, $C(3) = c(3) = \frac{1}{2}c_1 + \frac{1}{3}c_2 + \frac{1}{6}c_3 = \frac{1}{27}(2a + 25c_1)$ and $c = c_4$. We obtain the price $p(3,4) = \frac{1}{2}\frac{31a + 77c_1 + 27(a - c_1)\alpha}{54}$ and outsider's output and profit, $q_4(3,4) = \frac{1}{2}\frac{(31 - 81\alpha)}{54}\mu$ and $\Pi_4(3,4) = \frac{1}{4}\frac{(31 - 81\alpha)^2}{54^2}\varphi$. We also obtain also cartel's output and profit: $\sum_{i=1}^{3} q_i(3,4) = \frac{(23 + 27\alpha)}{54}\mu$ and $\sum_{i=1}^{3} \Pi_i(3,4) = \frac{1}{2}\frac{(23 + 27\alpha)^2}{54^2}\varphi$, both being shared out

[5] The changed pattern between total gains and individual gains is because of the fourth operator.

with $\lambda_1(3)=\frac{1}{2}$, $\lambda_2(3)=\frac{1}{3}$ and $\lambda_3(3)=\frac{1}{6}$. Table 5 indicates what the results are depending on the value of α.

Fourth operator's costs equal those of:	$p(3,4)$	Partial cartel: Orange, SFR, Bouygues Telecom		Fringe: the fourth operator	
		$\sum_{i=1}^{3} q_i(3,4)$	$\sum_{i=1}^{3} \Pi_i(3,4)$	$q_4(3,4)$	$\Pi_4(3,4)$
Orange: $\alpha = 0$, $c_4 = c_1$	$\frac{1}{2}\frac{31a+77c_1}{54}$	.4259 μ	.0907 φ	.287 μ	.0824 φ
SFR: $\alpha=\frac{1}{9}$, $c_4 = c_2$	$\frac{17a+37c_1}{54}$	.4815 μ	.1159 φ	.2037 μ	.0415 φ
Bouygues Telecom: $\alpha=\frac{2}{9}$, $c_4 = c_3$	$\frac{1}{2}\frac{37a+71c_1}{54}$	.537 μ	.1442 φ	.1204 μ	.0145 φ

Table 5. Outputs and profits of the monopolistic cartel of Orange, SFR and Bouygues Telecom but the fourth operator in the fringe

Result 6. Passing from a four-way Cournot competition to a partial cartel of Orange, SFR and Bouygues Telecom against the fourth operator is unlikely because Orange experiences negative incentives.

Proof. The proof is given in Table 6 which indicates the percentages $\frac{\Pi_i(3,4)-\overline{\Pi}_i(4)}{\overline{\Pi}_i(4)}$ for Orange, SFR and Bouygues Telecom. Orange always experiences losses; the higher α is, the lower they are. The other operators benefit from the operation; however, the higher α is, the lower their incentives are. This table could be usefully compared to Table 3 where Orange experienced losses only for $\alpha=\frac{2}{9}$.•

Fourth operator's unit costs equal those of:	Incentives (%) Orange	SFR	Bouygues Telecom
Orange: $\alpha = 0$, $c_4 = c_1$	**-36.22**	+24.95	+665.34
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	**-30.56**	+22.25	+334.67
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	**-25.51**	+20.17	+204.18

Table 6. Gains when the operators move from four-way Cournot competition to a partial cartel of three

Second, it is also possible to examine what happens if the fourth operator is introduced in the market dominated by the monopolistic cartel of Orange, SFR and Bouygues Telecom, while these three remain cartelized. Here, the quotas in the partial cartel are logically not proportional to the market shares in four-way Cournot competition but to the market shares in three-way Cournot competition, while the monopolistic cartels of three and four have been calculated above.

Result 7. When the fourth operator enters a market dominated by the monopolistic cartel of Orange, SFR and Bouygues Telecom, these three operators will not continue forming a cartel. Hence, the monopolistic cartel of Orange, SFR and Bouygues Telecom is unstable.

Proof. Compared to the monopolistic cartel of three, each member of the partial cartel of three, Orange, SFR and Bouygues Telecom, experiences a loss. The result is shown in Table 7. •

Result 8. Passing to a partial cartel of Orange, SFR and Bouygues Telecom by excluding the fourth operator from the monopolistic cartel of four is losing. Hence, the monopolistic cartel of four is has some form of stability.

Proof. With respect to the monopolistic cartel of four, only Bouygues Telecom has positive incentives: the higher α is, the greater the incentive is. The results are shown in Table 7. •

Fourth operator’s unit costs equal those of:	Incentives in a partial cartel (%): Compared to the monopolistic cartel of Orange, SFR and Bouygues Telecom	Compared to the monopolistic cartel of four: Orange	SFR	Bouygues Telecom
Orange: $\alpha = 0$, $c_4 = c_1$	**-57.68**	**-46.2**	**-38.51**	+1.08
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	**-45.92**	**-31.25**	**-21.43**	+1.38
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	**-32.72**	**-14.47**	**-2.25**	+1.71

Table 7. Gains of the partial cartel of Orange, SFR and Bouygues Telecom compared to the monopolistic cartel of three and of four

5.2. Partial cartel SFR, Bouygues Telecom and the fourth operator

Result 4 suggests that it could be wise to examine what happens if SFR, Bouygues Telecom and the fourth operator form a partial cartel that excludes Orange (the “little guys cartel”). We assume that SFR, Bouygues Telecom and the fourth operator choose the quotas of their partial cartel proportionally to the market shares they had or would have in four-way Cournot competition.

Result 9. Passing from four-way Cournot competition to a partial cartel of SFR, Bouygues Telecom and the fourth operator is unlikely.

Proof. As the market shares of the last row of Table 1 are maintained, Orange has a market share of $\frac{14}{31}$ while the partial cartel (SFR, Bouygues Telecom and the fourth operator) has $\frac{17}{31}$ of the market after Table 1, which implies that SFR has a quota of $\frac{9}{17}$, Bouygues Telecom of $\frac{4}{17}$ and the fourth operator of $\frac{4}{17}$ also. Hence $C(3) = \frac{9}{17} c_2 + \frac{4}{17} c_3 + \frac{4}{17} c_4 = \frac{1}{153}(25a + 128 c_1)$ and $c = c_1$. The proof is given by Table 9: the gains $\frac{\Pi_i(3,4) - \overline{\Pi}_i(4)}{\overline{\Pi}_i(4)}$ are indicated as percentages. We have added the results for $\alpha = 0$ (quotas of $\frac{1}{3}$, $\frac{2}{21}$ and $\frac{4}{7}$ with $C(3) = \frac{1}{189}(11a + 178 c_1)$) and $\alpha = \frac{1}{9}$ (quotas of $\frac{8}{19}$, $\frac{3}{19}$ and $\frac{8}{19}$ with $C(3) = \frac{1}{171}(14a + 157 c_1)$) to allow a comparison. Hence, when the fourth operator has the same unit costs as Bouygues Telecom, i.e. $\alpha = \frac{2}{9}$, SFR has negative incentives to form a partial cartel with Bouygues Telecom and the fourth operator. For $\alpha = 0$ and $\alpha = \frac{1}{9}$, at least one operator loses out; by contrast, Orange wins in all three cases.

Fourth operator's costs equal those of:	$p(3,4)$	Partial cartel: SFR, Bouygues Telecom and the fourth operator		Fringe: Orange	
		$\sum_{i=2}^{4} q_i(3,4)$	$\sum_{i=2}^{4} \Pi_i(3,4)$	$q_1(3,4)$	$\Pi_1(3,4)$
Orange: $\alpha = 0$, $c_4 = c_1$	$\frac{211a + 545c_1}{756}$	.4418 μ	.0976 φ	.2791 μ	.0779 φ
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	$\frac{199a + 485c_1}{684}$	.4181 μ	.0874 φ	.2909 μ	.0846 φ
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	$\frac{203a + 409c_1}{612}$	.3366 μ	.0567 φ	.3317 μ	.011 φ

Table 8. Outputs and profits of the monopolistic cartel of SFR, Bouygues Telecom and the fourth operator but Orange in the fringe

	Incentives (%)			
	Fringe	Partial cartel		
Fourth operator's unit costs equal those of:	Orange	SFR	Bouygues Telecom	Fourth operator
Orange: $\alpha = 0$, $c_4 = c_1$	+9.54	+34.44	+370.54	**-21.58**
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	+1.42	**-50.97**	+30.76	**-50.97**
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	+13.67	**-25.02**	+68.70	+68.70

Table 9. Incentives to form a partial cartel between SFR, Bouygues Telecom and the fourth operator, compared to four-way Cournot competition

6. Conclusion

It has been announced in the beginning of the year 2009 that a fourth operator may enter the French phone market in addition to Orange, SFR and Bouygues Telecom, as is planned by the French government since some months.

Under simple and reasonable hypotheses (linear demand, constant unit costs deduced from the market shares observed in Cournot competition), we have demonstrated that, if the operators are in four-way Cournot competition, (i) minimizing the operators' total profit fails to maximize the consumer surplus or even the total surplus as wished and (ii) the most realistic price fall is only of 1.11% compared to three-way Cournot competition: disappointing…

We have also shown that the overall incentives for forming a monopoly are positive and larger when the fourth operator has the same costs as SFR. However, when the fourth operator's costs are equal to those of Bouygues Telecom, that is rather high, there will be no move from a three-way Cournot competition to a monopolistic cartel of four because Orange experiences negative financial incentives. It is unlikely there will be a move from a monopolistic cartel of three to a monopolistic cartel of four.

Finally, we have examined two cases of partial cartels. First, that of Orange, SFR and Bouygues Telecom against the fourth operator: (i) moving from four-way Cournot competition to a partial cartel formed by Orange, SFR and Bouygues Telecom is unlikely because unanimity is not achieved, (ii) when the fourth operator enters a market dominated by the monopolistic cartel of Orange, SFR and Bouygues Telecom, these three operators will not continue forming a cartel and (iii) excluding the fourth operator from the monopolistic cartel of four is also impossible. Second, the "small guys cartel" formed by SFR, Bouygues Telecom and the fourth operator by excluding Orange is never credible either.

7. Annexes

7.1. Data

Fourth operator's costs equal those of:	Profits under Cournot competition: Orange	SFR	Bouygues Telecom	Fourth Operator	Total profit
Orange: $\alpha = 0$, $c_4 = c_1$	.0711 φ	.0242 φ	.0020 φ	.0711 φ	.1684 φ
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	.0835 φ	.0316 φ	.0044 φ	.0316 φ	.1511 φ
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	.0968 φ	.04 φ	.0079 φ	.0079 φ	.1526 φ

Table 10. Operators' profit under Cournot competition with four operators

Fourth operator's unit costs equal those of:	$c(4)$	Total output	Total profits: Monopolistic cartel	Total profits: Cournot competition	Global incentives (%)
Orange: $\alpha = 0$, $c_4 = c_1$	$\frac{a + 26c_1}{27}$	.1605 μ	.2318 φ	.1684 φ	37.67%
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	$\frac{11a + 133c_1}{144}$	.1539 μ	.2139 φ	.1511 φ	41.11%
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	$\frac{25a + 254c_1}{279}$	.1517 μ	.2072 φ	.1526 φ	35.79%

Table 11. Weighted average cost,
total profit under monopolistic cartel, total profit under four-way Cournot competition
and incentives for forming a monopolistic cartel

Fourth operator's unit costs equal those of:	Profits in a monopolistic cartel of four: Orange	SFR	Bouygues Telecom	Fourth Operator
Orange: $\alpha = 0$, $c_4 = c_1$	.0843 φ	.0492 φ	.0141 φ	.0843 φ
SFR: $\alpha = \frac{1}{9}$, $c_4 = c_2$	.0866 φ	.0533 φ	.02 φ	.0533 φ
Bouygues Telecom: $\alpha = \frac{2}{9}$, $c_4 = c_3$	.0936 φ	.0602 φ	.0267 φ	.0267 φ

Table 12. Operators' profit under a monopolistic cartel of four

7.2. Cournot competition: theory

We consider Cournot competition among *n* firms (here, $n = 3$ or $n = 4$). We assume that all firms produce the same homogenous commodity, a mobile phone service. We denote $\overline{q}(n)$ the total output in Cournot competition, $\overline{p}(n)$ the price in Cournot competition, *n* the number of operators (that is, three today, possibly four in the future), *a* and *b* fixed non-negative parameters, $\overline{q}_i(n)$ the output of operator *i* in Cournot competition, $\lambda_i(k)$ is the market share of operator *i*, and c_i the fixed unit costs of operator *i* which do not change when one moves from Cournot competition to a monopolistic cartel; $i = 1$ for Orange, $i = 2$ for SFR and $i = 3$ for Bouygues Telecom. Each of the *n* firms produces $\overline{q}_i(n)$; their unit costs are constant, that is, the costs are equal to $c_i\,\overline{q}_i$,[6] the total output is $\overline{q}(n) = \sum_{i=1}^{n} \overline{q}_i(n)$; demand is linear: $\overline{p}(n) = a - b\,\overline{q}(n)$. Each firm *i* earns a profit of $\overline{\Pi}_i(n) = \overline{p}(n)\,\overline{q}_i(n) - c_i\,\overline{q}_i(n)$; we have $\overline{\Pi}_i = (a - c_i)\,\overline{q}_i(n) - b\,\overline{q}_i^2(n) - b\,\overline{q}_i(n)\sum_{j=1, j\neq i}^{n} \overline{q}_j(n)$. The model is the Cournot oligopoly: each firm *i* maximizes its profits, $\overline{q}_j(n)$ being given, that is, $\frac{\partial \overline{\Pi}_i(n)}{\partial \overline{q}_i(n)} = 0 \Leftrightarrow$

[6] See Bittlingmayer (1989) for a discussion of the dramatic effect of fixed costs on competition and Donsimoni et al. (1986) on quadratic costs.

(2) $$\overline{q}_i(n) = \frac{a - c_i}{2\,b} - \frac{\sum_{j=1, j \neq i}^{n} \overline{q}_j(n)}{2} \text{ for all } i$$

Summing this expression over *i*, gives $\overline{q}(n) = \frac{n\,a - \sum_{i=1}^{n} c_i}{2\,b} - \frac{n-1}{2}\overline{q}(n)$ and after rearranging we deduce the size of the market $\overline{q}(n)$:

(3) $$\overline{q}(n) = n\frac{a - c_m(n)}{(n+1)\,b}$$

where $c_m(n)$ denotes the mean cost $\frac{1}{n}\sum_{i=1}^{n} c_i$. To prevent a negative market size it is necessary to impose $a \geq c_m(n)$; by substitution, it turns out that this constraint is equivalent to $c_1 \leq a$.

By substituting into the inverse demand function, the price is given by

(4) $$\overline{p}(n) = \frac{a + n\,c_m(n)}{n+1}$$

Hence, by substituting $\sum_{j=1, j \neq i}^{n} \overline{q}_j(n) = \overline{Q}(n) - \overline{q}_i(n)$ in (2), we have after solving in $\overline{q}_i(n)$: $\overline{q}_i(n) = \frac{a - (n+1)\,c_i + n\,c_m(n)}{(n+1)\,b}$ for all *i*. To avoid negative outputs we have to posit $a \geq (n+1)\,c_i - n\,c_m(n)$, which is also equivalent to $c_i \leq a$. Each firm *i* has a market share equal to:

(5) $$\lambda_i(k) = \frac{\overline{q}_i(n)}{\overline{Q}(n)} = \frac{a - (n+1)\,c_i + n\,c_m(n)}{n\,(a - c_m(n))} \text{ for all } i$$

Now we are able to deduce the profit of each firm *i*: $\overline{\Pi}_i(n) = \frac{(a - (n+1)\,c_i + c_m(n))^2}{(n+1)^2\,b}$ for all *i*, the total profit being:

(6) $$\overline{\Pi}(n) = \frac{n\,(a - (n+1)\,c_i + n\,c_m(n))^2}{(n+1)^2\,b}$$

7.3. Monopolistic cartel: theory

The monopolistic cartel of *n* operators (here, $n=3$ or $n=4$) maximizes the sum of cartel members' profits, the price being the monopoly price. The profit of the monopolistic cartel is equal to the monopoly profit. Demand remains $p(n)=a-b\,q(n)$.

Following the general theory of cartels, each member's output is such that its marginal cost equals those of others, the cartel producing a total quantity such that the marginal revenue is equal to the marginal cost. Cartel's profit writes as $\Pi(q)=p(q)\,q-q\,C(q)$, where $C(q)$ is cartel's cost which is equal to the sum of members cost. Maximizing $\Pi(q)$ by respect to *q* gives $R'(q)=C'(q)$, where $C'(q)$ denotes cartel's marginal cost, what gives the optimal output *q*:

(7) $$q=\frac{a-C'(q)}{2b}$$

$C'(q)$ is found by first summing over *i* the inverse marginal cost functions:

(8) $$q_i=\mathrm{f}_i^{-1}(C_i')$$

where f_i denotes the function of marginal cost of firm *i*, i.e., $C_i'=\mathrm{f}_i(q_i)$, and by positing $C_i'=C'$ for all *i* in (8), that is,

(9) $$q=\mathrm{g}(C')=\sum_i \mathrm{f}_i^{-1}(C')$$

and then by inversing this function:

(10) $$C'=\mathrm{g}^{-1}(q)$$

Each member's output is given by $q_i=\mathrm{f}^{-1}(C')$. However, in this paper, each operator has c_i as constant unit cost; it is impossible to compute the inverse of the functions of marginal cost (8): the members' output is undetermined. This is why we must have another rule. The cartel's members could go beyond and try to reach the minimum cartel's marginal cost; however, this would oblige the cartel to produce only with the operator which has the lower unit cost (France Telecom), what would mean that the other members are automatically eliminated: the industry turns out to be a one-firm monopoly. Hence, we assume that each operator produces a share

$\lambda_i(n)$ of the monopoly output: $q_i(n)=\lambda_i(n)\,q(n)$. Hence, as the profit of each operator is $\Pi_i(n)=p(n)\,q_i(n)-c_i\;q_i(n)$, the cartel's profit is $\Pi(n)=p(n)\,q(n)-q(n)\,c(n)$ where $c(n)$ denotes $\sum_{i=1}^{n}\lambda_i(n)\,c_i$; the individual cost functions are averaged by the weights $\lambda_i(n)$ in the cartel and play no role later. The monopolistic cartel maximizes its profit; at the optimum, we have $\frac{d\Pi(n)}{dq(n)}=0$, which allows us to deduce the market size:

(11) $$q(n)=\frac{a-c(n)}{2b}$$

and $q_i(n)=\lambda_i(n)\frac{a-c(n)}{2b}$. Equation (11) allows us further to deduce $p(n)=\frac{a+c(n)}{2}$. Each cartel member receives a share $\lambda_i(n)$ of the monopoly profit:[7]

(12) $$\Pi_i(n)=\lambda_i(n)\frac{(a-c(n))^2}{4\,b}$$

and then:

(13) $$\Pi(n)=\frac{(a-c(n))^2}{4\,b}$$

7.4. Partial cartel of three operators among four: theory

The model is that of the oligopoly with a Cournot competitive fringe as exposed in Shaffer (1995, p. 745) but the possibility of different costs between insiders and outsiders will be taken into account. The model will be expounded for four operators, three of them being insiders and one an outsider: a subset K of three operators, with $card(K)=3$, form a partial cartel but we do not specify which operators are insiders or outsiders for more generality. The inverse function of demand remains the same as above: $p(3{,}4)=a-b\,(Q(3{,}4)+q(3{,}4))$ where $p(3{,}4)$ denotes the

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[7] The share of the total profit produced by a cartel member differs from the share of the total output received by the same cartel member only if an explicit payment mechanism among members exists. This would be obvious proof of collusion for the control authorities…, even if these transfers may be hidden by using tax havens. We ignore this case.

price when a partial cartel of three operators is formed among four, $Q(3,4)$ and $q(3,4)$ the cartel's output and follower's output respectively for the same partial cartel and *a* and *b* are non-negative parameters. If *c* is the outsider's unit cost, the outsider's profit is $P(3,4) = p(3,4)\, q(3,4) - c\, q(3,4)$. Outsider *i* maximizes its profit by taking the production of the cartel and those of other outsiders as given: $\max_{q(3,4)} P(3,4)$ $\Rightarrow \frac{\partial P(3,4)}{\partial q(3,4)} = 0 \Leftrightarrow q(3,4) = \frac{1}{2}\left(a - c - b\, Q(3,4) - 2b\, q_i(3,4)\right)$ which rearranged gives:

(14) $$q(3,4) = \frac{p(3,4) - c}{b}$$

The cartel adopts a monopolistic behavior with respect to the residual demand curve, which implies the following inverse demand function:[8]

(15) $$p(3,4) = \frac{a - b\, Q(3,4) + c}{2}$$

The cartel's profit is $\Pi(3,4) = p(3,4)\, Q(3,4) - \sum_{i \in K} C_i\, Q_i(3,4)$, C_i being the unit cost of insiders *i*, which can be rewritten $\Pi(3,4) = p(3,4)\, Q(3,4) - Q(3,4)\, C(3)$, where $C(3)$ denotes the insiders' weighted mean cost $\sum_{i \in K} \lambda_i(3)\, C_i$; again, it is as if the costs were averaged by the quotas. This profit can be maximized with respect to $Q(k,n)$, which gives after rearranging $Q(3,4) = \frac{a - 2\, C(3) + c}{2\, b}$ and by substituting this expression into (15) we obtain:

(16) $$p(k,n) = \frac{a + c}{4}$$

Each insider *i* produces a share $\lambda_i(3)$ of this output with $\sum_{i \in K} \lambda_i(3) = 1$: $Q_i(3,4) = \lambda_i(k) \frac{a - 2\, C(3) + c}{2\, b}$, which requires $C(3) < \frac{a + c}{2}$. Substituting (16) into (14) gives

[8] Other possible definitions of the residual demand curve may affect the result following Donsimoni at al. (1986) and Shaffer (1995).

$q_i(3,4)=\frac{a+2C(3)-3c}{4b}$. Finally, $\Pi(k,n)=\frac{(a-2C(3)+c)^2}{16b}$ of which each member receives a share $\lambda_i(3)$: $\Pi_i(3,4)=\lambda_i(3)\Pi(3,4)=\lambda_i(3)\frac{(a-2C(3)+c)^2}{16b}$ and $P(3,4)=\frac{(a+2C(3)-3c)^2}{64b}$, which requires $C(3)\geq\frac{-a+3c}{2}$.

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