# Is the French mobile phone cartel really a cartel?†

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**Abstract**. France Telecom (FT), SFR and Bouygues Telecom (BT) have been fined by France's Conseil de la Concurrence (CC) for organizing a mobile phone cartel with stable market shares (one-half, one-third and one-sixth respectively) and for directly exchanging commercial information. While not contesting the legal decision, it is argued here that the economic reasoning is flawed. 1) As the CC made much of the firms' stable market shares, we have first followed this line of reasoning by considering that the market shares are quotas under uniform costs. Even if there is a general incentive to form a monopolistic cartel, BT was too small for it to be worth its while to join it; it is not necessary to exchange information directly to coordinate market shares and prices effectively; all partial cartels are unlikely. 2) We then considered that the non-uniform market shares are explained by the costs in Cournot competition which can be deduced from the observed market shares by assuming that the costs are kept the same when switching from Cournot competition to any form of cartel. We deduced that market shares cannot be other than stable and non-uniform; any monopoly is unlikely to come about, because FT has negative incentives to form a monopolistic cartel; no partial cartels of two operators are viable because at least one member would lose out. The paper also shows that Stackelberg competition is unlikely as well as Bertrand-Edgeworth competition. In conclusion, Cournot competition is the only arrangement that guarantees no losses to all operators.

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# 1. Introduction

France Telecom (later, Orange), SFR and Bouygues Telecom, the three mobile phone operators (GSM) in France, have recently been fined[1] for forming a cartel[2] to maintain high prices and stable market shares, in proportions of roughly one-half for France Telecom, one-third for SFR and one-sixth for Bouygues Telecom for having exchanged data.[3] The purpose of this paper is not to discuss the truth of the evidence laid before the French anti-trust watchdog, the *Conseil de la Concurrence*[4]—that is no subject for an economics paper—but to discuss the economic plausibility of the claims: the economic arguments that will be set out in this paper[5] seem to have been missed by operators and by the *Conseil de la Concurrence* alike.

The *Conseil de la Concurrence*'s arguments turn on there being stable market shares, which is judged to be evidence of collusion. The implication is that market shares are purely exogenous quotas and are independent of production costs. This in turn suggests that the production costs of

[1] The case is not closed: after an unsuccessful appeal, the three operators were fined a total of €442 million; see the decision of the *Cour d'Appel de Paris* (2006). The operators went to the French supreme court; see the decision of the *Cour de Cassation* (2007). The *Cour de cassation* has referred the point about the exchange of information, for which the fine was €92 million, back to a second appellate court. That decision is pending.

[2] In France, the term 'cartel' is often replaced by the vaguer term 'entente'. An entente may be implicit, so opening the way to systematic fines for any group of firms practicing the same prices. However, in most sectors of the French economy, prices are uniform without triggering off legal proceedings.

[3] An historical explanation for unequal market shares is that France Telecom, which historically had a monopoly of wired telephone services, was well established in the telephone market before the *GSM* (Global System for Mobile communications) was devised; SFR joined the market at that time and Bouygues Telecom later. This suggests a dynamic side to the question which is ignored here: this would entail consideration of the market, demand and its inertia, etc. Like many papers on cartels, the focus here is on the supply side alone.

[4] The *Conseil de la Concurrence* is an independent administrative agency, specialized in preventing anti-competitive practices and in investigating how markets operate. Its job is to police the economic sector and ensure consumers have access to the broadest possible range of goods and services at competitive prices as a result of unrestricted operation of the market (see *Conseil de la Concurrence* 2008).

[5] This paper makes no comment on the legal aspects of the issue.

the three operators are equal. This is a very common hypothesis in theoretical studies of cartels.[6] On the basis of this hypothesis, we will discuss: (i) whether there is any point in forming a cartel with stable market shares—with a noticeable result: Bouygues Telecom was too small to have anything to gain from joining a cartel; (ii) whether such market shares can be stabilized and; (iii) whether there is any advantage in directly exchanging information. This will form the second section.

The *Conseil de la Concurrence*'s implicit assumptions of exogenous quotas and uniform costs will be challenged by considering that costs account for market shares, so allowing costs to be inferred from market shares. By making three assumptions, namely: (i) The unit costs are fixed; this hypothesis is particularly appropriate for the telecommunication sector where both decreasing returns and increasing returns can be suspected.[7] (ii) The quotas under the cartel are equal to market shares in Cournot competition. (iii) The costs do not change between Cournot competition and a monopolistic cartel, some disturbing arguments can be proved: principally, that the cartel was not in the best interests of France Telecom. This will form the third section.

The first section is this introduction while the fourth section is the conclusion and the fifth section is an annex that recalls the models for Cournot competition, monopolistic cartels, and partial cartels with a Cournot competitive fringe.

## 2. On the *Conseil de la Concurrence*'s arguments: uniform costs

Even if it has been shown that collusion cannot be identified by examining price and quantity data alone (Kühn 2001; Harrington 2008), the operators have been fined, among other things, for stabilizing their market shares at one-half, one-third and one-sixth for France Telecom,

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[6] Usually, costs are assumed to be equal because firms are numerous. In this paper, we posit that unit costs are fixed and the inverse demand function is linear; the model of competition is the very familiar Cournot oligopoly.

[7] For example, if $n$ is the number of individuals that phone, the number of possible pairs is equal to $\frac{1}{2}n(n-1)$; hence, the investments, to serve a market that approximately grows with $n^2$, depend largely on $n$ for the terminals but the depend on $n^2$ for the switches. On the other hand, serving all the population obliges to invest far from the center of the country, in the countryside, with decreasing returns. These remarks, that could be the basis of a complete paper, are beyond the scope of this one.

SFR and Bouygues Telecom, respectively.[8] The level of costs are not discussed by the Conseil de la Concurrence: if costs are unknown, it is better to assume, at least implicitly, that they are uniform, following the Bernoulli-Laplace principle. If costs are uniform, unequal market shares are in themselves a sign of collusion, because one might expect each firm would naturally tend to become as big as the others.[9] This implies that the non-uniform market shares are pure quotas, exogenously given by the cartel, independently of any other considerations. The implication is that the quotas are independent of the costs, that is, we posit the following assumption so as to follow the *Conseil de la Concurrence*'s line of argument:

**Hypothesis 1.** The quotas in monopolistic cartel are exogenously given and the unit costs are uniform ($c_i = c$ for all *i*, where $c_i$ is the unit cost of operator *i*, with $i = 1$ for France Telecom, $i = 2$ for SFR and $i = 3$ for Bouygues Telecom, and *c* is the uniform unit cost).

This hypothesis is not completely satisfactory, and we will challenge it in the next section, but it allows to be close to the Conseil de la Concurrence (implicit) way of reasoning.

We assume also the inverse function of demand to be linear. In order to compare with the possible cartels we consider the most common model of oligopolistic competition, Cournot competition, recalled in annex; see Bloch (2002).[10] This model is symmetric what is in conformity with Hypothesis 1: under this hypothesis, following equations (5) and (6), in Cournot competition all firms have the same size $\overline{q}_i = \frac{a-c}{4b}$ for all *i* and the same market share of one-

[8] Such cases are referred to as collusive networks by Bellaflamme and Bloch (2004); here we ignore the network side of the question because of the small number of firms.

[9] Quota coordination is discussed later: it is very difficult to achieve in practice.

[10] It could be useful to remind that the literature considers three forms of oligopoly: (i) Cournot-Nash: the variable is the output; both firms are followers. (ii) Stackelberg: the variable is the output; one is follower, one is leader; if both are followers, the duopoly degenerates into a Cournot-Nash duopoly; if both are leaders (Bowley's hypothesis), an indefinite struggle begins; this form of oligopolistic competition is clearly asymmetric. (iii) Bertrand-Edgeworth: the variable is the price, the quantities are given; the prices fall up to the marginal cost as in perfect competition unless the firms understand that it is better not to begin the price war, what is the case in France's mobile telephony market.

third. The choice of Cournot competition implies that the price is a dependant variable and that competitors fight on quantities rather than on prices.[11]

Stackelberg competition could be also discussed. A firm is the leader: logically, the leader should be France Telecom, the "historical" state-owned operator, first settled in the market (the possibility for SFR or Bouygues Telecom to be a leader is absolutely not realistic) because it is in contradiction with the Hypothesis 1; the model is exposed in annex. Unlike Cournot competition, Stackelberg competition does not lead to uniform market shares when the costs are uniform:[12] equation (6) indicates that if $c_i = c$ for all $i$, the market shares are of three-fifth for France Telecom, one-fifth for SFR as well as for Bouygues Telecom. However, Stackelberg competition is not completely coherent in the context of uniform costs of Hypothesis 1: it is hard to assume that one operator is leader if it has the same costs than the others. Bertrand-Edgeworth competition is also not considered because it conducts prices to be equal to the marginal costs, that is, perfect competition.[13] However, it is not realistic to assume that real prices are equal to the marginal cost in France's mobile telephony market; the proof is simple: the French government thinks to add a fourth operator in order to increase competition; he won't do that if perfect competition were firmly settled among the three operators; the fear of price war –which cannot be considered as a collusive behavior *per se*– has probably generated similar but high prices.[14]

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[11] Even if the paper is not game-theory orientated, it should be noticed that the game between operators is essentially static here but it is known that in Cournot competition, an infinitely repeated Cournot game give the same Nash equilibrium than the repetition of a single-period Cournot game (Kip Viscusi et al., 1995, chap. 5). Additionally, we do not consider that payoffs may vary over time as in a supergame (Friedman 1986).

[12] As underlined before, Stackelberg competition which assumes that one firm is leader is not really compatible with the hypothesis of uniform costs. However, equality of costs is now only a reference, not a true hypothesis unlike in the section 2.

[13] Notice that Hypothesis 1 prevents the quantities to be determined under Bertand-Edgeworth competition because marginal costs are constant.

[14] Edgeworth price cycles are always possible in a dynamic model (Maskin and Tirole 1988; Eckert and West 2004). However, considering the objective of the paper, discussing Conseil de la Concurrence's decision, we remain, at least in the context of this paper, in the Conseil de la Concurrence's static perspective.

Nonetheless, Hypothesis 1 implies the following result:

**Result 1.** If Hypothesis 1 holds, there is a general incentive to form a monopolistic cartel by respect to Cournot competition.

Proof. Denote by $\Pi(3)$ the total profit with a monopolistic cartel and $\overline{\Pi}(3)$ the total profit with Cournot competition. As $c_i = c$ for all $i$ we have $\Pi(3) = .25\frac{(a-c)^2}{b}$ following (16) and $\overline{\Pi}(3) = .1875\frac{(a-c)^2}{b}$ following equation (8) in the annex.[15] Hence $\frac{\Pi(3)}{\overline{\Pi}(3)} = \frac{16}{12} > 1$ which is always true, whatever *a*, *b* and *c* are; there is no need to posit $\lambda_i(3) = \frac{1}{3}$ for all $i$ to obtain this result. •

We will demonstrate that it was probably not in the cartel members' interests to stabilize market shares at a non-uniform level, and particularly for those whose share deviated from one-third.

## *2.1. The point in forming a cartel with stable market shares*

**Result 2.** If Hypothesis 1 holds, all three operators must have a market share of more than one-quarter to make the monopolistic cartel attractive; Bouygues Telecom is under this limit and the monopolistic cartel is unlikely.

Proof. Denote by $\overline{\Pi}_i(3)$ and $\Pi_i(3)$ the profit of operator $i$ in Cournot competition and in a monopolistic cartel. As $c_i = c$ for all *i*, we have $\overline{\Pi}_i(3) = .0625\frac{(a-c)^2}{b}$ for all $i = 1,...,3$ following (7) and $\Pi_i(3) = \frac{\lambda_i(3)}{4}\frac{(a-c)^2}{b}$ following (15). So $\Pi_i(3) > \overline{\Pi}_i 3 > 1 \Leftrightarrow \lambda_i(3) > \frac{1}{4}$ whatever *a*, *b* and *c* are; this always holds if $\lambda_i(3) = \frac{1}{3}$ for all *i*. •

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[15] So that results can be compared easily, profits are all expressed in terms of decimal fractions of $\frac{(a-c)^2}{b}$ although they could be given in terms of rational fractions.

As $\Pi_1(3) = .125\,\frac{(a-c)^2}{b}$, $\Pi_2(3) = .0833\,\frac{(a-c)^2}{b}$ and $\Pi_3(3) = .0417\,\frac{(a-c)^2}{b}$ following (15), France Telecom has an incentive of 100%, measured by the ratio $\frac{\Pi_i(3) - \overline{\Pi}_i 3}{\overline{\Pi}_i 3}$, to join a monopolistic cartel, SFR of 33% and Bouygues Telecom of -33%. Notice that $\Pi_1(3) > \Pi_2(3) > \overline{\Pi}_i(3) > \Pi_3(3)$.

Hence, it is not in Bouygues Telecom's best interests to join the monopolistic cartel: the whole case collapses! However, it must be said that membership of a cartel prevents it from increasing its market share from one-sixth toward one-third: its market share cannot grow and Bouygues Telecom is frozen by the cartel. However, it is fair to say that the cartel prevents it from being driven out of the market.

France Telecom has a strong incentive to join a monopolistic cartel. However, there are some other arguments. A cartel could prevent France Telecom's market share from falling to one-third but its strong position would allow it to enjoy lower costs than its competitors, thereby allowing a degree of price competition to check this movement. Arguing that France Telecom might have accepted a market share of one-half only to avoid anti-trust actions from the regulation authority (ARCEP[16]) for having eliminated Bouygues Telecom is untenable because a regulation authority is there to prohibit anti-competitive behavior.

With a market share of one-third, SFR is rather in a neutral position, even if it has positive incentives; the main advantage of a cartel is that it may seem to bring stability (if it is itself stable).

From all the above developments it can be deduced that the existence of stable market shares is not proof of the existence of a cartel in the French mobile phone market. However, the possibility that they truly did form a cartel is not ruled out: they could have formed a coalition even if it were not to their advantage to do so, say through ignorance or out of fear of competition: both are irrational arguments, something that invariably troubles economists. Is irrationally a mitigating circumstance? That is for the *Conseil de la Concurrence* to decide!

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[16] *Autorité de Régulation des Communications Electroniques et des Postes*, the Electronic Communications and Postal Service Regulation Authority.

In their defense, the operators argued that their market shares were highly variable in the short term. It is not a bad argument in itself but the fact is that the three operators could have developed fierce competition among themselves so generating variable market shares or they could equally have maneuvered to make the market shares variables in order to make the detection of the cartel difficult.

## *2.2. Can market shares be stabilized really?*

In a general cartel, equations (9) to (12) indicate that the market shares depend on the marginal costs, which are generally considered stable in the short term: the market shares are stable what is considered as a sign of collusion. It an erroneous idea because the market shares can be stable even in other market organizations, like Cournot competition: in this case, it is sufficient to have stable costs!

In a cartel where market shares are exogenous quotas, it is much more complicated. Equations (14) and (15) indicate that the profits are shared proportionally to market shares in the monopolistic cartel. One may wonder how a firm can make its market share lower when it is too high. By refusing to sell goods and services? That is prohibited by commercial legislation. By breaking off contracts unilaterally? That is prohibited in contract law. By refusing to extend a contract that comes to an end? That cannot be a serious suggestion because of the amplified adverse effects on the operator's image. By lowering the quality of their service so as to discourage customers from renewing their contracts? That cannot be a serious suggestion either for the same reasons but also because all customers would be affected. By cutting back on advertising? There is no way to be sure about the effect it will have and it takes time, as advertising is itself contractual (except when the operator's website is the advertising medium). The only available means is to manipulate prices for new customers, without affecting prices for existing customers. The operators have two options: they can either raise the price of the contracts themselves, or they can raise the price of the "contract + mobile" packages (and generally, by offering less attractive mobile phones).

Now, how can operators make their market shares higher when they are too low? Generally, there are two means to sell more: by cutting prices, but this is dangerous as it could

lead to a price war; or by advertising more, but again the effect is uncertain and it could take time (a month or so) except for do-it-yourself advertising.

Overall, pricing is the easiest and most effective way to increase or decrease market share. But no allegations of price manipulation, with similar or stable prices, or even equal prices, were made as evidence of a hidden agreement between France Telecom, SFR and Bouygues (while the operators have argued that prices fell: this point is debated, depending of which product is considered: communications, SMS, MMS, contracts, etc.).[17] Again this is an obvious flaw in the prosecution's case.

### *2.3. Is the direct exchange of information a valid argument?*

The investigations revealed that the three operators communicated directly about monthly market shares, which is *per se* proof of overt collusion. Whether this is true or not is of no concern to us here. Our economic question is whether or not direct communication is useful. For the *Conseil de la Concurrence*, the operators disclosed information to each other so as to stabilize their market shares. Let us turn the question around: in order to stabilize the market shares of three operators in a cartel, is the direct exchange of information necessary?

Kühn and Vives (1995), Carlton, Gertner and Rosenfield (1997) and Kühn (2001) have examined the question of whether an information exchange is a violation of antitrust laws in itself; mainly, information exchanges that are not concerned with the future conduct of firms must not be taken as an evidence of a cartel, while information exchanges have important efficiency effects, principally in vertically related markets (Caffarra and Kühn 2006).

Beyond that, collusion may, of course, be tacit. If we consider the prisoner's dilemma, in the case of an infinitely repeated game, it is well known that, since firms *A* and *B* both know that they will earn less by cheating than by playing fair, neither will begin to cheat because the other would then do so: both will opt to be loyal. Even so, this means the firms must be sufficiently informed to avoid being involuntarily aggressive: each firm must observe the other, which is akin to the idea of barometric firm. The three operators are (probably) cognizant with economic theory; they are (probably) aware that price competition, *à la* Bertrand-Edgeworth, leads to price

[17] SMS: Short Message Service; MMS: Multimedia Message Service.

war. Consider a duopoly with two equivalent firms that have the same price level above the marginal cost. If one firm cuts its price to just a little below the common price, the other will follow suit: both lose out; the game will be repeated and the price war will continue until the marginal cost is reached: this is known in the economic literature as the *Bertrand paradox*. However, because each firm knows this story, it can be expected that prices will tend to remain equal and stable, with no price war, and at a level well above the marginal cost; this does not need any direct communication between the firms, that is, no explicit collusion is required. Consequently, high and stable prices are secured by the fear of a price war, which implies that stable and high prices are perfectly acceptable in a duopoly: there is nothing wrong about being scared.

A good distribution network—dealers, branded stores, etc.—can rapidly alert the marketing department by means of the computerized reporting tools now available about price changes on contracts of other operators or about the success or failure of a new mobile phone package; it is easy enough to adapt prices quickly. So, for an operator, it is easy to observe market prices and then to change its own prices quickly to keep pace with other operators, if necessary, with no need for direct communication: imitation is sufficient to explain why the three agents may have had the same pricing behavior. To take a concrete example, it has been proved that the operators favored contracts over pay-as-you-go cards or all decided to bill per 30-second increments after a minimum first minute (*Conseil de la Concurrence* 2005b). There is no need for direct disclosure of information to imitate a competitor's price strategy. Even without directly exchanging information, it takes just a day for an operator to become aware of the new strategy adopted by another operator, just a day or two to centralize the information and decide what to do, and just a week to set a new price (and put it into the computerized contract system so it is operationally available to dealers) and somewhere between one week to one month to organize a communication campaign depending on the advertising medium... In this context, exchanging information directly seems a minor sin!

### *2.4. Partial cartels under uniform costs*

It is very interesting to examine whether Cournot competition may develop into a partial cartel of two operators against the third or whether the monopolistic cartel is able to degenerate

into such a partial cartel. The partial cartel model is set out in the annex. The following result obtain:

**Result 3.** Under uniform costs (Hypothesis 1), no partial cartels are viable because at least one member would lose out!

Proof. There are three operators, two in the partial cartel and one outsider. Following the model in the annex, the cartel's profit is $\Pi(2,3)=.125\frac{(a-c)^2}{b}$ to be shared following the quotas $\lambda_i(2)$ which are proportional to the market shares in Cournot competition or in a monopolistic cartel; the outsider's profit is $P(2,3)=.5\frac{(a-c)^2}{b}$. See Table 1, Table 2 and Table 3 that indicate the result for each of the three possible partial cartels in the event of Cournot competition and of the monopolistic cartel. •

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | Bouygues Telecom | France Telecom $\lambda_1(2)=\frac{3}{5}$ | SFR $\lambda_2(2)=\frac{2}{5}$ | *Result* |
| Relative to Cournot Competition | 0 | +20.00 | -20.00 | *Unlikely* |
| Relative to the monopolistic cartel | +50.00 | -40.00 | -40.00 | *Unlikely* |

**Table 1.** Gains in the partial cartel of France Telecom and SFR under Hypothesis 1

It is interesting to discuss the partial cartel of France Telecom and SFR. France Telecom's real opponent is Bouygues Telecom mainly because France Telecom's market share has fallen to one-half because of Bouygues' market entry: for France Telecom, it would be irrational to form a cartel with Bouygues Telecom. However, SFR would be interested in joining a partial cartel of two with France Telecom only to increase its market share from one-third to, say, one-half, which would lead to Bouygues Telecom's elimination. In this case, what is the point for France Telecom in forming a cartel with SFR if eliminating Bouygues Telecom benefits SFR exclusively? France Telecom might accept to form a cartel with SFR if it were to gain some of the market share left open by Bouygues Telecom's demise, for example by going to a market share of $\frac{1}{2}+\frac{1}{12}=\frac{7}{12}$, and SFR obtaining a market share of $\frac{1}{3}+\frac{1}{12}=\frac{5}{12}$ (assuming they divide up the

spoils from Bouygues Telecom equally, that is $\frac{1}{12}$ each, in a Nash solution). However, $\frac{7}{12}$ and $\frac{5}{12}$ are not far from $\frac{1}{2}$: the situation would still be similar to a perfect duopoly, which is of no interest to France Telecom, as it is always preferable for a firm that dominates half of the market to face two weak competitors (SFR with one-third and Bouygues Telecom with one-sixth) rather than one strong competitor (SFR with $\frac{5}{12}$). Nonetheless, this reasoning is largely false insofar as Bouygues Telecom is not killed off by the partial cartel of France Telecom and SFR; on the contrary, Bouygues Telecom benefits from being excluded as proved by Table 1![18]

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | SFR | France Telecom $\lambda_2(2)=\frac{3}{4}$ | Bouygues Telecom $\lambda_3(2)=\frac{1}{4}$ | *Result* |
| Relative to Cournot Competition | 0 | +50.00 | -50.00 | *Unlikely* |
| Relative to the monopolistic cartel | -25.00 | -25.00 | -25.00 | *Unlikely* |

**Table 2.** Gains in the partial cartel of France Telecom and Bouygues Telecom under Hypothesis 1

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | France Telecom | SFR $\lambda_2(2)=\frac{2}{3}$ | Bouygues Telecom $\lambda_3(2)=\frac{1}{3}$ | *Result* |
| Relative to Cournot Competition | 0 | +33.33 | -33.33 | *Unlikely* |
| Relative to the monopolistic cartel | -50.00 | 0 | 0 | *Indifferent* |

**Table 3.** Gains in the partial cartel of SFR and Bouygues Telecom under Hypothesis 1

We can also examine what happens when one sole operator is leader and the two others are in the Fringe. However, considering that one operator is leader while it has no cost advantage is

[18] Remark that this particular result partially contradicts those of Eckbo (1976) who shows that the cartel success is increasing with the market share of the cartel (here, it is very large: five sixth).

not completely satisfactory: the leadership is here based only on a mental ascendant. Hence, France Telecom must be logically the leader as it is the "historic" operator, first arrived in the market.

**Result 4.** Under uniform costs (Hypothesis 1), if France Telecom is leader, SFR and Bouygues Telecom followers, France Telecom has three-fifth of the market, SFR and Bouygues Telecom one-fifth. Passing from Cournot competition to this configuration is possible as the leader benefits but passing from the monopolistic cartel to this situation is unlikely.

Proof. See equations (21) to (29).

| Gains in % | Leader | Fringe | | *Result* |
|---|---|---|---|---|
| | France Telecom | SFR | Bouygues Telecom | |
| Relative to Cournot Competition | +33.33 | -55.55 | -55.55 | *Possible* |
| Relative to the monopolistic cartel | -33.33 | -66.66 | -33.33 | *Unlikely* |

**Table 4.** Gains in the partial cartel of SFR and Bouygues Telecom under Hypothesis 1

# 3. Going further: unequal endogenous costs

So as to follow the *Conseil de la Concurrence*'s reasoning, we considered above that the non-uniform market shares were exogenous quotas, decided on by the cartel, independently of unit costs; these were implicitly equal. We shall now demonstrate, though, that non-uniform market shares can be endogenously determined by the non-uniform structure of production costs. This will allow us to demonstrate some surprising new results. We will consider first Cournot competition and then Stackelberg competition shortly in order to demonstrate that this one is unlikely in France's mobile phone market, while Bertrand-Edgeworth is discarded for the reason explained in the previous section.

## *3.1. Endogenous costs deduced from Cournot competition*

### 3.1.1. Deducing the unit costs from the market shares

The above developments raise the question of where these potentially unequal quotas come from? If the three firms were identical, and if Cournot competition were perfect, with a homogenous product, each of the three firms in the oligopoly would tend to have one-third of the market (there is no reason why anyone firm should prevail). Hence, market shares in Cournot competition can only be non-uniform if costs are non-uniform: this allows to deduce the costs from the market shares. Hence, if we abandon the hypothesis of uniform unit costs, we can compute the unit costs that should correspond to the observed market shares of France Telecom, SFR and Bouygues Telecom, namely $\lambda_1(3)=\frac{1}{2}$, $\lambda_2(3)=\frac{1}{3}$ and $\lambda_3(3)=\frac{1}{6}$, when they are in Cournot competition.

**Result 5.** The unit costs of SFR and Bouygues Telecom are the following when they are expressed in terms of France Telecom's unit cost in Cournot competition:

(1) $c_2=\frac{1}{9}(a+8c_1)$ and $c_3=\frac{1}{9}(2a+7c_1)$

Proof. Writing formula (6) in the annex for each of the three operators and positing $\lambda_1(3)=\frac{1}{2}$, $\lambda_2(3)=\frac{1}{3}$ and $\lambda_3(3)=\frac{1}{6}$ yields the following system of equations:

$$\left\{\frac{a-4c_1+\sum_{j=1}^{3}c_j}{3a-\sum_{j=1}^{3}c_j}=\frac{1}{2};\frac{a-4c_2+\sum_{j=1}^{3}c_j}{3a-\sum_{j=1}^{3}c_j}=\frac{1}{3};\frac{a-4c_3+\sum_{j=1}^{3}c_j}{3a-\sum_{j=1}^{3}c_j}=\frac{1}{6}\right\}$$

This system is under-determined. One variable must be chosen as a parameter: we choose $c_1$ the costs of France Telecom, the "historical operator", to deduce $c_2$ and $c_3$, which leads to (1). •

Notice that it is only when $c_1=a$ that $c_2$ and $c_3$ are equivalent to $c_1$: it is necessary to have $c_1<a$. The mean cost is $\frac{1}{3}\sum_{i=1}^{3}c_i=\frac{1}{9}(a+8c_1)$ while the weighted mean cost, which will be useful later, is:

(2) $\sum_{i=1}^{3}\lambda_i(3)c_i=\frac{1}{2}c_1+\frac{1}{3}c_2+\frac{1}{6}c_3=\frac{1}{27}(2a+25c_1)$

This is always lower than the mean cost. SFR, which has the neutral market share of one-third, has a unit cost that is unsurprisingly equal to the mean cost; Bouygues Telecom' is higher, France Telecom's is lower: $c_1 < c_2 = \frac{1}{3}\sum_{i=1}^{3} c_i < c_3$. This is depicted in Figure 1. By substituting equation (1) in (7), one deduces that, France Telecom has $\frac{9}{14}$, SFR $\frac{4}{14}$ and Bouygues Telecom $\frac{1}{14}$ of the total profit under Cournot competition what is rather realistic.

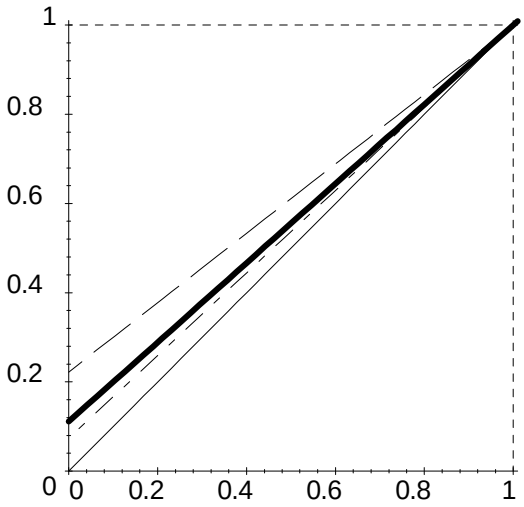


**Figure 1.** $c_1$ (thin line), $c_2$ and mean cost (thick line), $c_3$ (dashed line) and mean weighted cost (dotted-dashed line) relative to $c_1$ for $a = 1$ (and $c_1 \leq a$).

**Result 6.** If the costs are unequal and endogenous (equation (1)), the market shares in Cournot competition cannot be anything else than stable in the short term, without requiring any collusion.

The outputs follow form a Nash equilibrium. There is no reason explaining why a produced would divert from this equilibrium if the all costs are themselves stable. Moreover, if we look at equation (4), we see that the operators need some information from their competitors under the form of the market size: this even if they are in Cournot competition. These argument are close to those of Phlips cited by Dang Nguyen (1995, p. 286-290) for a Nash competitive equilibrium, even if Phlips thinks in terms of competition on prices on the case of the paper pulp imported in the CEE between 1975 and 1981.

Proof. Following equation (1), the market shares depend exclusively on the unit costs. It is reasonable to consider that the costs vary slowly; hence the market shares vary slowly also and seem stable over time; there is nothing wrong in that. •

### 3.1.1. Monopolistic cartels under unequal endogenous costs

Two assumptions may be made:

**Hypothesis 2.** The fixed quotas are equal to the non-uniform market shares ($\lambda_1(3)=\frac{1}{2}$, $\lambda_2(3)=\frac{1}{3}$ and $\lambda_3(3)=\frac{1}{6}$) the operators enjoyed before joining the cartel, that is, in Cournot competition as given by equation (1).[19]

This hypothesis is logical: the cartel members allocate the quotas depending on what they observed before forming the cartel. The existence of quotas, that is, the stable market shares, is one of the charges held against the operators by the *Conseil de la Concurrence*. If a stable monopolistic cartel exists on the basis of unequal market shares, then the members must have accepted to stabilize their market shares at the level they were at before forming the cartel, in Cournot competition. This suggests that differences between quotas in a monopolistic cartel are brought about by differences in unit costs in Cournot competition if costs do not change during the passage from competition to monopolistic cartel: we posit an additional assumption.

**Hypothesis 3.** Unit costs do not change individually when operators pass from Cournot competition to a cartel.

These two hypotheses allow deriving the results for the monopolistic cartel from equations (13) to (16).

**Result 7.** Assume that Hypothesis 2 and Hypothesis 3 hold. The monopolistic cartel makes more profit by using the differentiated quotas ($\frac{1}{2},\frac{1}{3},\frac{1}{6}$) than by using uniform ones ($\frac{1}{3},\frac{1}{3},\frac{1}{3}$).

Strong suspicions of collusion would arise in this case.

Proof. Using uniform quotas ($\frac{1}{3},\frac{1}{3},\frac{1}{3}$) amounts to using the mean cost instead of the weighted mean cost: the total profit is equal to $\Pi^u(3)=.1975\frac{(a-c_1)^2}{b}$ ; hence, $\Pi(3)=\frac{625}{576}\Pi^u(3)$.

However, France Telecom is an exception that challenges the above result, as shown by the following result:

---

[19] Osborne (1976a) discusses unequal market shares in general terms. This paper caused some controversy: see Mills & Elzinga (1976), Holahan (1976), Osborne (1976b).

**Result 8.** Consider that Hypothesis 2 and Hypothesis 3 hold. In switching between Cournot competition and a monopolistic cartel, France Telecom's profit falls by 3.55%, SFR's profit grows by 44.68% and Bouygues Telecom's by 189.35%: France Telecom has negative incentives to form a monopolistic cartel.

Proof. Substituting (2) into (16) gives $\Pi(3) = .2143\frac{(a-c_1)^2}{b}$; this aggregate profit is shared between the operators in accordance with the quotas $\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$. Hence: $\Pi_1(3) = .9645\,\overline{\Pi}_1(3)$, $\Pi_2(3) = 1.4468\,\overline{\Pi}_2(3)$ and $\Pi_3(3) = 2.8935\,\overline{\Pi}_3(3)$. •

If France Telecom has negative incentives to form a monopolistic cartel, no monopolistic cartel can come about! The whole case collapses, in spite of all other arguments, unless France Telecom is considered non-rational. However, this idea logically leads us to examine the possibility of a partial cartel of two operators, SFR and Bouygues Telecom, leaving France Telecom out in the cold. This is examined in the next subsection.

### 3.1.2. Partial cartels under unequal endogenous costs

**Result 9.** Under Hypothesis 2 and Hypothesis 3, no partial cartels of two operators against the third one are viable because at least one member loses out!

Proof. Two operators are in the partial cartel facing one outsider. The quotas are proportional to the market shares in Cournot competition or in a monopolistic cartel. See Table 5, Table 6 and Table 7 that indicate the result for the insiders; $\Pi(2,3)$ denotes the cartel's profit and $P(2,3)$ the outsider's profit. •

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | Bouygues Telecom | France Telecom $\lambda_1(2)=\frac{3}{5}$ | SFR $\lambda_2(2)=\frac{2}{5}$ | *Result* |
| Relative to Cournot Competition | -9.75 | -13.30 | +30.05 | *Unlikely* |
| Relative to the monopolistic cartel | -10.11 | -10.11 | -68.81 | *Unlikely* |

**Table 5.** Gains in the partial cartel of France Telecom and SFR under Hypothesis 2 and Hypothesis 3

$$\Pi(2,3)=.1606\,\frac{(a-c_1)^2}{b} \text{ and } P(2,3)=.0111\frac{(a-c_1)^2}{b}$$

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | SFR | France Telecom $\lambda_1(2)=\frac{3}{4}$ | Bouygues Telecom $\lambda_3(2)=\frac{1}{4}$ | *Result* |
| Relative to Cournot Competition | -23.44 | -15.63 | +153.13 | *Unlikely* |
| Relative to the monopolistic cartel | -47.08 | -12.52 | -12.52 | *Unlikely* |

**Table 6.** Gains in the partial cartel of France Telecom and Bouygues Telecom under Hypothesis 2 and Hypothesis 3

$$\Pi(2,3)=.125\,\frac{(a-c_1)^2}{b} \text{ and } P(2,3)=.0378\frac{(a-c_1)^2}{b}$$

| Gains in % | Fringe | Partial cartel | | |
|---|---|---|---|---|
| | France Telecom | SFR $\lambda_2(2)=\frac{2}{3}$ | Bouygues Telecom $\lambda_3(2)=\frac{1}{3}$ | *Result* |
| Relative to Cournot Competition | -5.48 | -16.44 | +67.13 | *Unlikely* |
| Relative to the monopolistic cartel | -2.00 | -56.68 | -42.24 | *Unlikely* |

**Table 7.** Gains in the partial cartel of SFR and Bouygues Telecom under Hypothesis 2 and Hypothesis 3

$$\Pi(2,3)=.0619\,\frac{(a-c_1)^2}{b} \text{ and } P(2,3)=.105\frac{(a-c_1)^2}{b}$$

In the partial cartels, two firms are leader and the third one is follower. Why not to examine what happens if France Telecom adopts the behavior of leader while SFR and Bouygues Telecom become followers, knowing that the role of leader is not for SFR and Bouygues Telecom alone.

**Result 10.** Consider that Hypothesis 2 and Hypothesis 3 hold. If France Telecom is leader while SFR and Bouygues Telecom are followers, then Bouygues Telecom is eliminated while SFR has one-seventh of the market and one-thirteenth of the total profit only.

Proof. Equations (21) to (29) in annex combined with equation (1) show that $\hat{q}_1 = \frac{2}{3}\frac{a-c_1}{b}$, $\hat{q}_2 = \frac{1}{9}\frac{a-c_1}{b}$ and $\hat{q}_3 = 0$ while $\hat{\Pi}_1 = \frac{4}{27}\frac{(a-\hat{c}_1)^2}{b}$, $\hat{\Pi}_2 = \frac{1}{81}\frac{(a-\hat{c}_1)^2}{b}$ and $\hat{\Pi}_3 = 0$, the hat denoting variables in Stackelberg competition. •

This leads to examine Stackelberg competition in the next section as it has been done for Cournot competition in this section.

## *3.2. Endogenous costs deduced from Stackelberg competition when France Telecom is the sole leader*

Stackelberg competition seems a priori compatible with non-uniform market shares: France Telecom is naturally the leader, while SFR and Bouygues Telecom are the followers in the competitive fringe. Hence, it is conceivable to deduce the costs from Stackelberg competition when France Telecom is leader, as done above from Cournot competition.

### 3.2.1. Deducing the unit costs from the market shares

We compute the unit costs that should correspond to the observed market shares of France Telecom, SFR and Bouygues Telecom, namely $\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$, if they are in Stackelberg competition dominated by France Telecom.

**Result 11.** If France Telecom is Stackelberg leader and SFR and Bouygues Telecom followers, the unit costs of SFR and Bouygues Telecom are the following when they are expressed in terms of France Telecom's unit cost:

(3) $\quad c_2 = \frac{1}{7}\left(-a + 8c_1\right)$ and $c_3 = c_1$

The cost functions are depicted in Figure 2; notice that if $c_1 = a$, then $c_3 = a$ and $c_2 = a$ also: it is necessary to have $c_1 < a$.

Proof. Writing formulae (24), (25) and (26) in the annex for each of the three operators and positing $\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$ yields the following system of equations:

$$\left\{3\frac{a - 3\hat{c}_1 + \hat{c}_2 + \hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3} = \frac{1}{2}; \frac{a + 3\hat{c}_1 - 5\hat{c}_2 + \hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3} = \frac{1}{3}; \frac{a + 3\hat{c}_1 + \hat{c}_2 - 5\hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3} = \frac{1}{6}\right\}$$

This system is again under-determined: we choose again $c_1$ as parameter to deduce $c_2$ and $c_3$, what leads to (3). •

**Corollary 1.** Stackelberg competition, with France Telecom as leader, SFR and Bouygues as followers and costs deduced from the observed market shares (equation **(**3), is unlikely.

Proof. SFR's unit costs are below France Telecom's and Bouygues Telecom's. This surprising result can be explained easily: in Stackelberg competition, SFR should have a relatively low market share. As it has a rather large market share for a follower, the only explanation, following the equations, is that its unit costs are low. However, they are so low that they are below those of France Telecom, what is not credible at all considering the very large experience effect of which France Telecom benefits and the strong cutbacks in its manpower these last years; moreover, it would be surprising to see a follower to have lower costs than the leader! Furthermore, SFR takes 50% of the total profit, more than France Telecom that has only 37.5% (while Bouygues Telecom has 12.5%). •

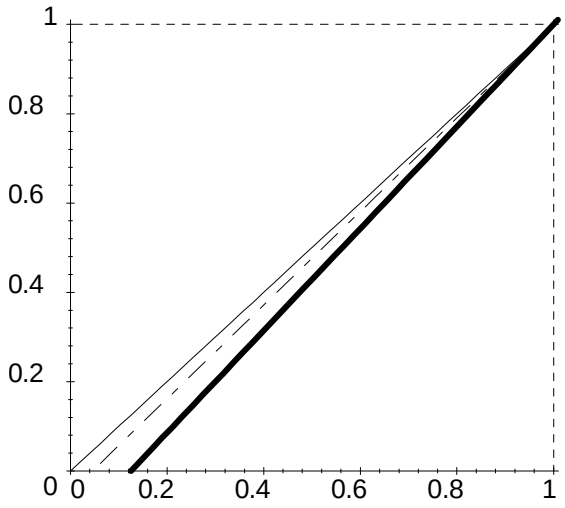


**Figure 2.** $c_1 = c_3$ (thin line), $c_2$ (thick line)
and mean weighted cost (dotted-dashed line) relative to $c_1$
for $a = 1$ (and $c_1 \leq a$).

Nevertheless, it is always possible to examine what should happen if a monopolistic cartel is formed from Stackelberg competition by posing two hypotheses:

**Hypothesis 4.** The quotas in monopolistic cartel are equal to the market shares ($\lambda_1(3) = \frac{1}{2}$, $\lambda_2(3) = \frac{1}{3}$ and $\lambda_3(3) = \frac{1}{6}$) the operators enjoyed before joining the cartel, that is, in Stackelberg competition as given by equation (3).

**Hypothesis 5.** Unit costs do not change individually when operators pass from Stackelberg competition to a cartel.

Again, the cartel members allocate the quotas depending on what they observed before, what allows us to infer immediately that the differences in market shares are caused by differences in unit costs alone, if costs are stable, that is, if we posit again Hypothesis **4** and Hypothesis 5.

**Result 12.** Consider that Hypothesis 4 and Hypothesis 5 hold. In switching between Stackelberg competition and monopolistic cartel, the monopolistic cartel is plausible. All operators' profit makes headway: France Telecom gains 124.07% as Bouygues Telecom but SFR gains only 12.04%.

This result is not surprising as SFR's profit were very high in Stackelberg competition, while France Telecom's were relatively low.

Proof. The mean unit cost *c* is equal to $\frac{22c_1 - a}{21}$. It is sufficient to use equations (15), (27), (28) and (29) and to compute the relative gains. •

We do not go further as Stackelberg competition is unlikely.

# 4. Conclusion

France Telecom, SFR and Bouygues Telecom have been fined for forming a cartel in the mobile phone sector in France on the grounds that (a) their market shares were stable, (b) they directly exchanged commercial information. The *Conseil de la Concurrence* took both arguments as proof of the existence of a three-firm cartel. This papers has not discussed whether France Telecom, SFR and Bouygues Telecom truly formed a cartel and has not discussed the truth of the charges against the three operators: those are matters for the investigators and for the *Conseil de la Concurrence*. However, this paper has examined whether the arguments are economically relevant and has yielded results based on reasonable but simple hypotheses (linear demand, fixed unit costs, uniform costs).

As the *Conseil de la Concurrence* made much of the operators' stable market shares, we have first followed this line of reasoning by considering that the market shares are quotas, which implies that they are independent of costs, implicitly assumed to be uniform. It has been shown that even if there is a general incentive to form a monopolistic cartel, Bouygues Telecom was too small to have any interest in joining it. Hence, it was not in the interests of France Telecom, SFR and Bouygues Telecom to maintain market shares fixed at one-half, one-third and one-sixth respectively in a monopolistic cartel. It has been emphasized that it is not necessary to exchange information directly to coordinate market shares and prices effectively, while stable uniform high prices may simply ensue from the fear of a price war.

We then departed from the point of view of the *Conseil de la Concurrence* to consider that the non-uniform market shares are explained by the costs in Cournot competition. This allowed us to deduce the unit costs from the observed markets shares: we assumed that the costs are kept the same when the operators switch from Cournot competition to any form of cartel. We have

been able to compute the profits in Cournot competition. We deduced that (i) Market shares will be naturally stable, contrary to the *Conseil de la Concurrence*'s opinion, because they depend on costs, which are obviously stable. (ii) The monopolistic cartel makes greater profits by using unequal quotas $(\frac{1}{2},\frac{1}{3},\frac{1}{6})$ than by using uniform shares $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$. (iii) However, the monopoly is unlikely to come about because France Telecom has negative financial incentives to form a monopolistic cartel. This argument is more powerful than for Bouygues Telecom above: if an operator like France Telecom, historically the first and the leading operator, has negative incentives, it can be considered it is impossible to form a monopolistic cartel. (iv) No partial cartels of two operators against the third one are viable because at least one member would lose out. (v) Stackelberg competition with France Telecom as leader, SFR and Bouygues Telecom as followers, is unlikely as well as Bertand competition.

We deduce finally that compared to the monopolistic cartel or to any of the partial cartels, Cournot competition is the only arrangement that guarantees no losses to all operators! Hence, the operators should not have been fined on the basis of arguments (a) and (b) unless it can be proved that their behavior was irrational, which would probably lead to other legal problems. All this shows that anyone accusing the three operators, France Telecom, SFR and Bouygues Telecom, of forming a lasting cartel is treading on thin ice.

# 5. Annex: the models

We assume that all firms produce the same homogenous good: in this instance, it is a mobile phone service. Practically, measuring a market share is not so easy because many criteria can be chosen: number of subscriptions, number of minutes of communications sold, etc. The simple solution is to measure total sales in units of money. What follows is well established but needs to be recalled here at is will serve as reference material for other results.

### *5.1. Cournot competition*

Each of the three firms produces $\bar{q}_i$; their unit costs are constant, that is, the costs are equal to $c_i\,\bar{q}_i$,[20] the total output is $\bar{q} = \sum_{i=1}^{3} \bar{q}_i$; the inverse demand function is assumed to be linear: $\bar{p}(\bar{q}) = a - b\,\bar{q}$. Each firm *i* earns a profit of $\bar{\Pi}_i = \bar{p}(\bar{q})\,\bar{q}_i - c_i\,\bar{q}_i$; this gives $\bar{\Pi}_i = (a - c_i)\,\bar{q}_i - b\,\bar{q}_i^2 - b\,\bar{q}_i \sum_{j=1,\,j\neq i}^{3} \bar{q}_j$. The model is the Cournot oligopoly: each firm *i* maximizes its profits, $\bar{q}_j$ being given, that is, $\frac{\partial \bar{\Pi}_i}{\partial \bar{q}_i} = 0 \Leftrightarrow \bar{q}_i = \frac{a - c_i}{2\,b} - \frac{\sum_{j=1,\,j\neq i}^{3} \bar{q}_j}{2}$ for all *i*. Given that $\sum_{j=1,\,j\neq i}^{3} \bar{q}_j = \bar{q} - \bar{q}_i$, it ensues that:

(4) $$\bar{q}_i = \frac{a - c_i}{b} - \bar{q}$$

If we sum this expression over *i,* we obtain the market size which is $\bar{q} = \sum_{i=1}^{3} \bar{q}_i : \bar{q} = \frac{3a - \sum_{j=1}^{3} c_j}{4\,b}$. To prevent market size being negative it is necessary to impose $a \geq \frac{1}{3} \sum_{i=1}^{3} c_i$; by substitution, it turns out that this constraint is equivalent to $c_i \leq a$. By substituting into the inverse demand function, the price follows: $\bar{p} = \frac{a + \sum_{i=1}^{3} c_i}{4}$. Then, substituting $\bar{q}$ into (4) gives:

(5) $$\bar{q}_i = \frac{a - 4\,c_i + \sum_{j=1}^{3} c_j}{4\,b} \text{ for all } i$$

To avoid negative outputs we have to posit $a \geq 4\,c_i - \sum_{j=1}^{3} c_j$, which is also equivalent to $c_i \leq a$. Each firm *i* has a market share equal to:

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[20] Bittlingmayer (1989) discusses of the dramatic effect of fixed costs on competition. Donsimoni et al. (1986) have examined quadratic costs.

(6) $$\lambda_i = \frac{\overline{q}_i}{\overline{q}} = \frac{a - 4\,c_i + \sum_{j=1}^{3} c_j}{3a - \sum_{j=1}^{3} c_j} \text{ for all } i$$

Now we can deduce the profit of each firm *i*:

(7) $$\overline{\Pi}_i = \frac{\left(a - 4\,c_i + \sum_{j=1}^{3} c_j\right)^2}{16\,b} \text{ for all } i$$

the total profit being

(8) $$\overline{\Pi} = \frac{3\left(a - 4\,c_i + \sum_{j=1}^{3} c_j\right)^2}{16\,b}.$$

### *5.2. Monopolistic cartel*

The monopolistic cartel of three operators acts as a monopoly and maximizes the sum of cartel members' profits, the price being the monopoly price. Demand remains $p(q) = a - b\,q$, where *q* is the market size, and *a* and *b* parameters. Usually, each member produces up to the point where its marginal cost equals those of others, the cartel producing a total quantity such that the marginal revenue is equal to the marginal cost. Cartel's profit is $\Pi(q) = p(q)\,q - q\,C(q)$, where $C(q)$ is cartel's cost, equal to the sum of members cost. At the maximum of $\Pi(q)$ one has $R'(q) = C'(q)$ what gives the optimal output *q*:

(9) $$q = \frac{a - C'(q)}{2b}$$

where $C'(q)$ denotes cartel's marginal cost; it is found by first determining the inverse marginal cost functions $q_i = \mathrm{f}_i^{-1}(C_i')$ for $q_i \geq 0$ for all *i*, where $\mathrm{f}_i$ denotes the function of marginal cost of firm *i*, i.e., $C_i' = \mathrm{f}_i(q_i)$, then by positing $C_i' = C'$ for all *i* and finally by summing the functions

(10) $$q_i = \max\left(\mathrm{f}_i^{-1}(C'), 0\right) \text{ for all } i$$

over *i* that is,

(11) $q = g(C') = \sum_i \max\left(f_i^{-1}(C'), 0\right)$

and then by inversing this function: [21]

(12) $C' = g^{-1}(q)$

Each member's output is given by $q_i = f^{-1}(C')$. Notice that coordinating such a cartel is hard: cheating could be a general behavior.

In this paper, each firm has $c_i$ as its unit cost. However, for constant unit costs, marginal costs are constants; hence, it is impossible to compute the inverse of the functions of marginal cost in (10): the members' output is undetermined, as it is always the case when marginal costs are constant. Hence, the way the cartel is presented here must differ a little to what is usual. We assume that the members agree to practice a high price –the monopoly price– with the market shares that are not determined by the cartel's equations, for example, arbitrary markets shares (as in section 2) or the present market shares which come from Cournot competition (as in section 3). The members could go beyond and try to minimize the cartel's marginal cost, that is, to apply the general procedure above but equation (10) indicates that this would oblige the cartel to produce only with the firm which has the lower unit cost (it could be France Telecom, as we will see in section 3), what would mean that the other members are automatically eliminated and that the industry turns out to be an ordinary monopoly! This is traduced by the following hypothesis:

**Hypothesis 6.** The shares of total output are fixed, that is, are pure quotas: each member always produces a fixed share $\lambda_i$ of the monopoly output: $q_i = \lambda_i\, q$. [22]

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[21] To go on the mathematical side of the question, it is not granted that the individual marginal cost function (10) has an inverse: f must be a continuous one-to-one function. For example, if the cost function is quadratic polynomial, the inverse can be computed (it is linear) but it is not always the case when the cost function is polynomial of degree three or if the marginal cost is constant. The inverse of any forms of marginal cost function cannot be derived. Similarly, deriving the global marginal cost function by computing the inverse of (11) could be impossible. When networks effects occur, that is, when the marginal cost of one firm depends on those of the other firms, the difficulty is increased. Moreover, it is not granted that all members enjoy nonnegative profits: the mean cost of some members may be above the price even if the cartel's mean cost is under the price. It is interesting to note that the equations are the same for the multi-plant monopoly: the traditional theory of cartels lacks specificity.

This is not unrealistic because it is known that in cartels the instability of quotas is source of dissatisfaction for its members. The profit is shared between the cartel members. The profit of each firm is $\Pi_i = p\, q_i - c_i\, q_i$ and the cartel's profit is $\Pi = p(q)\, q - q\, c$ where $c$ is the cartel's cost. It follows from Hypothesis 6 that $c = \sum_{i=1}^{3} \lambda_i\, c_i$ ; the individual cost functions are averaged by the weights $\lambda_i$ for the cartel as a whole and play no further role. The monopolistic cartel maximizes its profit; at the optimum we have

(13) $$q = \frac{a-c}{2b}$$

and,

(14) $$q_i = \lambda_i \frac{a-c}{2b}$$

Equation (9) allows us also to deduce $p = \frac{1}{2}(a+c)$. Each cartel member receives a share $\lambda_i$ of the monopoly profit:[23]

(15) $$\Pi_i = \lambda_i \frac{(a-c)^2}{4\,b}$$

We also get

(16) $$\Pi = \frac{(a-c)^2}{4\,b}$$

In this model with constant unit costs, the market shares are stable by hypothesis even if the demand changes, what is in conformity to France's mobile phone market. However in the general model (equations (9) to (12)) the quotas are generally not fixed over time when the demand changes. For example, if the marginal costs are linear, the market shares vary as soon as the demand moves, what validates the hypothesis of fixed unit costs. Consider the following

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[22] This amounts to follow Bloch (2003), even if our model is a little more general.

[23] The share of the total profit received by a cartel member differs from the share of the total output produced by the same cartel member only if an explicit mechanism of payment among members exists. This would be obvious proof of collusion for the control authorities…, even if such transfers may be hidden by using tax havens. We ignore this case.

classroom example: if $C_1' = c_1 + \frac{1}{2}q_1$, $C_2' = c_2 + \frac{1}{2}q_2$ and $C_3' = c_3 + \frac{1}{2}q_3$ and $p = 27 - q$. Then the optimal output is 12, the optimal price is 15 and each operator 1, 2 and 3 produces 6, 4 and 2 respectively, with quotas of 1/2, 1/3 and 1/6 respectively. However, these proportions are different for any other optimal outputs –for example, after a small change in the demand– even if the marginal costs remain unchanged. For example, if the demand becomes $p = 14 - q$, with the same marginal costs, then the optimal output falls at 6, operators' output are 4, 2 and zero, with market shares 2/3, 1/3 and zero respectively. This means that small changes in demand will generate changes in markets shares, what complicates much more quota coordination: the linearity of the reaction to demand changes disappear, what is very disturbing for managers.

### *5.3. A partial cartel as leader, facing to one operator in the fringe*

The model is one of a Stackelberg oligopoly, the cartel being the leader and the fringe's firms the followers, called by Shaffer (1995, p. 745) "oligopoly with a Cournot competitive fringe", which is "Cournot" in that sense that the fringe takes the output of the leader as given (but the inverse is not true). The inverse function of demand remains the same as above: $\tilde{p}(\tilde{q}) = a - b\tilde{q}$ where $\tilde{q} = \tilde{q}_c + \tilde{q}_f$, $\tilde{q}_c$ being cartel's output, $\tilde{q}_f$ the output of the firm in the fringe and $\tilde{p}$ denoting the price when a partial cartel of two operators is formed among three. The possibility of different costs between insiders and outsiders will be taken into account: $\tilde{c}_f$ denotes the follower's unit cost and $\tilde{c}_{c_i}$ unit cost of insider *i*; denote by *K* the subset of insiders. Outsider's profit is written $\tilde{\Pi}_c(\tilde{q}_c, \tilde{q}_f) = \tilde{p}(\tilde{q})\,\tilde{q}_f - \tilde{c}_f\,\tilde{q}_f$. The outsider maximizes its profit by taking the cartel's production as given: $\max_{\tilde{q}_f} \tilde{\Pi}_f(\tilde{q}_c, \tilde{q}_f)$ what gives:

$$(17) \qquad \tilde{q}_f = \frac{\tilde{p}(\tilde{q}) - \tilde{c}_c}{b}$$

The cartel adopts monopolist behavior with respect to the residual demand curve, which implies the following inverse demand function by reporting (17) into the inverse function of demand:[24]

[24] Donsimoni at al. (1986) and Shaffer (1995) emphasize that there are other possible definitions of the residual demand curve that affect the result.

(18) $\tilde{p}(\tilde{q}_c) = \frac{1}{2}(a - b\,\tilde{q}_C + \tilde{c}_c)$

The cartel's profit is $\tilde{\Pi}_c = \tilde{p}(\tilde{q}_c)\,\tilde{q}_c - \sum_{i=1}^{k} \tilde{c}_{c_i}\,\tilde{q}_{c_i}$, which can be rewritten $\tilde{\Pi}_c = \tilde{p}(\tilde{q}_c)\,\tilde{q}_c - \tilde{q}_c\,\tilde{c}_c$, where $\tilde{c}_c$ denotes $\sum_{i \in K} \lambda_i(k)\,\tilde{c}_{c_i}$ (see above); again, it is as if the costs were averaged by the quotas. This profit can be maximized with respect to $\tilde{q}_c$, which gives after rearranging $\tilde{q}_c = \frac{a - 2\tilde{c}_c + \tilde{c}_f}{2b}$ and substituting this expression into (18) gives: $\tilde{p} = \frac{1}{4}(a + 2\tilde{c}_c + \tilde{c}_f)$. By substituting it into (17) we get $\tilde{q}_f = \frac{a + 2\tilde{c}_c - 3\tilde{c}_f}{4b}$. Each insider *i* produces a share $\lambda_i(k)$ of the output with $\sum_{i=1}^{2} \lambda_i = 1$: $\tilde{q}_{c_i} = \lambda_i \frac{a - 2\tilde{c}_c + \tilde{c}_f}{2b}$. Finally $\tilde{\Pi}_c = \frac{(a - 2\tilde{c}_c + \tilde{c}_f)^2}{8b}$, of which each member receives a share $\lambda_i$, that is, $\tilde{\Pi}_{c_i} = \lambda_i\,\tilde{\Pi}_c = \lambda_i \frac{(a - 2\tilde{c}_c + \tilde{c}_f)^2}{8b}$ and $\tilde{P} = \frac{(a + 2\tilde{c}_c - 3\tilde{c}_f)^2}{16b}$.

Notice that if $\lambda_1 = \lambda_2 = \frac{1}{2}$ then $\tilde{q}_{c_i} = \tilde{q}_f = \frac{a - 2\tilde{c}_c + \tilde{c}_f}{4b}$ and $\tilde{\Pi}_{c_i} = \tilde{\Pi}_f = \frac{(a - 2\tilde{c}_c + \tilde{c}_f)^2}{16b}$, which requires $\tilde{c}_c < \frac{1}{2}(a + \tilde{c}_f)$ and $\tilde{c}_c \geq \frac{1}{2}(-a + 3\tilde{c}_f)$.

### *5.4. France Telecom as leader, SFR and Bouygues Telecom in the fringe*

If we consider the possibility of a leader firm, France Telecom is the good choice because it is the "historical" operator, the first in the market, the biggest, etc. The two other operators, SFR and Bouygues Telecom will be considered as competitive followers: the fringe is also competitive à la Cournot. The inverse function of demand remains the same as above: $\hat{p}(\hat{q}) = a - b\,\hat{q}$ where $\hat{p}(\hat{q})$ denotes the price, $\hat{q}_i$ denote the operators' output, $\hat{q} = \sum_i \hat{q}_i$, $\hat{c}_i$ denotes the operators' unit cost. Followers' profit is written $\hat{\Pi}_i = \hat{p}(\hat{q})\,\hat{q}_i - \hat{c}\,\hat{q}_i$ for $i = 2,3$. The followers maximize their profit by taking the production of the leader as given: $\max_{\hat{q}_i} \hat{\Pi}_i$ for $i = 2,3 \Leftrightarrow \hat{q}_i = \frac{1}{2}(a - \hat{c}_i - b\,\hat{q} - 2b\,\hat{q}_i)$ for $i = 2,3$. Rearranged, this gives:

(19) $\hat{q}_i = \frac{\hat{p}(\hat{q}) - \hat{c}_i}{b}$ for $i = 2,3$

The leader adopts monopolist behavior with respect to the residual demand, which implies the following inverse demand function:

(20) $\hat{p}(\hat{q}_1) = \frac{1}{3}(a - b\,\hat{q}_1 + \hat{c}_2 + \hat{c}_3)$

The leader's profit is $\hat{\Pi} = \hat{p}(\hat{q}_1)\,\hat{q}_1 - \hat{c}_1\,\hat{q}_1$, what gives, after maximizing with respect to $\hat{q}_1$:

(21) $\hat{q}_1 = \frac{a - 3\hat{c}_1 + \hat{c}_2 + \hat{c}_3}{2\,b}$

and substituting this expression into (20) gives $\hat{p} = \frac{1}{6}(a + 3\hat{c}_1 + \hat{c}_2 + \hat{c}_3)$, what reported into (19) allows to find:

(22) $\hat{q}_2 = \frac{a + 3\hat{c}_1 - 5\hat{c}_2 + \hat{c}_3}{6\,b}$

(23) $\hat{q}_3 = \frac{a + 3\hat{c}_1 + \hat{c}_2 - 5\hat{c}_3}{6\,b}$

The total output is equal to $\frac{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3}{6\,b}$ and the market shares are:

(24) $\lambda_1 = 3\frac{a - 3\hat{c}_1 + \hat{c}_2 + \hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3}$

(25) $\lambda_2 = \frac{a + 3\hat{c}_1 - 5\hat{c}_2 + \hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3}$

(26) $\lambda_3 = \frac{a + 3\hat{c}_1 + \hat{c}_2 - 5\hat{c}_3}{5a - 3\hat{c}_1 - \hat{c}_2 - \hat{c}_3}$

Finally the profits are the following:

(27) $\hat{\Pi}_1 = \frac{(a - 3\hat{c}_1 + \hat{c}_2 + \hat{c}_3)^2}{12\,b}$

(28) $\hat{\Pi}_2 = \frac{(a + 3\hat{c}_1 - 5\hat{c}_2 + \hat{c}_3)^2}{36\,b}$

$$(29) \quad \hat{\Pi}_3 = \frac{\left(a + 3\hat{c}_1 + \hat{c}_2 - 5\hat{c}_3\right)^2}{36\,b}$$

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