On the impossibility of calculating the product technology in the Supply-Use model

Louis de Mesnard
University of Burgundy and CNRS
(*Laboratoire d'Economie et de Gestion*, UMR 5118)
and *Regional Economics Application Laboratory*

Address. Faculty of Economics, University of Burgundy, 2 B^d Gabriel, F-21000 Dijon, FRANCE. E-mail: *louis.de-mesnard@u-bourgogne.fr*.

Abstract. The Supply-Use input-output model of the SNA and Eurostat is examined. For the product-by-product IO tables, two hypothesis are possible: "product technology", largely adopted (Eurostat A) and examined here, and "industry technology" (Eurostat B). One examines the calculability of the model. Negatives are an issue; (i) they are systematical in the inverse supply matrix: negative probabilities are impossible, Markov chains become impossible; (ii) negative flows are nonsense in symmetric matrix of coefficients or in inverse matrices. Matrices must be square what removes much of their interest. Same conclusions can be transposed to industry-by-industry IO tables under fixed-industry-sales-structure assumption (Eurostat C).

Keywords. Input-Output, SNA, Eurostat, Supply-Use, Make-Use.

JEL Classification. C67, D57.

1 Introduction

The two-matrices-input-output model, so-called Supply-Use or Make-Use, is the basis for most charts of national accounting as the SNA or *System of National Accounts* (United Nations (1968, 1993, 1999, 2001),[1] but it is also considered as very useful and more realistic than the traditional input-output model for regional or interregional modeling (Oosterhaven 1984). This model is based on two matrices because the one-to-one correspondence sector/product is abandoned to the benefit of the distinction between industries and commodities, a same product being able to be produced by many industries, and reciprocally. One encounters:

- The *Use* matrix which is analogous to the Leontief matrix and which describes a linear production function with complimentary inputs.
- The *Make* or *Supply* matrix which describes which industry produces which commodity and reciprocally.

Actually, Eurostat (2008) considers two types of tables, the product-by-product input-output tables by making an assumption on the technology, and the industry-by-industry input-output tables by making the assumption of “fixed sales structure”. In this paper, we focus on the product-by-product input-output tables, Eurostat (2008, p. 310) saying that “product-by-product input-output tables are believed to be more homogeneous in terms of cost structures” while the industry-by-industry input-output tables “are close to statistical sources and more heterogeneous in terms of input structures. It remains to be seen in empirical research which type of tables is the better option for comparisons across nations...” (Eurostat 2008, p. 310). The demonstrations could be transposed to the industry-by-industry tables; see Rueda-Cantuche and ten Raa (2008) for a fine analysis of the axiomatics of those tables.

Two hypotheses are set by the SNA 1993 to transfer outputs and associated inputs:[2]

- The *technology based on commodities*, also called *product-technology assumption* , which corresponds to Eurostat’s Model A: “Each product is produced in its own specific way, irrespective of the industry where it is produced” (Eurostat 2008, p. 297). This

[1] See also Blades (1989), Van Bochove and Bloem (1987), Vanoli (1994), Lawson (1997) and Guo et al. (2002).

[2] This excerpt from the SNA 1993 (United Nations 2001, item 15.144 and 15.145):
“*The mathematical methods used when transferring outputs and associated inputs hinge on two types of technology assumptions:*
(a) Industry (producer) technology, assuming that all products produced by an industry are produced with the same input structure;
(b) Product (commodity) technology, assuming that a product has the same input structure in whichever industry it is produced.
The importance of the role played by the assumptions depends on the extent of secondary production, which depends not only on how production is organized in the economy, but also on the statistical units and the industry breakdown in the tables. More secondary production will appear with institutional units than with establishments, and more secondary production will inevitably be found in more detailed tables”.
These two alternative models can also be combined into mixed models (Gigantes, 1970; ten Raa, Chakraborty and Small, 1984; Miller & Blair 1985); for a review see ten Raa and Rueda-Cantuche 2003. Konijn and Steenge (1995) have described a model that uses von Neumann’s activities: we do not examine it here.
Stone (1961, pp. 107-108) is at the origin of these two hypotheses.

hypothesis is recommended by the new SNA even it generates negatives: "Economically, the commodity technology assumption makes more sense than the industry technology assumption" (United Nations 1999, p. 87).[3]

- Or the *technology based on industries*, also called *industry technology assumption*, which corresponds to Eurostat's Model B: "Each industry has its own specific way of production, irrespective of its product mix" (Eurostat 2008, p. 297). This hypothesis was recommended by the former SNA 1968 (United Nations 1968) but the present SNA 1993 considers that it is incoherent because it leads to incoherent "cooking recipes" (United Nations 1999, p. 99; Almon 2000). Unlike the product-technology assumption, it violates the last three of the following list of four desirable axioms: materiel equilibrium, financial equilibrium, price invariance, scale invariance (Kop Jansen and Thijs Ten Raa 1990; ten RAA and Rueda-Cantuche 2003; ten RAA 2005) and (United Nations 1999, pp. 100-103).[4] Following ten Raa (2005), this is an obvious reason to abandon the hypothesis based on industries.

This paper wants to examine the question of the negatives in the Supply-Use model when the hypothesis of the *technology based on commodities is chosen*. Various approaches have been proposed by some authors to eliminate the negative terms (Almon 1970, 2000; Armstrong, 1975; Rainer 1989; Steenge 1990; Rainer and Richter 1992; Mattey 1993; Mattey and ten Raa 1997; for a review, see ten Raa and Rueda-Cantuche (2003)). Even if they allow solving the problem of negatives, they cannot be more than a stopgap because the negative terms are not caused by some errors or by the presence of different technologies or by heterogeneous classifications, but are inherent to the nature of the *technology based on commodities* in the Supply-Use model. Indeed, they are necessarily produced by the inversion of the matrix of output proportions of industries, and are present at least one by row and one by column in the inverse of this matrix (de Mesnard, 2004). Intuitively, one understands that these terms are bothering but beyond of this, it is necessary to ponder about their economic and mathematical meaning. This will allow demonstrating that SNA's

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[3] For the SNA 1993 (United Nations 2001, item 15.147),
"*… the product (commodity) technology model seems to meet the most desirable properties, i.e., the axioms of material balance, financial balance, scale invariance and price invariance. It also appeals to common sense and is found a priori more plausible than the industry technology assumption. While the product technology assumption thus is favoured from a theoretical and a common sense viewpoint, it may need some kind of adjustment in practice. The automatic application of this method has often shown results that are unacceptable, insofar as input-output coefficients sometimes appear as extremely improbable or even impossible*".

[4] For the SNA 1933 (United Nations 2001, item 15.146),
"*On theoretical grounds, … by referring to certain axioms of desirable properties one may come somewhat closer to a choice between these two technology assumptions. On this basis, the industry technology assumption performs rather poorly, as being:*
(a) Highly implausible;
(b) Not price invariant, which means that values at current prices are affected;
(c) Not scale invariant, due to its fixed market share property, which means that the coefficients that follow may vary without change in technique;
(d) Not maintaining financial balance, which means that the axiom of revenue being equal to cost plus value added for each commodity is not met;
*(e) The Leontief material balance (total output = input-output coefficients * total output + final demand) is however met*".

approach to fix the problem is wrong: the difficulty cannot be solved by arranging the data or by creating a mixed hypothesis.[5]

2 The product-technology hypothesis

In the rectangular models such as SNA, one considers two rectangular homogeneous matrices. The Use matrix, noted **U**, indicates which quantity of each product each industry[6] buys in order to produce: u_{ij} is the quantity of input *i* used by industry *j*. For example, for 2 industries and 3 products:

$$(1)\qquad \begin{matrix} \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix} & \begin{matrix} e_1 & q_1 \\ e_2 & q_2 \\ e_3 & q_3 \end{matrix} & \text{Commodities} \\ \begin{matrix} w_1 & w_2 \\ x_1 & x_2 \end{matrix} & & \\ \text{Industries} & & \end{matrix}$$

where x_i is the output of industry *i* ($x_i > 0$ for all *i*), w_j is the value added of industry *j* ($w_j > 0$ for all *j*), q_i is the total production of commodity *i* ($q_i > 0$ for all *i*), e_i is the amount of commodity *i* sold to final demand ($e_i > 0$ for all *i*).

The Supply (or Make) matrix, noted **V**, indicates which quantity of each product each industry produces, where v_{ij} is the quantity of good *j* produced by industry *i*. For example:

$$(2)\qquad \begin{matrix} \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} & \begin{matrix} x_1 \\ x_2 \end{matrix} & \text{Industries} \\ \begin{matrix} q_1 & q_2 & q_3 \end{matrix} & & \\ \text{Commodities} & & \end{matrix}$$

This table is similar to those of Miller & Blair (1985, p. 160) but it is transposed by respect to Eurostat (2008, p. 311). Four accounting identities are given, **s** being the sum or identity vector, i.e., $\mathbf{s}' = (1 \ldots 1)$, prime denoting the transposition:

$$(3)\qquad \mathbf{x} = \mathbf{V}\,\mathbf{s}$$

$$(4)\qquad \mathbf{x} = \mathbf{U}'\,\mathbf{s} + \mathbf{w}$$

$$(5)\qquad \mathbf{q} = \mathbf{U}\,\mathbf{s} + \mathbf{e}$$

$$(6)\qquad \mathbf{q} = \mathbf{V}'\,\mathbf{s}$$

Technical coefficients are defined by:

$$(7)\qquad \mathbf{B} = \mathbf{U}\,\hat{\mathbf{x}}^{-1}$$

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[5] For the SNA (United Nations 2001, item 15.148):
"*Further improvement of the input-output tables can be made in the following ways:*
(a) Make proper adjustments to the basic data so as to obtain a supply and use table of good quality, since this will in fact mean more to the quality of the symmetric tables than the choice of technology assumption;
*(b) Introduce other models like mixed technology models whenever modifications of the basic input-output model are to be made, however complicated to implemen*t".

[6] Industries are called "establishments" by Stone (1961, p. 107).

By combining (5) and (7), one obtains:

(8) $\mathbf{q} = \mathbf{B}\,\mathbf{x} + \mathbf{e}$

In connection with the way in which matrix **V** must be read, two alternative assumptions are posed, that of product technology, examined here, and that of industry-based technology, recalled in annex. The derivation of models' solution will follow the standard presentation of Miller & Blair (1985 pp. 159 -…).[7] It is possible to present the complete solution of these models: each assumption generates two accounting identities (commodities / commodities and industries / industries) and four inverse matrices (commodities / commodities, commodities / industries, industries / industries and industries / commodities).

While noting by $\mathbf{A}(\mathbf{U},\mathbf{V})$ the matrix of the direct requirements in intermediate goods formed when one of the two polar assumptions is chosen, the four axioms of Kop Jansen and ten Raa (1990), ten Raa and Rueda-Cantuche (2003) are as follows (the hat over a vector indicates the diagonal matrix formed from this vector):

- material balance, $\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}'\mathbf{s} = \mathbf{U}\mathbf{s}$,
- financial balance, $\mathbf{s}'\mathbf{A}(\mathbf{U},\mathbf{V})\,\mathbf{V}' = \mathbf{s}'\mathbf{U}$,
- price invariance, $\mathbf{A}(\hat{\mathbf{p}}\mathbf{U},\mathbf{V}\hat{\mathbf{p}}) = \hat{\mathbf{p}}\mathbf{A}(\mathbf{U},\mathbf{V})\,\hat{\mathbf{p}}$ for all price vector $\mathbf{p} > 0$
- scale invariance, $\mathbf{A}(\mathbf{U}\hat{\mathbf{k}},\hat{\mathbf{k}}\mathbf{V}) = \mathbf{A}(\mathbf{U},\mathbf{V})$ for all scale factor vector $\mathbf{k} > 0$.

In the product-technology hypothesis, as Miller & Blair (1985, p. 165) say: "…the total output [x_i] of any industry [*i*] is composed of goods [*j*] in fixed proportions", and the input structure of a product does not depend on the industry which produces really this commodity; that is, the matrix **C** is fixed:[8] [9]

(9) $c_{ij} = \dfrac{v_{ij}}{x_i}$ or $\mathbf{C} = \mathbf{V}'\,\hat{\mathbf{x}}^{-1}$

For Miller & Blair (1985), this assumption is applicable to secondary products but for Rainer (1989), it is not suitable for some secondary products as mineral oil industry. The 1993 System of National Accounts prescribes the product-technology hypothesis (United Nations 1999, p. 98-99), mainly because it fulfills the four desirable axioms cited above (materiel equilibrium, financial equilibrium, price invariance, scale invariance).

From (6) and (9) one obtains

(10) $\mathbf{x} = \mathbf{C}^{-1}\,\mathbf{q}$

what indicates how the goods are produced by industries but requires calculating the inverse of **C**. Remember that **C** is invertible because it is the product of **V** and of an invertible matrix from (9)), **V** being invertible from ten Raa's theorem 7.1 (ten Raa and van der Ploeg 1989, p. 89). Combining (10) with (8) gives:

(11) $\mathbf{q} = \mathbf{B}\,\mathbf{C}^{-1}\,\mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^{C}(\mathbf{U},\mathbf{V})\,\mathbf{q} + \mathbf{e}$

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[7] See also Aidenoff (1970), United Nations (1999, pp. 86-103), Gilchrist et al. (2000); Shao and Miller (1990) have focused on the multiregional case; there is a remarkable survey in Guo et al. (2002).

[8] As for Stone (1961, p. 108) who says: "… on the assumption that the output of each industry is made up of the different products in **fixed** proportions…".

[9] Matrix **C** is defined as in Miller & Blair (1985), while it is transposed by respect to de Mesnard (2004) or to the SNA 1993 (United Nations 1999, 2001).

by denoting

(12) $\mathbf{A}^{C}(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{C}^{-1}$

the matrix of direct intermediary consumption of commodities in the product-technology hypothesis; this matrix is a matrix of constant as **B** and **C** are. Note that

(13) $\mathbf{A}^{C}(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\mathbf{V}'^{-1}$

by using (7) and (9). The solution is:

(14) $\mathbf{q} = \left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{e} = \left(\mathbf{I} - \mathbf{A}^{C}(\mathbf{U},\mathbf{V})\right)^{-1}\mathbf{e}$

where $\left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}$ is called the commodity - commodity inverse matrix. The final demand addressed to industries is:

(15) $\mathbf{e} = \mathbf{C}\,\mathbf{f}$

thus (14) transforms directly into

(16) $\mathbf{q} = \left(\left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{C}\right)\mathbf{f}$

where $\left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{C}$ is the commodity - industry inverse matrix. From (10) one draws $\mathbf{q} = \mathbf{C}\mathbf{x}$, what carried into (14) gives $\mathbf{C}\mathbf{x} = \left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{e} \Leftrightarrow \left(\mathbf{I} - \mathbf{B}\,\mathbf{C}^{-1}\right)\mathbf{C}\mathbf{x} = \mathbf{e}$ $\Leftrightarrow (\mathbf{C} - \mathbf{B})\,\mathbf{x} = \mathbf{e} \Leftrightarrow \mathbf{C}^{-1}(\mathbf{C} - \mathbf{B})\,\mathbf{x} = \mathbf{C}^{-1}\,\mathbf{e} \Leftrightarrow \left(\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B}\right)\mathbf{x} = \mathbf{C}^{-1}\,\mathbf{e}$ and:

(17) $\mathbf{x} = \left(\left(\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B}\right)^{-1}\mathbf{C}^{-1}\right)\mathbf{e}$

where $\left(\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B}\right)^{-1}\mathbf{C}^{-1}$ is the industry - commodity inverse matrix. Finally, from (15) it comes $\mathbf{C}^{-1}\,\mathbf{e} = \mathbf{f}$ and by using (17):

(18) $\mathbf{x} = \left(\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B}\right)^{-1}\mathbf{f}$

where $\left(\mathbf{I} - \mathbf{C}^{-1}\,\mathbf{B}\right)^{-1}$ is the industry - industry inverse matrix.

Note that in all cases the number of goods must be equal to the number of industries so that the inverse of matrix **C** can be calculated as of the use of equation (9): Supply and Use matrices must be square, which is highly restrictive and will be discussed in the next section. It is known that this technology, if it does not lead to absurd recipes of production (no chocolate into cheese as Almon says (2000)…) generates negative terms in $\mathbf{C}^{-1}$, which, even if they are small, are not interpretable economically and cannot be avoided. It is on this point that we will insist: this will also be discussed in the next section.

3 The negatives

3.1 Where are the negatives?

The existence of negative terms in the Supply-Use model under the product-technology hypothesis has been badly understood in the past. The negative terms may be very small in absolute value in national accounting matrices: this is why most authors or scholars tend to neglect them or try to remove them by some process even theoretically unsatisfactory. A complete review can be found in ten Raa and Rueda-Cantuche (2003). Most authors have thought that the negatives are caused by nonhomogeneities (Rainer 1989) or by measurement errors (Steenge 1990). They have tried to eliminate them by various methods that are absolutely correct in themselves as. For example one may quote the simple method of the SNA (United Nations 1999, p. 97)[10], the sophisticated Almon's iterative method (Almon 2000) or ten Raa and van der Ploeg's statistical adjustment (1989) (even if they reject the product-technology hypothesis) or the non-negativity constraints (Ten Raa 2005, p. 96).[11] Alternately, a transition matrix between **B** and **C** has been proposed (Steenge 1990), what is a matter for another category of methods. Rainer (1989) lists three methods to alleviate negatives: set the negatives to zero, set the negatives to zero iteratively as done by Almon (1970) again, or set the negatives to zero by replacing some by a positive value as done by Armstrong (1975). Most of these methods takes us away from the input-output model. The SNA 1993 thinks that over-specification, misclassification, differences between secondary products and products, and above all, errors in data, are the cause of the negative terms; the following quotation of the SNA 1993 is enlightening (United Nations 2001, item 15.147):

"*There are even numerous examples of the method leading to negative coefficients which are clearly nonsensical from an economic point of view. Improbable coefficients may partly be due to errors in measurement and partly to heterogeneity (product-mix) in the industry of which the transferred product is the principal product*".

This one is also interesting (United Nations 1999, p. 96-97):

"*… one can see that as the inputs required for secondary products are removed from total input, the derived technical coefficient can be negative if one of the following occur: (i) There is the over-specification of the secondary products, i.e. the output of the secondary product in the make matrix (the supply table), in our example, product 2 produced in product 1, is misclassified; (ii) The secondary product is not exactly the same as the product produced as a primary product elsewhere; it requires less inputs than assumed; (iii) There are errors in data. … The solution to the problem of negative coefficients is to recheck data themselves. Significant secondary products and their associated inputs must be transferred by using the redefinition method on the basis of the information provided by establishments producing only these kinds of secondary products or collected by special surveys*".

However, even if one or all of the items of this quotation are true –(i), (ii) or (iii)–, the negative terms in $\mathbf{C}^{-1}$ are unavoidable as soon as the **V** matrix is not diagonal. The following theorem can be found in (de Mesnard, 2004), for $\mathbf{C}^{-1}$.[12] In de Mesnard (2004) matrix **C** was assumed indecomposable. However, by respect to de Mesnard (2004), it is necessary to add the particular case where matrix **C** is quasi-diagonal, that is, decomposable with at least one diagonal block: this configuration corresponds to real Make matrices that are large with a few terms out of the diagonal. Almon (2000, p. 30) gives the following example where **C** is decomposable and the block formed by the last three sectors is diagonal:

[10] "*In cases where negative coefficients are very small in comparison to other coefficients in the same columns, practitioners may set them to zero and balance the tables by the RAS method*".

[11] This method is absolutely correct in itself but the terms that should be negative will tend to accumulate themselves on the border of the set delimited by the non-negativity constraints, that is, are replaced by zeros, what could be not realistic to some extent.

[12] Obviously, the same holds for $\mathbf{D}^{-1}$ even if it is not a big issue: see annex.

(19) $$\mathbf{C} = \begin{bmatrix} .7 & .1 & 0 & 0 & 0 \\ .3 & .9 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It is true that the more aggregated the data are the more the nondiagonal blocks are large:

"*Heterogeneity results from working on aggregated data with a high occurrence of non-characteristic products. This might be overcome by making adjustments based on supplementary information or exploiting informed judgment to the fullest extent possible*" (United Nations 2001, item 15.147).

<u>Theorem</u>. Consider a nonnegative matrix **C** which has one or more nondiagonal blocks (**C** may be composed only of one nondiagonal block, that is, not decomposable). The nondiagonal blocks in matrix $\mathbf{C}^{-1}$ have at least one negative term per row and column, that is, per industry and commodity.

Example (coming from de Mesnard (2004)):

(20) $$\mathbf{C} = \begin{bmatrix} .5 & .1 & .1 \\ .3 & .6 & .2 \\ .2 & .3 & .7 \end{bmatrix} \text{ Commodities}$$

1 1 1

Industries

so:

(21) $$\mathbf{C}^{-1} = \begin{bmatrix} 2.25 & -.25 & -.25 \\ -1.0625 & 2.0625 & -.4375 \\ -.1875 & -.8125 & 1.6875 \end{bmatrix} \text{ Industries}$$

1 1 1

Commodities

In the example (20)-(21), all nondiagonal terms are negative. In the inverse of matrix of example (19) all nondiagonal terms of the nondiagonal block are negative:

(22) $$\mathbf{C}^{-1} = \begin{bmatrix} 1.5 & -.1667 & 0 & 0 & 0 \\ -.5 & 1.1667 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Proof.

Matrix **C** is nonnegative: $c_{ij} \geq 0$ for all i, j. Consider the k^{th} nondiagonal block $\mathbf{C}^k$ of matrix **C**: it is such that for all diagonal element c_{ii}^k, there is at least one strictly positive term in the row i or in the column i:

(23) $$\forall i \left\{ \left(\exists j / c_{ij}^k > 0 \right) \text{or} \left(\exists j / c_{ji}^k > 0 \right) \right\}$$

As **C** is not negative by assumption, and as $\mathbf{C}^k\left(\mathbf{C}^k\right)^{-1}=\mathbf{I}$, then for the nondiagonal terms of I, one can pose the following formula: $\sum_p c_{ip}^k \sigma_{pj}^k = 0$ for all *i* and *j*, where σ_{pj}^k is the term $\{p, j\}$ of $\left(\mathbf{C}^k\right)^{-1}$. Therefore, at the very least, there is one *k* such that $\sigma_{pj}^k < 0$ for all *j*; hence there is thus one negative term per column of $\left(\mathbf{C}^k\right)^{-1}$, that is, per commodity. But one could have written $\left(\mathbf{C}^k\right)^{-1}\mathbf{C}^k=\mathbf{I}$ in an equivalent way: there is also one negative term per row of $\left(\mathbf{C}^k\right)^{-1}$, that is, per industry. •

The negative terms are thus systematic in **C** in their non diagonal blocks. Remember that a completely diagonal Make matrix makes the model completely equivalent to the single-matrix model of Leontief. Hence, the diagonal block can be considered as trivial in the context of the Supply-Use model, even if, in real Make matrices, there could be many diagonal blocks. This theorem is important because it indicates that scholars[13] and the SNA 1993 (United Nations 1999), are completely misleading on this point.[14] All the cases have obliged to calculate the inverse of matrix **C**:

$$\left\{\begin{array}{l}\left\{\begin{array}{c}\mathbf{U}\\ \mathbf{x}\end{array}\right\}\rightarrow\mathbf{B}\\ \left\{\begin{array}{c}\mathbf{V}\\ \mathbf{x}\end{array}\right\}\rightarrow\mathbf{C}\\ \mathbf{C}\rightarrow\mathbf{C}^{-1}\end{array}\right\}\rightarrow\left\{\begin{array}{c}\left(\mathbf{I}-\mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\\ \left(\mathbf{I}-\mathbf{B}\,\mathbf{C}^{-1}\right)^{-1}\mathbf{C}\\ \left(\mathbf{I}-\mathbf{C}^{-1}\mathbf{B}\right)^{-1}\mathbf{C}^{-1}\\ \left(\mathbf{I}-\mathbf{C}^{-1}\mathbf{B}\right)^{-1}\end{array}\right\}$$

However, for the product-technology model, the linking of mathematical derivations indicates in Figure 1 indicates that the negative terms of $\mathbf{C}^{-1}$ generated by the combination of equations (6) and (9) pollute all cases: all the cases contain negative terms and the first appearance of a negative term orders the others.

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[13] Except ten Raa (1988) who saw that the negatives are not caused by errors in the data but by the model, what must lead to abandon the model based on commodities.
[14] What's more, the SNA 1993 (United Nations 1999, p. 87) indicates that:
"*The more prevalent methods are (i) setting all negatives to zero and using the RAS technique … to balance the table and (ii) optimization such as minimization of variances under constraints to generate positive values. However, the latter is also questioned on other grounds such as an economic justification for a specific form of the objective function*".
This is obviously awful: nothing indicates that negatives are small and above all, they are not accidental but completely systematic.

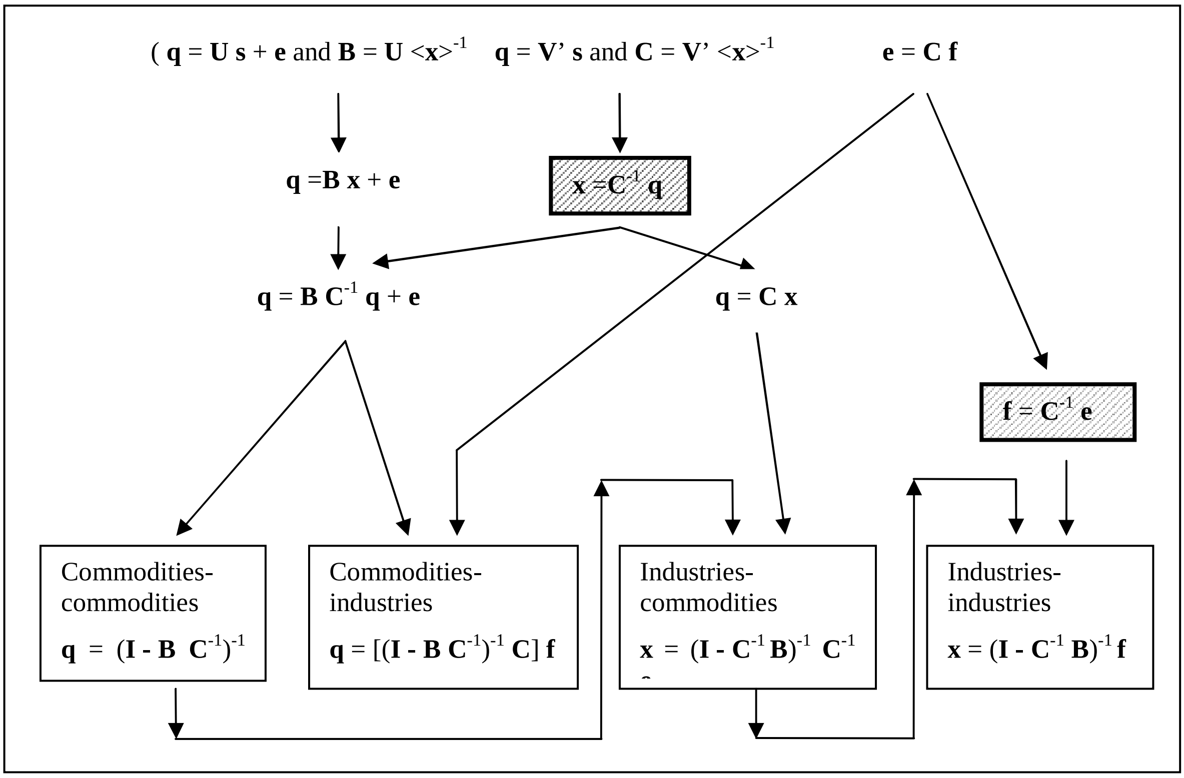

Figure 1. Linking of mathematical derivations

3.2 Why negative technical coefficients are an issue

As the inverse of matrix **V** necessarily contains negative terms, the matrix $\mathbf{A}^C(\mathbf{U},\mathbf{V})$ given by (13) may contain negative terms. Ten Raa and van der Ploeg explain when on a 2x2 example (1989, p. 89): if the diagonal terms of **U** are large. They have not explored larger matrices but it is sufficient to know that $\mathbf{A}^C(\mathbf{U},\mathbf{V})$ may contain negatives.[15] The negative terms in $\mathbf{A}^C(\mathbf{U},\mathbf{V})$ are economic flows of which subtle explanation is complex and unconfirmed.[16] Consider an eventual couple $\{i, j\}$ such that the flow of product *i* bought by sector *j* is negative, that is, $a_{ij}^c < 0$.

- Either this means that *j* buys a negative quantity of commodity *i* to sector *i*. What interpretation for this negative quantity?
- Or sector *j* itself sells commodity *i* to sector *i*. In this case, sector *j*, which normally produces commodity *j*, becomes also manufacturer of commodity *i*. Strange. Moreover it violates the identity sector-product.

If we consider the example of car industry for *j* and of steel for *i*, then in the first case, the car sector buys a negative quantity of steel: what does mean a negative input? In the

[15] Remark that $\mathbf{A}^I(\mathbf{U},\mathbf{V})$ never contains negatives even if equation (33) in annex –one of the four cases of the industry-based model– could lead to negatives.
[16] The 1993 SNA (United Nations 1999, p. 83) explains that it is impossible to make the symmetric-table model functioning when by-products lead to introduce negatives (in the Stone's "negative transfer method"): the net output of by-product cannot be equal to zero if the final demand increases. It is a completely different reason to explain why negatives are impossible.

second case, the car sector sells a positive quantity of steel to the steel sector itself, that is, the steel sector becomes a manufacturer of steel entirely sold to the car industry, without itself consuming steel: if we think on the balance of opposed flows, the car industry is becoming a net seller of steel to the steel industry by selling more to this sector than it buys: very unlikely. In any case, these interpretations that makes the car industry in a situation of being multiproduct which is excluded by the square Leontief model.

3.3 Why negatives in C^{-1} are an issue: the Supply-Use model as Markovian probabilistic model

Assume that there are no negatives in $\mathbf{A}^C(\mathbf{U},\mathbf{V})$, or these negatives are very small, even if there are in $\mathbf{C}^{-1}$: we could say that we don't care of the problem of negatives. This is the position of the large majority of the authors. However, negatives in the inverse matrices $\mathbf{C}^{-1}$ –which is only an intermediary result– is a very important problem even when there are no negatives in $\mathbf{A}^C(\mathbf{U},\mathbf{V})$. Having one forbidden mathematical object inside a mathematical derivation is not allowed even if the result is allowed. I take an example in the probability theory. The probability of having a six with 6-faces dice is 1/6. Hence, the probability of having a double-six with two 6-faces dices is $(1/6)\times(1/6)=+1/36$. Now, assume that we are the Devil with a special dice, impossible for humans, where the probability of having a 6 is negative, say $-1/6$, the other probabilities (of having 1 to 5) being equal to 1.16666/5. What is the probability of having a double 6? It is $(-1/6)\times(-1/6)=+1/36$, of course, an ordinary positive probability: acceptable. However, will humans accept this result even if it falls between zero and one? Certainly not because $-1/6$ and the probabilities greater than 1 are impossible.

In the product-technology model, even when there are no negative terms in the results of the model (equations (14), (16), (17) and (18)), there are always in $\mathbf{C}^{-1}$. However one has been obliged to pass by an illegal operation, the inversion of **C** to obtain these results. For most authors and scholars, having negatives in $\mathbf{C}^{-1}$ is not an issue even though the result given by the inverse matrix is not negative: they neglect everything except the direct matrices **U**, **V** and **B**, and the inverse matrices. In their mind, the input-output models are only interesting for deriving multipliers from inverse matrices. The core of the question is there. Beyond the question of the realistic character of the negative terms in input-output analysis, some arguments against the negative terms in $\mathbf{C}^{-1}$ can be exposed following the idea that the input-output model can be understood in terms of iterative intermediary transmission of economic impacts. The idea of circuit is exposed in (de Mesnard, 2004) but another idea is possible: Markovian chains. Some other arguments come even if one considers only the final result of inverse matrices.

We develop here a probabilistic interpretation of the Supply-Use model in terms of Markovian chains, what follows from the interpretation of input-output analysis in terms of probability (Jackson and West 1989). If we follow the probabilistic interpretation in terms of Markovian chains, coefficient b_{ij} is the probability that industry *j* spends one unit of money in commodity *i*, d_{ij} is the probability of producing commodity *j* by industry *i* and c_{ij} is the probability for industry *i* to produce commodity *j*.[17] As $\sum_i b_{ij} < 1$, there are "leakages" and **B**

[17] One way wonder what the associated probability distributions are; they are multinomial (draw with put back); see Choukroun (1975).

is a sub-Markovian matrix, but as $\sum_i b_{ij} + l_j = 1$, where z_{ij}/x_j is denoted l_j, the terms b_{ij} and l_j are yet probabilities: the matrix **B** becomes a Markovian matrix when it is completed by the row vector of the l_j is Markovian.

Hence for the hypothesis of technology based on the industries (see annex), matrix $\mathbf{A}^I(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{D}$ is generated by the following reasoning. Commodity *j* has a probability d_{ij} of being produced by industry *i*: **D** is a Markovian matrix of probability. Then when a quantity $q_j^{(k)}$ of good *j* is produced, the expectancy of the output of industry *i* is $d_{ij}\, q_j^{(k)}$ of *j*. For all commodities *j*, the expectancy of the output of industry *i* is $E\left(\mathbf{x}^{(k+1)}\right) = \mathbf{D}\,\mathbf{q}^{(k)}$. As the probability of industry *i* to buy an intermediate commodity *j* is b_{ij}, the expectancy of industry *i* to buy good *j* is equal to $b_{ij}\; E\left(x_i^{(k+1)}\right)$. The expectancy of all industries *i* to buy commodity *j* is equal to $E\left(\mathbf{q}^{(k+1)}\right) = \mathbf{B}\; E\left(\mathbf{x}^{(k+1)}\right)$. Hence, $E\left(\mathbf{q}^{(k+1)}\right) = \mathbf{B}\,\mathbf{D}\,\mathbf{q}^{(k)}$. If we make the hypothesis that the true value of $\mathbf{q}^{(k+1)}$ tends toward $E\left(\mathbf{q}^{(k+1)}\right)$, then the Markovian cycle starts again at the next step. The model is fundamentally compatible with an interpretation in terms of Markovian matrix even if matrix $\mathbf{D}^{-1}$ appears when one of the four inverse matrices is computed: the negative terms of $\mathbf{D}^{-1}$ produced by equation (31) affect only one of the four cases but not at all the three others.

Let's look now at the product-technology model. The Markovian interpretation of the product-technology hypothesis is never possible because $\mathbf{C}^{-1}$ cannot be a matrix of probabilities: even in the square case of the hypothesis of technology based on commodities, an interpretation in terms of probability is impossible. The probability of producing commodity *j* by industry *i* is denoted σ_{ij}. A term σ_{ij} of $\mathbf{C}^{-1}$ is the probability for a commodity *j* to be manufactured by a given industry *i*, knowing that $\sum_i \sigma_{ij} = 1$ for all *j*. Then consider a couple $\{i, j\}$ such that the corresponding term σ_{ij} is negative. What does $\sigma_{ij} < 0$ mean? This negative term means that, in plain English, the probability of industry *i* to manufacture commodity *j* is negative. This has no meaning since these shares cannot be negative, as indicated above when the interpretation in terms of probabilities has been done. Hence, as this so-called probability may be negative in some cases as demonstrated in the theorem recalled above (there is at least one negative term per row and per column), it is not a probability. In conclusion, even if **C** is a Markovian probability matrix, $\mathbf{C}^{-1}$ is not and the derivation done as above is impossible for the product-technology model.

It was the same for the interpretation in terms of circuit developed by de Mesnard (2004). The plausibility of the two alternative hypotheses depend on the possibility of building a circuit: either the circuit is plausible and the solution of the model is economically meaningful or it is not. De Mesnard has demonstrated (2004) that the product-technology Make-Use model cannot form a circuit if it is demand-driven even if the matrices are square; but it can if the model is supply-driven even if the matrices are rectangular.

3.4 Same number of commodities and industries: epistemological discussion

Mathematical derivations from the product-technology hypothesis are possible only if the number of commodities is equal to the number of industries (square matrices). The following quotation of the SNA (United Nations 1999, p. 95) is very clear (with our notations):

"*The relationship* $\mathbf{A}^C(\mathbf{U},\mathbf{V}) = \mathbf{B}\mathbf{C}^{-1}$ *implies a strong restrictiveness of the commodity technology assumption, i.e.,* **C** *is invertible only if* **C** *is square or the number of industries must equal the number of products. This mathematical requirement is unrealistic since the number of industries needs not equal the number of products unless statisticians make it so by aggregatio*n".

Can we go further?

- Either the number of commodities is necessarily equal to the number of industries: there is a one-to-one correspondence between commodities and industries, that is, commodities are defined according to industries and reciprocally. It is what Miller et Blair (1985) suggest when they indicate that each industry is named from the main product that they produce, what gives a Supply matrix with a strong main diagonal (after having sorted adequately the rows and columns). In this case, one is very close to the French idea of "sector". This point of view poses some additional problems. For example, what happens if two sectors have the same main product? They are merged, but this leaves aside an orphan product. Or what happens if a sector has two main products, placed equal first? There is no mean for deciding.

  At the same time, when the number of commodities is necessarily equal to the number of industries, one is very close to the Leontief square model (commodities being defined at a very fine level and hence being rather homogenous) but now the nondiagonal elements indicate only the secondary products, what remove much interest to the Supply matrix.

- Or the number of commodities and industries is equal by chance, but without any correspondence between industries and commodities. This cannot be anything else than a fortuitous coincidence. The main interest of the Stone model is to be close to the idea of firms by considering that industries may have many products, and that a product may be produced by many industries. However, in the real life, the number of firms is not equal to the number of products.
  - In some sectors, there will be much more products than firms; this will be the case in industry and service sectors (example: there is a virtually infinite variety of cars, models and variants in a same car manufacturer or the virtually infinite types of service contracts).
  - In other sectors, there will be much more firms than products (for example in agriculture with the very large number of farmers producing a very homogenous good, as wheat, corn, fruits, etc.).
  - And in some rare cases, there will not be necessarily the same number of firms and products, but these numbers will be of the same order of magnitude (for example, in the wine sector in France).

  Moreover, modern companies are multidivisional (Aoki's M form), each division often producing a different main product. Hence the modern company cannot belong to a particular industry but to many industries at the same time: it is nonsense to say what the main product of such a company is.

  A last argument: even if one admits that a firm must belongs to the industry of its main products, this leaves aside the very large number of secondary products. It is impossible to say that all secondary products are main products of other industries and conversely; some secondary products may remain into the air. What to do with them? One concludes that the philosophy of the model must be fundamentally rectangular, with an unequal number of commodities and industries.

- Ten Raa and van der Ploeg (1989, p. 95-96) discuss what happens when there are more commodities than sectors and conversely.
  - The case of more commodities than sectors occurs "when input-output data are aggregated into national accounts": there are more technical coefficients than equations; the coefficients are impossible to infer.
  - The case of more sectors than commodities occurs "when input-output data are in raw form", that is, these data come from establishments: there are more equations than technical coefficients. Ten Raa suggests to introduce an error term, $\mathbf{U} = \mathbf{A}^{C}(\mathbf{U},\mathbf{V})\,\mathbf{V}'+\boldsymbol{\varepsilon}$ and to estimate these equations econometrically. This is interesting but we are far beyond the mathematics of input-output analysis: if we begin to use econometrics, one might as well introduce non linear coefficients, etc., that is, abandon input-output economics to the benefit of a computable general equilibrium model, of which purpose is largely different.

4 Conclusion

After recalling what the product-technology model is, we have demonstrated five things.

i. The negatives are systematical in matrix $\mathbf{C}^{-1}$, at least one per row and one per column in each of its nondiagonal blocs.

ii. In the product-technology model, all of the four inverse matrices are affected by the negative terms of $\mathbf{C}^{-1}$ (while in the industry-based version only one of the four inverse matrices is affected by those of $\mathbf{D}^{-1}$). When deriving the product-technology model, if negatives appear in the technological matrix $\mathbf{A}^{C}(\mathbf{U},\mathbf{V})$ or in the inverse matrices, they are embarrassing because they are impossible to interpret economically.

iii. Even if the technological matrix $\mathbf{A}^{C}(\mathbf{U},\mathbf{V})$ or the results in the inverse matrices are non-negative and hence acceptable, this model must be rejected because it needs to use a forbidden intermediary matrix, $\mathbf{C}^{-1}$.

iv. Unlike the industry-based model, the product-technology model cannot be interpreted in terms of Markovian matrices while it was yet established that it cannot be in terms of circuit.

v. Requiring the matrices **U** and **V** to be square is a strong assumption that has very bad epistemological consequences which remove much of the interest of the model.

Hence the negative terms have baneful consequences much more serious than those generally considered. Thus, trying to eliminate them is not adequate: the whole model in its *product technology* version is **false** and must be rejected, even if it fulfils the four axioms proposed by Kop Jansen and ten Raa (Kop Jansen and ten Raa 1990; ten Raa 2005). The *product-technology hypothesis* must be abandoned. To the benefit of what? The alternative model, the *industry technology*, even if this one is disappointing from the point of view the four axioms since this technology fulfils only the first axiom and even if it leads to use "production recipes" that are sometimes absurd? Perhaps, unless another model can be developed.

The same conclusions should be able to be transposed, *mutatis mutandis*, to the industry-by-industry input-output tables when the *fixed industry sales structure* assumption is posed –Eurostat's model C: "Each industry has its own specific sales structure, irrespective of its product mix" (Eurostat 2008, p. 297)–: the inverse of matrix **V** has also to be computed.[18]

One last epistemological remark. Even if the linear models are the simplest in Economics, this particular model poses some insuperable difficulties. One may wonder if making it non-linear as in the models of computable general equilibrium changes anything.

5 Annex. The industries-based technology assumption in the product-by-product tables

In the industries-technology model "…we assume that the total output [q_j] of a commodity [*j*] is provided by industries [*i*] in fixed proportions", as said by Miller & Blair (1985, p. 165). The input structure of an industry does not depend of the goods that it produces; that is, the matrix **D** is fixed:[19]

(24) $$d_{ij} = \frac{v_{ij}}{q_j} \text{ or } \mathbf{D} = \mathbf{V}\,\hat{\mathbf{q}}^{-1}$$

This assumption corresponds to a fixed market share of all industries (realistic in the short run and to the by-products). Combining (3) and (24) gives

(25) $$\mathbf{x} = \mathbf{D}\,\mathbf{q}$$

what reported in (8) gives the model:

(26) $$\mathbf{q} = \mathbf{B}\,\mathbf{D}\,\mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^I(\mathbf{U},\mathbf{V})\,\mathbf{q} + \mathbf{e}$$

by denoting

(27) $$\mathbf{A}^I(\mathbf{U},\mathbf{V}) = \mathbf{B}\,\mathbf{D}$$

the matrix of direct consumption of commodities in the industry-based hypothesis; this matrix is fixed as **B** and **D** are. Note that

(28) $$\mathbf{A}^I(\mathbf{U},\mathbf{V}) = \mathbf{U}\,\langle \mathbf{V}\,\mathbf{s}\rangle^{-1}\,\mathbf{V}\,\langle \mathbf{V'}\,\mathbf{s}\rangle^{-1}$$

by using (7), (24), (3) and (6). The solution is:

(29) $$\mathbf{q} = (\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}\mathbf{e}$$

where $(\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}$ is called the commodity - commodity inverse matrix. From (25) and (29) the following solution comes:

(30) $$\mathbf{x} = \left[\mathbf{D}(\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}\right]\mathbf{e}$$

$\mathbf{D}(\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}$ is called the industry - commodity inverse matrix. The final demand of commodities addressed to the industries writes as:

(31) $$\mathbf{f} = \mathbf{D}\,\mathbf{e}$$

From (25) and (26) it comes $\mathbf{x} = (\mathbf{I} - \mathbf{D}\,\mathbf{B})^{-1}\mathbf{D}\,\mathbf{e}$ and by using (31) the following solution:

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[18] The *fixed product sales structure* hypothesis –Eurostat's model D: "Each product has its own specific sales structure, irrespective of the industry where it is produced" (Eurostat 2008, p. 297)– is not affected by negatives.

[19] As for Stone (1961, p. 107) who says: "… on the assumption that each industry produces a **fixed** proportion… of each product…".

(32) $\mathbf{x} = (\mathbf{I} - \mathbf{D}\,\mathbf{B})^{-1}\mathbf{f}$

where $(\mathbf{I} - \mathbf{D}\,\mathbf{B})^{-1}$ is the industry - industry inverse matrix. Finally by writing (31) under the form $\mathbf{e} = \mathbf{D}^{-1}\,\mathbf{f}$ –which obliges to calculate the inverse of **D** and implies that **D** is rectangular, unlike what is asserted in (United Nations 1999, p. 99)– and by deferring that in (29) it follows the solution:

(33) $\mathbf{q} = \left((\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}\mathbf{D}^{-1}\right)\mathbf{f}$

$(\mathbf{I} - \mathbf{B}\,\mathbf{D})^{-1}\mathbf{D}^{-1}$ is the commodity - industry inverse matrix.[20] Note that ten Raa's theorem 7.1 (ten Raa and van der Ploeg 1989, p. 89) demonstrates that **V** is invertible; hence, we deduce of it that **D** is also invertible (from (24), **D** is the product of **V** and of an invertible matrix).

It must be recalled that the industries based hypothesis violates three of the four axioms cited above: financial balance, price invariance and scale invariance and respects only the axiom of material balance.

In the industry-based technology, only the last case obliges to calculate the inverse of **D**. One has:

$$\left\{\begin{array}{l}\left\{\begin{array}{c}\mathbf{U}\\ \mathbf{x}\end{array}\right\}\rightarrow\mathbf{B}\\ \left\{\begin{array}{c}\mathbf{V}\\ \mathbf{q}\end{array}\right\}\rightarrow\mathbf{D}\end{array}\right\}\rightarrow\left\{\begin{array}{c}(\mathbf{I}-\mathbf{B}\,\mathbf{D})^{-1}\\ \mathbf{D}\,(\mathbf{I}-\mathbf{B}\,\mathbf{D})^{-1}\\ (\mathbf{I}-\mathbf{D}\,\mathbf{B})^{-1}\end{array}\right\}$$

for the first three cases and

$$\left\{\begin{array}{l}\left\{\begin{array}{c}\mathbf{U}\\ \mathbf{x}\end{array}\right\}\rightarrow\mathbf{B}\\ \left\{\begin{array}{c}\mathbf{V}\\ \mathbf{q}\end{array}\right\}\rightarrow\mathbf{D}\\ \mathbf{D}\rightarrow\mathbf{D}^{-1}\end{array}\right\}\rightarrow\left\{(\mathbf{I}-\mathbf{B}\,\mathbf{D})^{-1}\,\mathbf{D}^{-1}\right\}$$

for the last case. It is thus seen that the inversion of **D** is here only one case among four, which conditions the three others by no means. There is thus no obligation for **U** and **V** to be

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[20] Remark that Gilchrist et al. (2000) derive the model by considering that the whole model is a unique matrix (as it is done in Miller & Blair (1985, p. 161)): $\begin{bmatrix}\mathbf{0} & \mathbf{U}\\ \mathbf{V} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{s}_C\\ \mathbf{s}_I\end{bmatrix}+\begin{bmatrix}\mathbf{e}\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\mathbf{q}\\ \mathbf{x}\end{bmatrix}$ where $\mathbf{s}_C$ and $\mathbf{s}_I$ are sum vectors of commodity and industry order respectively. As from (7) and (24), $\mathbf{U} = \mathbf{B}\,\hat{\mathbf{x}}$ and $\mathbf{V} = \mathbf{D}\,\hat{\mathbf{q}}$ respectively, this system turns out to be:

$$\begin{bmatrix}\mathbf{0} & \mathbf{B}\\ \mathbf{D} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \mathbf{x}\end{bmatrix}+\begin{bmatrix}\mathbf{e}\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\mathbf{q}\\ \mathbf{x}\end{bmatrix}\Leftrightarrow\begin{bmatrix}\mathbf{I}_C & -\mathbf{B}\\ -\mathbf{D} & \mathbf{I}_I\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \mathbf{x}\end{bmatrix}=\begin{bmatrix}\mathbf{e}\\ \mathbf{0}\end{bmatrix}$$

$$\Leftrightarrow\begin{bmatrix}\mathbf{q}\\ \mathbf{x}\end{bmatrix}=\begin{bmatrix}(\mathbf{I}_C-\mathbf{B}\,\mathbf{D})^{-1} & (\mathbf{I}_C-\mathbf{B}\,\mathbf{D})^{-1}\mathbf{B}\\ (\mathbf{I}_I-\mathbf{D}\,\mathbf{B})^{-1}\,\mathbf{D} & (\mathbf{I}_I-\mathbf{D}\,\mathbf{B})^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{e}\\ \mathbf{0}\end{bmatrix}$$

Carrying the multiplication in the right hand side yields $(\mathbf{I}_C - \mathbf{B}\,\mathbf{D})^{-1}\,\mathbf{e} = \mathbf{q}$, which is (29) and $(\mathbf{I}_I - \mathbf{D}\,\mathbf{B})^{-1}\,\mathbf{D}\,\mathbf{e} = \mathbf{x}$, which is (32); the two other equations, (30) and (31), are not produced by this way.

square, except for the case commodities-industries. For the industry-based model, the chain of mathematical derivations is indicated by Figure 2: only Commodities-by-Industries is polluted by the negative terms in $\mathbf{D}^{-1}$.

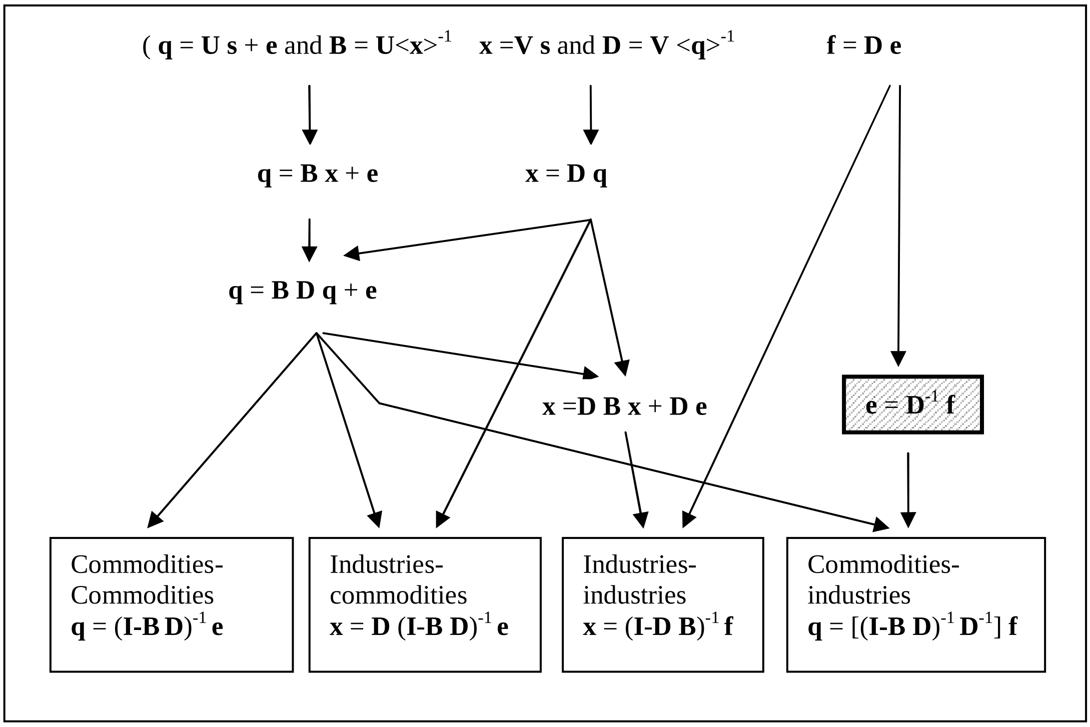

Figure 2. Industry-based model: linking of mathematical derivations

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