

About the Ghosh model: clarifications*

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JEL classification. C67, D46, D57.

Keywords. Input-Output, Ghosh, Cronin, Supply-driven, Demand-driven.

Abstract. We examine the consistency of the Ghosh supply-driven input-output model (SM) by respect to the traditional Leontief demand-driven input-output model (LM); the variants considered are: primal and dual, quantity and value; input prices are not considered. SM offers solutions of limited interest, being incapable to separate quantities and prices or values and price indexes. Comparing the dual value SM to the primal of LM is wrong. Even if the agents are forced to buy inputs in SM, the interpretation of SM as a centrally planned economy must be rejected but SM may serve for modeling interfirm relations or analyzing the structural interindustry change when the production function is not specified. SM may also serve for cost-push exercises but the dual of LM performs the same task in a much simpler and natural way. Cronin's mixed models do not mix demand-driven and supply-driven hypotheses actually.

Running head. About the Ghosh model: clarifications.

* The definitive version of this paper has been published in the *Journal of Regional Science* (Vol. 49, Issue 2, pp. 361-372, May 2009) under the title "Is the Ghosh model interesting?".

1. Introduction

The Leontief model (1936, 1986) assumes that all inputs are bought by producers in fixed proportions, the production function being of "complementary inputs" type; it is qualified as "demand-driven". The Ghosh model (1958) assumes that, in an input-output framework, each commodity is sold to each sector in fixed proportions; it is qualified as "supply-driven". If the Leontief model is well accepted by in some fields as national accounting or regional economics, the Ghosh model has its critics and its supporters:¹ if the quantity Ghosh model seems not plausible (Oosterhaven, 1988, 1989), (Dietzenbacher, 1989), for Dietzenbacher (1997) the model can be interpreted as a price model. On the other hand, the applied studies comparing the stability of technical or allocation coefficients are not fully concluding in favor of one model or to the other (Augustinovics, 1970; Giarratani, 1976, 1981; Helmstädter and Richtering, 1982; Cronin, 1984; Bon, 1986, 2000; de Mesnard, 1988, 1997, 2002).

However, in the literature, the models are explored in a special configuration because of the data available in real statistics: data are in value, that is, in currency units (e.g. in dollar, in euro, etc) and price indexes are considered.² It is a pity and gives a limited validity to the results, while it could be economically more correct to consider also data in physical quantities instead of data in value (that is, true data), and simple prices instead of price indexes. Instead of focusing the discussion on how the model can be deduced from an economic reasoning (cost minimization, revenue maximization, etc., as in Oosterhaven, 1989) or on how the axioms that serve as basis for the model can be established, we will examine the demand-driven and supply-driven models by considering the physical flow matrices and their associated accounting identities and the value flow matrices and their accounting

¹ See Helmstadter and Richtering (1982), Cronin (1984), Chen and Rose (1986, 1991), Oosterhaven (1988, 1989), Dietzenbacher (1989, 1997), Gruver (1989), Miller (1989), Rose and Allison (1989), Sonis and Hewings (1992), de Mesnard (1997, 2002), Park (2007).

² Price indexes are ratios of prices, **always** used when the data are computed in money terms instead of physical terms, as it is the case in the national accounting systems.

The demand-driven model uses the ordinary concept of price, the price of sector's output, also called "output price". An output price is the traditional "production price" in the Classical sense (Ricardo, Marx, Sraffa), that is the price of commodities sold by producers and affecting a whole row of the exchange table; for a review, see Seton (1993). However, the supply-driven model is sometimes presented in the literature with another concept of price, the price of sector's input, here called *input price* (Oosterhaven 1996). An input price is a price controlled by the buyer, affecting a whole column of the exchange table. The idea of input price is nonsense for many and it will be discarded.

identities. This will give two variants for both models. As each model can be solved either per rows in the *primal* or per columns in the *dual*, this yields four variants for both models, to the total. This will allow showing that the supply-driven model offers poor solutions, moreover of limited interest (except for *cost-push exercises*). Cronin (1984) has found four value models by crossing (i) what he calls “the analytical assumptions”, that is, the demand or allocation functions and (ii) what he calls the “causal ordering employed”, demand-driven or supply-driven; this could generate twice as much models: we will discuss it and reject these mix models.

The paper is organized as follows. Section 1 is this introduction. Section 2 recalls what the classical demand driven model is. Section 3 presents the supply-driven model. Section 4 discusses the supply-driven model by comparing it to the demand-driven model: model’s results, its economic interpretation and the supply-driven model as a price model for cost-push exercises. Section 5 discusses Cronin’s mixed models. Section 6 concludes.

2. Remind: the demand driven model

Even if the model is universally known, it is necessary to remind exactly how it is derived to allow a clear comparison with the supply-driven model.

The accounting identities

Consider n sectors, producing n commodities; each sector produces one and only one commodity; each commodity is produced by one and only one sector. $\bar{z}_{ij} \geq 0$ is the flow of physical input i –produced by sector i – to sector j ; sector j uses it to produce commodity j . \bar{x}_i denotes the output of sector i . For each commodity i , the total sold is equal to the total output \bar{x}_i :

$$(1) \quad \bar{\mathbf{x}} = \bar{\mathbf{Z}} \mathbf{s} + \bar{\mathbf{f}}$$

where $\bar{f}_i > 0$ denotes the final demand,³ \mathbf{s} is the sum vector, $\mathbf{s}' = (1 \dots 1)$, and prime denotes the transpose operator. Commodities i have a positive price p_i . As usual in the theory of production prices, the price of any commodity does not depend on who buys the commodity: it is the same along a row, that is, homogenous. All quantities and prices are set for the same

³ For simplicity, only one category of final demand is considered here. The results could be generalized easily.

period, called by commodity “the base year”. Prices are such that the model is at equilibrium per rows (primal), yielding the following accounting identity:

$$(2) \quad \hat{\mathbf{p}} \bar{\mathbf{Z}} \mathbf{s} + \hat{\mathbf{p}} \bar{\mathbf{f}} = \hat{\mathbf{p}} \bar{\mathbf{x}} \Leftrightarrow \bar{\mathbf{Z}} \mathbf{s} + \bar{\mathbf{f}} = \bar{\mathbf{x}}$$

by simplifying prices in both sides of the equation, where $x_i = \bar{x}_i p_i$ is sector i 's output in money terms, $z_{ij} = \bar{z}_{ij} p_i$ is the value of the flow from i to j and $f_i = \bar{f}_i p_i$ is the final demand of commodity i in value; the notation $\hat{\mathbf{p}}$ (as well as then notation $\langle \mathbf{p} \rangle$) denotes the diagonal matrix formed from the vector \mathbf{p} . At equilibrium, the model solves per columns (dual) as:

$$(3) \quad \mathbf{p}' \bar{\mathbf{Z}} + \mathbf{p}' \mathbf{v}^V \hat{\mathbf{v}} = \mathbf{p}' \hat{\mathbf{x}}$$

where $v_j = \bar{v}_j p_j^V$ is sector j 's value added measured in money terms, while $\bar{v}_j > 0$ is the amount of labor employed by sector j and p_j^V is the wage rate of sector j (profits are taken as the owner's remuneration) that vary depending on which sector is considered, what is the general case.⁴

Data in physical quantities

Even it is very well known, the demand driven model must be recalled rapidly because it serves as reference for the other models. It corresponds to one of the most popular in economic science, the Leontief model (Leontief was "Nobel prized" for creating it), but it also falls into a very general category, the linear models of production and exchange (Pasinetti 1977; Wheale 1985; Gale 1989); in this category one finds some of the most familiar and historic models, namely the Ricardo, Marx and Sraffa models. It is assumed that each sector buys each commodity in fixed proportions, but there are two possibilities: data may be defined in physical terms or in value terms; this real difficulty, often hidden, justifies recalling the Leontief model.

Coefficients are defined in physical terms: it is assumed that the matrix $\bar{\mathbf{A}} = \bar{\mathbf{Z}} \hat{\mathbf{x}}^{-1}$ is stable. Note that these coefficients are heterogeneous: one commodity is in the numerator but

⁴ As the price of labor is variable between sectors, there is implicitly either a hypothesis of labor heterogeneity, the labor employed by a sector being not the same than the labor employed by another sector, or a hypothesis of immobility of labor. Otherwise the price of labor would be uniform among sectors because the labor would transfer itself from low-paying sectors to high-paying sectors.

another in the denominator. The economy must be at equilibrium per rows and per columns. Per rows, the accounting identity (2) implies $\bar{\mathbf{A}} \bar{\mathbf{x}} + \bar{\mathbf{f}} = \bar{\mathbf{x}}$. This Cramer system has a non trivial solution only if the determinant $|\mathbf{I} - \bar{\mathbf{A}}|$ is not zero; the solution is found in physical terms $\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} \bar{\mathbf{f}}$. Per columns, in the dual, the accounting identity (3) becomes

$$(4) \quad \mathbf{p}' \bar{\mathbf{A}} + \mathbf{p}'^V \bar{\mathbf{L}} = \mathbf{p}'$$

where $\bar{\mathbf{L}} = \hat{\mathbf{v}} \hat{\mathbf{x}}^{-1}$ is the diagonal matrix of the input coefficients of labor (in quantity), $\bar{l}_j = \bar{v}_j / \bar{x}_j$. So, prices are found as a function of the input coefficients of labor multiplied by the price of labor:

$$(5) \quad \mathbf{p}' = \mathbf{p}'^V \bar{\mathbf{L}} (\mathbf{I} - \bar{\mathbf{A}})^{-1}.$$

Data in value

The demand-driven model in value is those of national accounting, at least for the countries who operate a square model as France (many countries operate a rectangular demand-driven model in value, the Stone model, which is based on two matrices –Make and Use– instead of one). As it is nonsense to multiply values by prices, one traditionally considers index prices that can validly multiply values. Consequently, the perspective is dynamic –that is, two periods– for what concerns prices. Considering the most simple type of price index,⁵ price indexes are defined as the ratio of the prices of a future year \mathbf{p}_* to those of the base year \mathbf{p} : $\pi = (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}}_*$ for all i and $\pi^V = (\hat{\mathbf{p}}^V)^{-1} \hat{\mathbf{p}}_*^V$ for all j , by posing:

$$(6) \quad \mathbf{p}_* = \mathbf{p}_0 + d\mathbf{p}$$

and

$$(7) \quad \mathbf{p}_*^V = \mathbf{p}^V + d\mathbf{p}^V$$

\mathbf{p}_* and \mathbf{p}_*^V are respectively the price vector of commodities and the price vector of labor in the sectors, in a future year; they are also called “shifted prices”. Per rows, the accounting identity is deduced from (2) by calculating its derivative and by using (6) and (7):

$$\hat{\mathbf{p}}_* \bar{\mathbf{Z}} \mathbf{s} + \hat{\mathbf{p}}_* \bar{\mathbf{f}} = \hat{\mathbf{p}}_* \bar{\mathbf{x}}$$

⁵ See Fisher and Shell (1997) for more sophisticated price indexes.

$$(8) \quad \hat{\mathbf{p}}_* (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}} \bar{\mathbf{Z}} \mathbf{s} + \hat{\mathbf{p}}_* (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}}_* \bar{\mathbf{f}} = \hat{\mathbf{p}}_* (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}}_* \bar{\mathbf{x}}$$

Equation (8) yields by using the definition of price indexes:

$$(9) \quad \hat{\boldsymbol{\pi}} \mathbf{Z} \mathbf{s} + \hat{\boldsymbol{\pi}} \mathbf{f} = \hat{\boldsymbol{\pi}} \mathbf{x} \Leftrightarrow \mathbf{Z} \mathbf{s} + \mathbf{f} = \mathbf{x}$$

where $\mathbf{Z} = \hat{\mathbf{p}} \bar{\mathbf{Z}}$ and $\mathbf{x} = \hat{\mathbf{p}} \bar{\mathbf{x}}$ are values (that is, quantities in value).

Coefficients are defined as ratio of values in money terms:

$$(10) \quad \mathbf{A} = \mathbf{Z} \hat{\mathbf{x}}^{-1}$$

what is assumed to be stable. Introducing (10) in equation (9) implies

$$(11) \quad \mathbf{A} \mathbf{x} + \mathbf{f} = \mathbf{x}$$

that is, the output \mathbf{x} in money terms is formed from final demand \mathbf{f} in money terms:

$$(12) \quad \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f}$$

Equation (11) is well known as the standard Leontief quantity model.

Per columns, the accounting identity (3) turns out to be

$$\mathbf{p}_* \bar{\mathbf{Z}} + \mathbf{p}_*^V \hat{\mathbf{v}} = \mathbf{p}_* \hat{\mathbf{x}} \Leftrightarrow \mathbf{p}_* (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}} \bar{\mathbf{Z}} + \mathbf{p}_*^V (\hat{\mathbf{p}}^V)^{-1} \hat{\mathbf{p}}^V \hat{\mathbf{v}} = \mathbf{p}_* (\hat{\mathbf{p}})^{-1} \hat{\mathbf{p}} \hat{\mathbf{x}}$$

what gives

$$(13) \quad \boldsymbol{\pi}' \mathbf{Z} + \boldsymbol{\pi}^V \hat{\mathbf{v}} = \boldsymbol{\pi}' \hat{\mathbf{x}}$$

Equation (13) combined with (10) implies that

$$(14) \quad \boldsymbol{\pi}' \mathbf{A} + \boldsymbol{\pi}^V \mathbf{L} = \boldsymbol{\pi}'$$

where $\mathbf{L} = \hat{\mathbf{v}} \hat{\mathbf{x}}^{-1}$. The price indexes, $\boldsymbol{\pi}$, are formed from the price index of labor, $\boldsymbol{\pi}^V$:

$$(15) \quad \boldsymbol{\pi}' = \boldsymbol{\pi}^V \mathbf{L} (\mathbf{I} - \mathbf{A})^{-1}$$

Equation (14) is known as the standard Leontief price model. The demand-driven model is at the same time rich in its results, credible (in the limits of the theory of production prices and of the hypothesis of linearity...) and useful. All this is well known.

It must be noted that equations (9) and (13) are accounting identities where the monetary quantities \mathbf{Z} , \mathbf{x} and \mathbf{v} belong to the period 0 and the prices indexes refer to periods 0

and t . Hence, equation (9) is valid for period 0; equation (13) is valid for period t but with the structure of period 0. This could seem annoying but we have no other mean to introduce prices validly (in order to follow Leontief but also the authors of production prices as Ricardo, Marx, Sraffa, Pasinetti, etc.). However, we are able to rewrite equation (13) for the period 0 only, what amounts considering that the prices are fixed, that is, $\boldsymbol{\pi} = \boldsymbol{\pi}^V = \mathbf{s}$, yielding

$$(16) \quad \mathbf{x}' = \mathbf{s}' \mathbf{Z} + \mathbf{v}'$$

This accounting identity is often used, but we may remember that we cannot derive prices or price indexes with it: the Leontief model becomes limited to a quantity model in its primal. Nevertheless, Cronin has used it to develop a mixed supply-demand-driven model: this will be discussed later.

3. The supply-driven model

Data in physical quantities

The familiar categories are considered: productive sectors, final demand \bar{f}_i of commodity i , value added \bar{v}_j of sector j , flow \bar{z}_{ij} of commodity i to sector j , price p_i of commodity i , price of labor p_j^V in sector j , etc. The idea is to build a supply-driven model on the same principles than the driven model defined above, that is, on the traditional economic framework of an exchange productive economy. This is what has been done by Dietzenbacher (1997), when he has exposed a vindication of the supply-driven model in terms of a price model, and by most of the authors. The accounting identities are again (2) and (3). The allocation coefficients are assumed to be stable for all i, j , that is,

$$(17) \quad \bar{\mathbf{B}} = \hat{\mathbf{x}}^{-1} \bar{\mathbf{Z}}$$

Unlike the demand driven model, these coefficients are intuitive as they are homogenous (the same commodity is in the numerator and in the denominator). Per rows, denoting $\bar{\mathbf{d}} = \hat{\mathbf{x}}^{-1} \bar{\mathbf{f}}$ the coefficient of final demand, equation (2) implies:

$$(18) \quad \bar{\mathbf{B}} \mathbf{s} + \bar{\mathbf{d}} = \mathbf{s}$$

This equation is a mathematical identity –an expression always true– that never provides prices as it might be expected to do at first glance. Per columns, (3) is transformed into:

$$(19) \quad \mathbf{x}' \bar{\mathbf{B}} + \mathbf{v}' = \mathbf{x}'$$

and only value outputs are found even if we are in the physical model:

$$(20) \quad \mathbf{x}' = \mathbf{v}' (\mathbf{I} - \bar{\mathbf{B}})^{-1}$$

while in the physical Leontief model, outputs in physical quantities and prices are found. Hence, the physical supply-driven model is not a quantity model as claimed by Oosterhaven (1988, p. 205) but it is false to say that it is a "price model" at least at this stage (this will be discussed later); it is a model which determines the product of quantities by prices, that is, values $\mathbf{x} = \hat{\mathbf{p}} \bar{\mathbf{x}}$, not quantities $\bar{\mathbf{x}}$ or prices \mathbf{p} separately. We will see now that things are a little more complicated with the value model.

Data in value

When data are defined in value, by including price indexes, the accounting identities remain (9)-(13). The stable coefficients are defined as

$$(21) \quad \mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{Z}$$

Per rows it holds from the accounting identity (9):

$$(22) \quad \mathbf{B} \mathbf{s} + \mathbf{d} = \mathbf{s}$$

where

$$(23) \quad \mathbf{d} = \hat{\mathbf{x}}^{-1} \mathbf{f}$$

Equation (22) is again a mathematical identity: nothing is found from this primal. Per columns, it comes from the accounting identity (13):

$$(24) \quad \mathbf{s}' \hat{\pi} \hat{\mathbf{x}} \mathbf{B} + \mathbf{s}' \hat{\pi}^V \hat{\mathbf{v}} = \mathbf{s}' \pi \hat{\mathbf{x}} \Leftrightarrow \tilde{\mathbf{x}}' \mathbf{B} + \tilde{\mathbf{v}}' = \tilde{\mathbf{x}}'$$

where

$$(25) \quad \tilde{\mathbf{x}} = \hat{\pi} \mathbf{x} = \hat{\mathbf{p}}_* \bar{\mathbf{x}}$$

and

$$(26) \quad \tilde{\mathbf{v}} = \hat{\pi}^V \mathbf{v} = \hat{\mathbf{p}}_*^V \bar{\mathbf{v}}$$

are values formed by the product of a value by an price index and can be interpreted as “shifted values”, that is, the product of a quantity at the base year by a shifted price. Finally, it comes

$$(27) \quad \tilde{\mathbf{x}}' = \tilde{\mathbf{v}}' (\mathbf{I} - \mathbf{B})^{-1}$$

Again, the value supply-driven model is not a quantity model, or a price model at least at this stage; it is a model which determines the products of values by price indexes, $\hat{\boldsymbol{\pi}} \mathbf{x}$, or quantities by shifted prices, $\hat{\mathbf{p}}_* \bar{\mathbf{x}}$.

4. Discussion

Richness of results

The supply-driven model is not the dual of the Leontief model and is poor regarding the solutions that it provides. The physical supply-driven model gives only an identity per rows whereas only values \mathbf{x} in money terms are found per columns, while normally prices \mathbf{p} and physical quantities $\bar{\mathbf{x}}$ would be hoped. The value model per rows is trivial and allows determining nothing. Per columns the value model allows to find only the product of value outputs by price indexes (or of physical outputs by prices), $\tilde{\mathbf{x}} = \hat{\boldsymbol{\pi}} \mathbf{x} = \hat{\mathbf{p}}_* \bar{\mathbf{x}}$, but it never allows to find separately values \mathbf{x} and price indexes $\boldsymbol{\pi}$ (or physical outputs $\bar{\mathbf{x}}$ and prices \mathbf{p}_*). As there is a dissymmetry between the primal and the dual of this supply-driven model, it is not the mathematical dual of the Leontief model (this one is symmetrical). All this is disappointing but the supply-driven model remains coherent: each sector i sells one commodity i , its homogenous output, and then buys multiple inputs j .

The paradox is that the coefficients of this model are perhaps more intuitive ($\bar{\mathbf{B}}$ and \mathbf{B} are both row-homogenous, as $\bar{\mathbf{Z}}$ is) than the coefficients of the demand driven model (only \mathbf{A} is column-homogenous, not $\bar{\mathbf{A}}$), even if these last ones are the coefficients of a production function. Moreover, as $\mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{Z} = \langle \hat{\mathbf{p}} \bar{\mathbf{x}} \rangle^{-1} \hat{\mathbf{p}} \bar{\mathbf{Z}} = \hat{\mathbf{x}}^{-1} \bar{\mathbf{Z}} = \bar{\mathbf{B}}$ holds (because \mathbf{Z} is homogenous by rows), the physical equation (19) could be rewritten

$$(28) \quad \mathbf{x}' \mathbf{B} + \mathbf{v}' = \mathbf{x}'$$

what implies

$$(29) \quad \mathbf{x}' = \mathbf{v}' (\mathbf{I} - \mathbf{B})^{-1}$$

It must be noted that equation (28) corresponds to the so-called "Ghosh model", the equation being transposed here by respect to Ghosh (1958, p. 61). This could have induces in error some scholars and practitioners: it seems that the value supply-driven model has a definite solution in terms of output in value, while it is the **physical model** which actually has one (remember that it is impossible to find $\bar{\mathbf{x}}$ and \mathbf{p} separately) and while the value supply-driven model is $\tilde{\mathbf{x}}' \mathbf{B} + \tilde{\mathbf{v}}' = \tilde{\mathbf{x}}'$ as indicated by (24). In other words, equation (28) is not the value Ghosh model and is not comparable to the seemingly similar equation of the Leontief model (11).

Economic interpretation of the supply-driven model

If the Ghosh model is not very rich, has it a valid economic interpretation that could allow interesting economic applications? Particularly, what is the interpretation of equations (28)-(29)? An explanation in terms of circuit can be done that allows reminding how the Leontief and Ghosh models "turn" (this is done for the value model but also holds for the physical model). This implies a round-by-round interpretation, which is not strange as it can be found in textbooks as Gale (1989, p. 300).

It is better returning first to the Leontief model: it's classical. At the first round, a sector j receives a new final demand Δf_j of commodity j that it **must** satisfy; it sells the equal quantity $\Delta x_j^{(0)} = \Delta f_j$ of commodity j it has in stock and earns the corresponding money; in order to rebuild its stocks of commodity j , sector j spends all this money buying a quantity $\Delta z_{ij}^{(1)}$ of each input and a quantity $\Delta v_j^{(1)}$ of labor following technical coefficients a_{ij} and l_j : $\Delta z_{ij}^{(1)} = a_{ij} \Delta x_j^{(0)}$ and $\Delta v_j^{(1)} = l_j \Delta x_j^{(0)}$; note that sector j buys the inputs depending of its own production function: the incentive comes from itself. Finally sector i receives an intermediate demand equal to $\Delta x_i^{(1)} = \sum_j a_{ij} \Delta x_j^{(0)}$. This is repeated from rounds to rounds: $\Delta x_i^{(k+1)} = \sum_j a_{ij} \Delta x_j^{(k)}$, that is, $\Delta \mathbf{x}^{(k+1)} = \mathbf{A} \Delta \mathbf{x}^{(k)}$ for all rounds k : this equation is the derivative of (11). This is very classical and corresponds to an economic behavior that is universally considered as acceptable: a sector must satisfy the demand, that is, has an obligation on itself.

For the Ghosh model (the supply-driven model), the same interpretation conducts to a surprising economic behavior: the interpretation in terms of circuit works but it is

economically implausible. We follow here de Mesnard (2004, p. 129). We have to consider an initial increase Δv_i in the value added of sector i that increases the costs of sector i of Δv_i . In the context of the Ghosh model, this generates an increase in the output of sector i equal to $\Delta x_i^{(0)} = \Delta v_i$. This increase in the output may be explained by the fact that the accounts are equilibrated per columns (as well as per rows); hence, all increase in the costs generates an increase in the output; however, beyond accounting necessities, this mechanism could seem strange. To rebuild its wealth, sector i **forces** all agents j to buy a quantity $\Delta z_{ij}^{(1)} = b_{ij} \Delta x_i^{(0)}$ of commodity i (that he has in stock) following the allocation coefficient b_{ij} ; the incentive do not come from itself but from the other sectors i : that is the great difference with the Leontief model. So, the costs of any sector j have increased of $\Delta x_j^{(1)} = \sum_i b_{ij} \Delta x_i^{(0)}$. This repeats from rounds to rounds:

$$(30) \quad \Delta x_j^{(k+1)} = \sum_i b_{ij} \Delta x_i^{(k)}, \text{ for all } i \text{ and } j, \Leftrightarrow \Delta \mathbf{x}^{(k+1)} = \Delta \mathbf{x}^{(k)} \mathbf{B}$$

for all rounds k .

In other words, any agent j is **forced to buy** by the other agents, and this implies that, at his turn, he must force all the agents to buy... It could seem rather strange to consider that a sector can force another sector to buy: one cannot understand why an agent j remains passive by respect to the agents i who force him to buy. Is it a behavior only valid in a centrally planned economy, as suggested by Ghosh? Unfortunately, the true historical centrally planned economies, mainly USSR, were not driven like that even they were authoritarian. The plan was computed following the Leontief model (or at least a model of production prices), for about ten thousand commodities, and not following the Ghosh model. And it is simpler to build a centralized economy where the sectors buy the quantities of input that are necessary for them, depending on their production function, instead of an economy where the sectors must buy the quantities of inputs that are decided by the other sectors: it is obvious that the adaptation to small variations around what has been planned, or the adaptation to take into account the products that are not in the nomenclature, are more easy.

In order to try saving the model, we have found two cases where the supply-driven model takes sense. (i) In the first case, the model is applied to interfirm relations (z_{ij} becomes the quantity sold by firm i to firm j and the model is equation (28). Here, the allocations coefficients can be considered as market shares: the companies may expect their

stability. The companies (denoted by j , in column) are not forced to buy the output of any company i , but each company i (in row) expect companies j to buy its output depending on what is indicated by the allocation coefficients b_{ij} . (ii) In the second case, Gruver (1989, p. 449) considers the transformation process between inputs and output as **productive**, the coefficients of the production function being variable and the production function being of the "substitutable inputs".⁶ This leads to apply the model to interindustry relations when the production function is not specified: the sectors may exchange following what is indicated by the technical coefficients in column, or by the allocation coefficients in row, or following both, meaning that the production function is of the complementary-inputs type (Leontief production function), or of the substitutable-inputs type, or of a mix of both. This allows measuring the structural change without predetermining if the model is demand-driven or supply-driven (unlike previous approaches as the founding work of Leontief (1936) or those of Carter (1970)), by the mean of a biproportional projector. This has been done by de Mesnard (1988, 1997) or later, slightly differently, by van der Linden and Dietzenbacher (1995, 2000), Dietzenbacher and Hoekstra (2003) and Walmsley and McDougall (2007). It remains to discuss another use of the Ghosh model: the cost-push exercises.

The supply-driven model as a price model for cost-push exercises

From equation (8) and (13), Dietzenbacher (1997) gives another interpretation of the supply-driven model. He explains how an increase of costs can be compensated, not by considering that a sector forces the other sectors to buy but by considering that this sector increases its prices: he shows that the supply-driven model remains useful to conduct *cost-push exercises*. As the supply-driven model never finds prices or price indexes in (13), he assumes quantities to be fixed –the variations of \mathbf{v} being composed only of variation of the factor's price– and demonstrates that the variations of \mathbf{x} in the Ghosh model are equivalent to the variation of $\boldsymbol{\pi}$ (which follows a variation of $\boldsymbol{\pi}^V$) in the Leontief model. Expression (30) means that an increase in the cost of any sector i at round k generates an increase in the costs of each sector j at round $k+1$: this comes because of the implicit hypothesis of *cost-plus pricing* that is behind the demand-driven model but also behind the supply-driven model.

⁶ In the Leontief model, agents j are productive sectors, producing and selling a unique output to many other sectors and buying multiple inputs: the transformation process is **productive** and the Leontief production function is of the "complementary inputs" type.

When (20) and (27) are considered, the model still allows to find price indexes and prices: when \mathbf{v} is assumed fixed, $\boldsymbol{\pi}$ can be deduced from $\boldsymbol{\pi}^V$ following (27); or when $\bar{\mathbf{v}}$ is fixed, \mathbf{p}_* can be deduced from \mathbf{p}_*^V following (20). However, in a dynamic perspective, it is not necessary to assume that \mathbf{v} or $\bar{\mathbf{v}}$ is fixed. To prove it, the following good sense axiom is necessary:

Axiom 1. In a dynamic perspective, that is, when two periods are considered, any change takes time. Consequently, are variable only the shifted things, that is, the quantities, values and prices for which the time index passes from the base year to the future year; the other variables are fixed.

Property 1. In a dynamic perspective (two periods), the value supply-driven model can serve only to study the effect of the cost variations (of the exogenous factor) on prices.

Proof. Following axiom 1, from equation (24), only $\boldsymbol{\pi}^V$ and $\boldsymbol{\pi}$ can be variable by respect to the base year (they indicate a price change from the base year to the future year), not \mathbf{v} and \mathbf{x} that both pertain to the base year; from (25) and (26), it is the same thing for \mathbf{p}_*^V and \mathbf{p}_* that can be considered as variable, and for $\bar{\mathbf{v}}$ or $\bar{\mathbf{x}}$ that cannot. Hence, only price indexes or prices can be considered as variable in (27), while in (20) or (29), nothing is variable (because these equations are purely static as they include \mathbf{p} and $\bar{\mathbf{x}}$ only). •

Overall, property 1 saves the credibility of the supply-driven model: the supply-driven model is a model of propagation of cost variations upon prices, what is realistic.

Dietzenbacher's approach (1997) is perfectly valid in a mathematical point of view but rather complicated, while property 1 makes things simple. He considers (Dietzenbacher 1997, p. 632) the equation $\mathbf{x}' = \mathbf{s}'\mathbf{Z} + \mathbf{v}'$ as accounting identity per columns, which is (13) if the price indexes are fixed. In other words, its table 1 which is (with our notations),

(31)

\mathbf{Z}	\mathbf{f}	\mathbf{x}
\mathbf{v}'		$\mathbf{v}'\mathbf{s}$
\mathbf{x}'	$\mathbf{s}'\mathbf{f}$	

is valid only if the prices are fixed, that is, for the base year (which he calls “period 0”). Then he considers that \mathbf{v} varies, the quantities remaining fixed:⁷ he considers the ratio $\mathbf{v}_*'\mathbf{v}^{-1}$ as being equal to the ratio of wages, that is, equal to $\boldsymbol{\pi}^{\mathbf{v}'}$, what allows to retrieve equation (15) from the Ghosh model. This allows him to implicitly introduce price variations. There is no mistake in this reasoning but, from property 1, \mathbf{v} and $\bar{\mathbf{v}}$ are **actually fixed** by nature. Hence, it should have been better to consider the dual value models, (14) and (24), the accounting identities $\boldsymbol{\pi}'\mathbf{Z} + \boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}} = \boldsymbol{\pi}'\hat{\mathbf{x}}$ (13) and $\hat{\boldsymbol{\pi}}\mathbf{Z}\mathbf{s} + \hat{\boldsymbol{\pi}}\mathbf{f} = \hat{\boldsymbol{\pi}}\mathbf{x} \Leftrightarrow \mathbf{Z}\mathbf{s} + \mathbf{f} = \mathbf{x}$ (9), and the corresponding table:

(32)

$\hat{\boldsymbol{\pi}}\mathbf{Z}$	$\hat{\boldsymbol{\pi}}\mathbf{f}$	$\hat{\boldsymbol{\pi}}\mathbf{x}$
$\boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}$		$\boldsymbol{\pi}^{\mathbf{v}'}\mathbf{v}$
$\boldsymbol{\pi}'\hat{\mathbf{x}}$	$\boldsymbol{\pi}'\mathbf{f}$	

to conduct cost-push exercises instead of the table in (31). Now, in the table of (32), if the price of labor increases at least in one sector, it is sufficient to pose $\boldsymbol{\pi}^{\mathbf{v}'} > \mathbf{s}$. Hence, the table of (32) is automatically and naturally equilibrated depending on what model is chosen, the Leontief model with technical coefficients (10):

(33)

$\hat{\boldsymbol{\pi}}\mathbf{A}\hat{\mathbf{x}}$	$\hat{\boldsymbol{\pi}}\mathbf{f}$	$\hat{\boldsymbol{\pi}}\mathbf{x}$
$\boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}$		$\boldsymbol{\pi}^{\mathbf{v}'}\mathbf{v}$
$\boldsymbol{\pi}'\hat{\mathbf{x}} = \boldsymbol{\pi}'\mathbf{A}\hat{\mathbf{x}} + \boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}$	$\boldsymbol{\pi}'\mathbf{f}$	

where $\boldsymbol{\pi}'\hat{\mathbf{x}} = \boldsymbol{\pi}'\mathbf{A}\hat{\mathbf{x}} + \boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}} \Leftrightarrow \boldsymbol{\pi}' = \boldsymbol{\pi}^{\mathbf{v}'}\mathbf{L}(\mathbf{I} - \mathbf{A})^{-1}$ (notice that this table is not identical to Dietzenbacher's table 10: $\boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}$ replaces \mathbf{v}_*'), or the Ghosh model with allocation coefficients (21):

(34)

$\hat{\boldsymbol{\pi}}\hat{\mathbf{x}}\mathbf{B}$	$\hat{\boldsymbol{\pi}}\mathbf{f}$	$\hat{\boldsymbol{\pi}}\mathbf{x}$
$\boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}$		$\boldsymbol{\pi}^{\mathbf{v}'}\mathbf{v}$
$\boldsymbol{\pi}'\hat{\mathbf{x}} = \boldsymbol{\pi}^{\mathbf{v}'}\hat{\mathbf{v}}(\mathbf{I} - \mathbf{B})^{-1}$	$\boldsymbol{\pi}'\mathbf{f}$	

⁷ "Suppose that in each sector the workers require a raise ... in their wages" (Dietzenbacher, 1997, p. 633).

However, the Ghosh model is redundant because the dual of the Leontief model – equations (4)-(14)– does the job as well in a simpler and natural way (it is a production prices model!): the tables in (33) and (34) are equivalent. To prove it, it is sufficient to demonstrate that the price variations given by cells $\{3,1\}$ of both tables are equal, that is,

$$(35) \quad \pi' = \pi^v' \hat{v} (\mathbf{I} - \mathbf{B})^{-1} \hat{x}^{-1} = \pi^v' \mathbf{L} (\mathbf{I} - \mathbf{A})^{-1}$$

As we have $\mathbf{B} = \hat{x}^{-1} \mathbf{A} \hat{x} \Leftrightarrow \mathbf{I} - \mathbf{B} = \hat{x}^{-1} (\mathbf{I} - \mathbf{A}) \hat{x} \Leftrightarrow (\mathbf{I} - \mathbf{B})^{-1} = \hat{x}^{-1} (\mathbf{I} - \mathbf{A})^{-1} \hat{x}$, it comes from (36): $\pi' = \pi^v' \hat{v} (\mathbf{I} - \mathbf{B})^{-1} \hat{x}^{-1} \Leftrightarrow \pi' = \pi^v' \hat{v} \hat{x}^{-1} (\mathbf{I} - \mathbf{A})^{-1} \Leftrightarrow \pi' = \pi^v' \mathbf{L} (\mathbf{I} - \mathbf{A})^{-1}$. QED. It would have been equivalent to demonstrate that the shifted values given by cells $\{3,1\}$ of both tables are equal. Notice that, as $\mathbf{f} = \mathbf{x} - \mathbf{Z}\mathbf{s}$ what combined with (21) $\mathbf{B} = \hat{x}^{-1} \mathbf{Z}$ yields $\mathbf{f} = \mathbf{x} - \hat{x} \mathbf{B} \mathbf{s} \Leftrightarrow \mathbf{f} = \hat{x} (\mathbf{I} - \mathbf{B}) \mathbf{s} \Leftrightarrow \mathbf{f} = \hat{x} (\mathbf{I} - \mathbf{B}) \mathbf{s}$, it comes a simple relation between the prices of outputs and the prices of labor: $\pi' \mathbf{f} = \pi^v' \hat{v} (\mathbf{I} - \mathbf{B})^{-1} \hat{x}^{-1} \hat{x} (\mathbf{I} - \mathbf{B}) \mathbf{s} = \pi^v' \mathbf{v}$.

5. On Cronin's mixed models

Cronin (1984, pp. 523-524) has proposed a completely different approach with two models that mix Ghosh and Leontief. Cronin finds equation (12) as solution of Leontief model (actually the standard Leontief quantity model) and (28) for the Ghosh model without introducing prices. Then, to put Leontief in Ghosh, he starts from the accounting identity (16): $\mathbf{x} = \mathbf{Z}'\mathbf{s} + \mathbf{v}$ (here, it is transposed). He introduces the technical coefficients $\mathbf{A} = \mathbf{Z} \hat{x}^{-1} \Leftrightarrow \mathbf{Z} = \mathbf{A} \hat{x}$ given by (10) what yields

$$(36) \quad \mathbf{x} = \hat{x} \mathbf{A}' \mathbf{s} + \mathbf{v}$$

In vectors, this is $x_j = \left(\sum_i a_{ij}\right) x_j + v_j \Leftrightarrow x_j = \left(1 - \sum_i a_{ij}\right)^{-1} v_j$ for all j , what writes in matrices as $\mathbf{x} = \left(\mathbf{I} - \langle \mathbf{s}' \mathbf{A} \rangle\right)^{-1} \mathbf{v}$. This derivation is rather exotic but perfectly valid. Formula (16) is supposed to be Ghoshian by Cronin.

Nevertheless, formula (16) is only the dual (i.e., per columns) accounting identity of the Leontief model, in value; and it is valid only for period 0 as it has no price indexes. Hence, Cronin could have solved his system following the Leontief model in equations (14)-(15) also: from (36) it comes after transposing: $\mathbf{s}' \hat{x} = \mathbf{s}' \mathbf{A} \hat{x} + \mathbf{v}' \Leftrightarrow \mathbf{s}' (\mathbf{I} - \mathbf{A}) \hat{x} = \mathbf{v}'$

$\Leftrightarrow \mathbf{s}' = \mathbf{v}' \hat{\mathbf{x}}^{-1} (\mathbf{I} - \mathbf{A})^{-1} \Leftrightarrow \mathbf{s}' = \mathbf{s}' \mathbf{L} (\mathbf{I} - \mathbf{A})^{-1}$ what is exactly the solution of the dual of the Leontief model if we pose $\boldsymbol{\pi} = \boldsymbol{\pi}^V = \mathbf{s}$ in (15). Nothing Ghoshian there, only fixed prices.

Similarly, to put Ghosh in Leontief, he starts from the right part of (9): $\mathbf{Z} \mathbf{s} + \mathbf{f} = \mathbf{x}$. He introduces the allocation coefficients $\mathbf{B} = \hat{\mathbf{x}}^{-1} \mathbf{Z}$ of (21) in (9) what yields

$$(37) \quad \mathbf{x} = \hat{\mathbf{x}} \mathbf{B} \mathbf{s} + \mathbf{f}$$

that is, in vectors $x_i = \left(1 - \sum_j b_{ij}\right)^{-1} f_i$ for all i , that is, $\mathbf{x} = \left(\mathbf{I} - \langle \mathbf{s}' \mathbf{B} \rangle\right)^{-1} \mathbf{f}$. Nevertheless, from (37), it comes $\hat{\mathbf{x}} \mathbf{s} = \hat{\mathbf{x}} \mathbf{B} \mathbf{s} + \mathbf{f} \Leftrightarrow \mathbf{s} = \mathbf{B} \mathbf{s} + \hat{\mathbf{x}}^{-1} \mathbf{f} \Leftrightarrow \mathbf{s} = \mathbf{B} \mathbf{s} + \mathbf{d}$, what is exactly the solution (22) of the primal of the Ghoshian model. Again, nothing “Leontiefian” there. Hence, Cronin has not really developed two mixed models, but only a limited rewriting of the Leontief and Ghosh models.

6. Conclusion

This paper has examined the consistency of the alternative input-output model, the supply-driven model developed by Ghosh. It has emphasized on the question of the treatment of quantities and prices, by examining the variant with data in quantities and prices and the variant with data in value and price indexes; each model has been solved in quantities (primal) and in prices (dual), that is, four variants. The comparison has been done with the Leontief model in the same basis, for its four variants. The following conclusions have been set.

The supply-driven model offers solutions of limited interest, being incapable to separate quantities and prices or values and price indexes. Comparing the primal of the Leontief model, $\mathbf{A} \mathbf{x} + \mathbf{f} = \mathbf{x}$, to the Ghosh value model, $\mathbf{x}' \mathbf{B} + \mathbf{v}' = \mathbf{x}'$, is wrong. Even if, in the supply-driven model, the agents are forced to buy by the suppliers, the interpretation of the supply-driven model as a centrally planned economy must be rejected. However, the Ghosh model may serve for modeling interfirm relations or analyzing the structural interindustry change when the production function is not specified. Dietzenbacher's interpretation –the model describes propagation of cost impacts and can serve for cost-push exercises– is fine but our methodology – introducing prices or price indexes by returning to accounting equations– has allows demonstrating that the familiar dual Leontief model of production prices performs the same task in a much simpler way. We have also shown that Cronin's mixed models do not mix demand-driven and supply-driven hypotheses actually.

Table 1 makes the synthesis of all results: the supply-driven model is credible but poor in its results (it does not separate output in quantities from prices and outputs in value from price indexes). Finally, only the Leontief demand-driven model is at the same time rich in its results and credible (it provides physical output and prices, or value outputs and price indexes)...

		Supply - Driven Model	Demand - Driven Model
Physical Model	Per rows (primal)	Identity	\bar{x}
	Per columns (dual)	x	p
Value Model	Per rows (primal)	Identity	x
	Per columns (dual)	\tilde{x}	π
Richness of results		Poor	Good
Usefulness		So-so	Yes
Credibility (production or transformation process)		So-so	Yes

Table 1. Typology and solutions of the models.

\bar{x} is a vector of total outputs or inputs in physical terms;
 x is the vector of total outputs in value; $\tilde{x} = \hat{\pi} x = \hat{p} \cdot \bar{x}$.

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