GROWTH AND SPATIAL DEPENDENCE IN EUROPE

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Abstract

Recent theoretical and empirical work generally often focus on the interdependence of nations and regions underlying that the economy of one country or region is not independent of the economies of others. However, these models generally ignores the impact of location and neighborhood in explaining growth. This paper presents an augmented Solow model that includes spatial externalities and spatial interdependence among economies. I obtain a spatial econometric reduce form which allows testing the effects of the rate of saving, the rate of population growth and the location on income per capita and on the conditional convergence process in Europe.

KEYWORDS: Solow growth model, technological interdependence, spatial externalities, spatial dependence, regional disparities

JEL: C31, R11, O4

1 Introduction

The convergence of European regions has been largely discussed in the empiric literature during the last decade. Two observations are often emphasized. First, the convergence rate among European regions appears to be very slow in the extensive samples considered (Barro and Sala-i-Martin 1991, 1995, Armstrong 1995, Sala-i-Martin 1996a, 1996b). Second, the tools used in the regional science literature show that the geographical distribution of European per capita GDP is highly clustered and so characterized by a strong evidence of global and local autocorrelation (Armstrong 1995, Ertur et al. 2004, López-Bazo et al. 1999 and Le Gallo and Ertur 2003a for UE15 regional database and Ertur and Koch (2004) for UE27 enlarged regional database). Many over studies show also that an evidence of global and local spatial autocorrelation as Rey and Montouri (1999) for US State data on per capita income throughout the period 1929-1994, Ying (2000) for growth rates of production in the Chinese provinces since the late seventies, and Conley and Ligon (2002) who develop an empirical approach that explicitly allows for interdependence among countries, and they underline the importance of cross-country spillovers in explaining growth using an international database.

Another empirical studies show also the importance of geography in the diffusion of knowledge and R&D as Keller (2002) who suggests that the international diffusion of technology is geographically localized, in the sense that the productivity effects of R&D decline with the geographic distance between countries. Audretsch and Feldman (1996), Jaffe (1989), Acs et al. (1992, 1994), Feldman (1994a, b) and Anselin et al. (1997) have identified the existence of spatially-mediated knowledge spillovers of R&D or academic research effects.

Therefore, this paper presents a spatially augmented Solow model that includes technological interdependence among regions in the structural model in order to take into account this global and local spatial autocorrelation and these neighborhood effects on growth and convergence. Thus, I consider the Solow model (Solow 1956, Swan 1956) with physical capital externalities suggested by Romer (1986), Krugman (1991a, b) and Grossman and Helpman (1991), among others, who have focused on the role that spillovers of economic knowledge across agents and firms play in generating increasing returns and ultimately economic growth. I add also spatial externalities in the model in order to take into account spatial knowledge spillovers and technological interdependence between regions.

More specifically, in Section 2, I suppose that the technical progress depends on the stock of physical capital per worker, which represents the stock of knowledge as in Romer (1986), in the home region and depends on the stock of knowledge in the neighboring regions which spills on the technical progress of the home region so as the regions are geographically close. This model leads to an equation for the steady state income level as well as a spatial conditional convergence equation. In Section 3, I present the database and the spatial weight matrix which is used to model spatial connections between all regions in the sample. In Section 4, I estimate the effects of investment rate, population growth and location on the real income per worker at steady state using a spatial econometric specification. I also estimate the magnitude of physical capital externalities at steady state which is usually not identified in the literature. In Section 5, I assess the role played by technological interdependence in growth and convergence processes. For this, I estimate a spatial version of the conditional convergence equation which leads to a convergence speed close to 2% as generally found in the literature. Finally, Section 6 concludes.

2 A spatially augmented Solow model

2.1 Production function and spatial externalities

In this section, I present the Ertur-Koch model (2005) of growth with physical capital externalities and spatial externalities which implies a technological interdependence in Europe between N regions denoted by $i = 1, ..., N$. Let us consider an aggregate Cobb-Douglas production function exhibiting constants returns to scale in labor and reproducible physical capital of the form, in region i at time t:

$$Y_i(t) = A_i(t)K_i^{\alpha}(t)L_i^{1-\alpha}(t) \tag{1}$$

with the standards notations: $Y_i(t)$ the output, $K_i(t)$ the level of reproducible physical capital, $L_i(t)$ the level of labor and $A_i(t)$ the aggregate level of technology:

$$A_i(t) = \Omega(t)k_i^{\phi}(t)\prod_{j\neq i}^{N} A_j^{\gamma w_{ij}}(t) \tag{2}$$

The function describing the aggregate level of technology $A_i(t)$ of any region i depends on three terms. First, as in the Solow model, I suppose that a part of technological progress is exogenous and identical to all regions: $\Omega(t) = \Omega(0)e^{\mu t}$ where μ is its constant rate of growth. Second, I suppose that each region's aggregate level of technology increases with the aggregate level of physical capital per worker $k_i(t) = K_i(t)/L_i(t)$ available in that region.[1] The parameter ϕ, with $0 < \phi < 1$, describes the strength of home externalities generated by the physical capital accumulation. Therefore, I have followed the well-known Arrow's (1962) and Romer's (1986) treatment of knowledge spillover from capital investment. In addition, in the third term, I assume that there are regional externalities emanating from knowledge accumulation in the other regions, which spills over from these neighboring regions j to the considered region i and improves its production efficiency. I suppose that the regional technological interdependence implied by these regional externalities, is given by the parameter γ, with $0 < \gamma < 1$. This parameter is assumed identical for each region but the net effect of these spatial externalities on the level of productivity of the firms in a region i depends on the relative spatial connectivity between this region and its neighbors. I represent the technological interdependence between a region i and all the regions belonging to its neighborhood by the spatial friction parameters w_{ij}, for $j = 1, ..., N$ and $j \neq i$. I assume that these parameters are non negative, non stochastic and finite; we have $0 \leq w_{ij} \leq 1$ and $w_{ij} = 0$ if $i = j$. I also assume that $\sum_{j \neq i}^{N} w_{ij} = 1$ for $i = 1, ..., N$.[2] The more a given region i is connected to its neighbors, the higher w_{ij} is and the more region i benefits from spatial externalities.

Resolving equation (2) for $A_i(t)$ and replacing the result in the production function (1) written per worker, we obtain:

$$y_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^{u_{ii}}(t) \prod_{j \neq i}^{N} k_j^{u_{ij}}(t) \tag{3}$$

with: $u_{ii} = \alpha + \phi \left(1 + \sum_{r=1}^{\infty} \gamma^r w_{ii}^{(r)}\right)$ and: $u_{ij} = \phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)}$ with $w_{ij}^{(r)}$ the element of the line i and the column j of the matrix W to the power of r, and $y_i(t) = Y_i(t)/L_i(t)$ the level of output per worker. We can note that if there is no physical capital externalities, that is $\phi = 0$, we have $u_{ii} = \alpha$ and $u_{ij} = 0$, and then the production function is written as usually.

Finally, in order to warrant the local convergence and then avoid explosive or endogenous growth, I suppose that there is decreasing social return: $\alpha + \frac{\phi}{1-\gamma} < 1$.[3]

As in the textbook Solow model, I assume that a constant fraction of output s_i is saved and that the labor exogenously grows at the rate n_i for a region i. I suppose also a constant and

[1] I suppose that all knowledge is embodied in physical capital per worker and not in the level of capital in order to avoid the scale effects (Jones, 1995).

[2] This hypothesis allows us to assume a relative spatial connectivity between all regions in order to underline the importance of the geographical neighborhood for economic growth. Moreover, it allows us to avoid spatial scale effects and then explosive growth.

[3] See Appendix for the proof.

identical annual rate of depreciation of physical capital for all regions, denoted by δ. We can derive the expression of the output per worker at the steady-state for an economy i:[4]

$$\begin{aligned}\ln y_i^*(t) &= \frac{1}{1-\alpha-\phi}\ln\Omega(t) + \frac{\alpha+\phi}{1-\alpha-\phi}\ln s_i - \frac{\alpha+\phi}{1-\alpha-\phi}\ln(n_i+g+\delta)\\ &- \frac{\alpha\gamma}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln s_j + \frac{\alpha\gamma}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln(n_j+g+\delta)\\ &+ \frac{\gamma(1-\alpha)}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln y_j^*(t) \qquad (4)\end{aligned}$$

This spatially augmented Solow model has the same qualitative predictions as the textbook Solow model[5] about the influence of the own saving rate and the own population growth rate on the real income per worker of a region i at steady-state. First, the real income per worker at steady state for a region i depends positively on its own saving rate and negatively on its own population growth rate. Second, it can also be shown that the real income per worker for a region i depends positively on saving rates of neighboring regions and negatively on their population growth rates.[6]

2.2 Transitional dynamic and local convergence

As the textbook Solow model, this model predicts that income per worker in a given region converges to that region's steady state value. Writing the fundamental dynamic equation of Solow including the production function (3), we obtain:

$$\frac{\dot{k}_i(t)}{k_i(t)} = s_i\Omega^{\frac{1}{1-\gamma}}(t)k_i^{-(1-u_{ii})}(t)\prod_{j\neq i}^{N} k_j^{u_{ij}}(t) - (n_i+\delta) \qquad (5)$$

The main element behind the convergence result in this model is also diminishing returns to reproducible capital. Physical capital externalities and technological interdependence only slow down the decrease of marginal productivity of physical capital, therefore the convergence result is still valid under the hypothesis $\alpha + \frac{\phi}{1-\gamma} < 1$, in contrast with endogenous growth models where marginal productivity of physical capital is constant.

[4] The balanced rate of growth is $g = \frac{\mu}{(1-\alpha)(1-\gamma)-\phi}$.

[5] Note that when $\gamma = 0$, we have the model elaborated by Romer (1986) with $\alpha + \phi < 1$ and when $\gamma = 0$ and $\phi = 0$, we have the Solow model.

[6] In fact, this equation is written in implicit form. When we write this equation in explicit form it is possible to evaluate elasticities of variables. See Ertur and Koch (2005) for more details about the predictions of this model.

In addition, our model makes quantitative predictions about the speed of convergence to steady state. As in the literature, the transitional dynamics can be quantified by using a log linearisation of equation (5) around the steady state, for $i = 1, ..., N$:

$$\begin{aligned} \frac{d \ln k_i(t)}{dt} &= -(1 - u_{ii})(n_i + g + \delta)[\ln k_i(t) - \ln k_i^*] \\ &+ \sum_{j \neq i}^{N} u_{ij}(n_i + g + \delta)[\ln k_j(t) - \ln k_j^*] \end{aligned} \tag{6}$$

We obtain a system of differential linear equations. Let us note $\chi_i(t) = [\ln k_i(t) - \ln k_i^*]$ and $\dot{\chi}_i(t) = \frac{d \ln k_i(t)}{dt}$, for $i = 1, ..., N$, we obtain in matrix form:

$$\dot{\chi}(t) = J\chi(t) \tag{7}$$

where:

$$J = -(1 - \alpha)diag(n + g + \delta) + \phi diag(n + g + \delta)(I - \gamma W)^{-1} \tag{8}$$

is the matrix of the system, with $diag(n_i+g+\delta)$ the diagonal matrix with the terms $(n_i+g+\delta)$.[7] The general solution of the system can be write in the following matrix form: $\chi(t) = VDb$, where D is the diagonal matrix with the terms $e^{\lambda_J t}$ with λ_J the eigenvalues of the matrix J, V the matrix of characteristic vectors associated with the eigenvalues of J and b a vector of constant which we can evaluate with the initial condition. Indeed, since the matrix J is d-stable, its eigenvalues are negatives and so: $\chi(0) = Vb$, then: $b = V^{-1}\chi(0)$. Finally the general solution can be written in the following form: $\chi(t) = VDV^{-1}\chi(0)$, or:

$$\ln k(t) - \ln k^* = VDV^{-1}[\ln k(0) - \ln k^*] \tag{9}$$

and subtracting both sides by $\ln k(0)$ and rearranging terms:

$$\ln k(t) - \ln k(0) = -(I - VDV^{-1}) \ln k(0) + (I - VDV^{-1}) \ln k^* \tag{10}$$

Replacing $\ln k^*$ by its expression (12) in matrix form:

$$\ln k^* = [(1 - \alpha)I - \phi(I - \gamma W)^{-1}][(I - \gamma W)^{-1}\Omega + S] \tag{11}$$

[7]See Appendix for a proof of local convergence.

where S is the $(N \times 1)$ vector of logarithms of saving rate divided by the effective rate of depreciation, we obtain after rearranging terms:

$$\begin{aligned} & \ln k(t) - \ln k(0) = -(I - VDV^{-1}) \ln k(0) \\ + & \frac{\phi}{1-\alpha}(I - VDV^{-1})(I - \gamma W)^{-1} \ln k(0) \\ + & \frac{1}{1-\alpha}(I - VDV^{-1})(I - \gamma W)^{-1}\Omega + \frac{1}{1-\alpha}(I - VDV^{-1})S \\ + & \frac{\phi}{1-\alpha}(I - VDV^{-1})(I - \gamma W)^{-1}(I - VDV^{-1})^{-1}[\ln k(t) - \ln k(0)] \end{aligned} \tag{12}$$

This equation shows that the convergence process of a region i is more complicated than the usual equation in the literature since it depends not only on usual variables as initial level of output per worker, the saving rate and the population growth rate, but also on the same variables in the neighboring regions. It depends also on the rate of growth of these neighboring regions reflecting global technological interdependence. However, we can note that if there is no physical capital externalities, that is $\phi = 0$, we can reduce this equation to those the traditional conditional convergence equation except for the constant term with exogenous technical progress. Another case is on interest: when we consider the case of unconditional convergence process, we have $n_i = n$ for all $i = 1, ..., N$, and then the eigenvalues of the matrix J can be rewrite in function of those of W matrix denoted by λ_W. Indeed, we have:

$$\lambda_J = -\left(1 - \alpha - \frac{\phi}{1 - \gamma\lambda_W}\right)(n + g + \delta) \tag{13}$$

3 Data and spatial weight matrix

All data are extracted from the Cambridge database. More precisely, I consider 204 European regions belonging to 17 countries over the 1977-2000 period at NUTS2 level for Belgium (11), Denmark (1), Germany (31), Greece (13), Spain (16), France (22), Ireland (2), Italy (20), Luxembourg (1), the Netherlands (12), Austria (9), Portugal (1), Finland (6), Sweden (8), United Kingdom (37), Norway (7), Switzerland (7). I measure n as the average growth rate of the working-age population (ages 15 to 64), real income per worker is measured by the GVA (Gross Value Added) divided by the number of worker, and finally the saving rate s is measured as the average share of gross investment in GVA.

The Markov-matrix W, containing the terms w_{ij}, corresponds to the so called spatial weight matrix commonly used in spatial econometrics to model spatial interdependence between regions or countries (Anselin 1988). More precisely, each region is connected to a set of neighboring regions by means of a purely spatial pattern introduced exogenously in W. The elements w_{ii} on the diagonal are set to zero whereas the elements w_{ij} indicate the way the region i is spatially connected to the region j. In order to normalize the outside influence upon each region, the weight matrix is standardized such that the elements of a row sum up to one. For the variable x, this transformation means that the expression Wx, called the spatial lag variable, is simply the weighted average of the neighboring observations.

Various matrices are considered in the literature: a simple binary contiguity matrix, a binary spatial weight matrix with a distance-based critical cut-off, above which spatial interactions are assumed negligible, more sophisticated generalized distance-based spatial weight matrices with or without a critical cut-off. The notion of distance is quite general and different functional forms based on distance decay can be used (for example inverse distance, inverse squared distance, negative exponential etc.). The critical cut-off can be the same for all regions or can be defined to be specific to each region leading in the latter case, for example, to $k-$nearest neighbors weight matrices when the critical cut-off for each region is determined so that each region has the same number of neighbors.

It is important to stress that the connectivity terms w_{ij} should be exogenous to the model to avoid the identification problems raised by Manski (1993) in social sciences. This is the reason why we consider pure geographical distance, more precisely great circle distance between centroid, which is indeed strictly exogenous; the functional form I consider is simply the $k-$nearest neighbors weight matrix $W(k)$ with the general term defined as follows in standardized form $[w(k)_{ij}]$:

$$w(k)_{ij} = w(k)^*_{ij} / \sum w(k)^*_{ij} \quad \text{with} \quad w(k)^*_{ij} = \begin{cases} 0 \text{ if } & i = j \\ 1 \text{ if } & d_{ij} \leqslant d_i(k) \\ 0 \text{ if } & d_{ij} > d_i(k) \end{cases} \tag{14}$$

where d_{ij} is the great circle distance between regional centroid and $d_i(k)$ is a critical cut-off distance defined for each region i. More precisely, $d_i(k)$ is the $k-$th order smallest distance between regions i and j so that each region i has exactly k neighbors. In this analyze, I consider $k = 10$.

4 Influence of saving rate and population growth on real income per worker distribution and growth

4.1 Empirical model and spatial econometric framework

In this section, I follow Mankiw et al. (1992) in order to evaluate the impact of saving, population growth and location on real income. Taking equation (4), we find that the real income per worker along the balanced growth path, at a given time ($t = 0$ for simplicity) is:

$$\begin{aligned}\ln\left[\frac{Y_i}{L_i}\right] &= \beta_0 + \beta_1 \ln s_i + \beta_2 \ln(n_i + g + \delta) + \theta_1 \sum_{j\neq i}^{N} w_{ij} \ln s_j \\ &+ \theta_2 \sum_{j\neq i}^{N} w_{ij} \ln(n_j + g + \delta) + \rho \sum_{j\neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \end{aligned} \tag{15}$$

where $\frac{1}{1-\alpha-\phi}\ln\Omega(0) = \beta_0 + \varepsilon_i$, for $i = 1, ..., N$, with β_0 a constant and ε_i a region-specific shock since the term $\Omega(0)$ reflects not just technology but also resource endowments, climate, institutions, and so on ..., and then it may differ across regions. We suppose also that $g+\delta = 0.05$ as used in the literature since Mankiw et al. (1992) and Romer (1989). We have finally the following theoretical constraints between coefficients: $\beta_1 = -\beta_2 = \frac{\alpha+\phi}{1-\alpha-\phi}$ and $\theta_2 = -\theta_1 = \frac{\alpha\gamma}{1-\alpha-\phi}$. Equation (15) is our basic econometric specification in this section.

In the spatial econometrics literature, this kind of specification, including the spatial lags of both endogenous and exogenous variables, is referred to as the spatial Durbin model (see Anselin, 1988, 2001), we have in matrix form:

$$y = X\beta + WX\theta + \rho Wy + \varepsilon \tag{16}$$

here y is the $(N \times 1)$ vector of logarithms of real income per worker, X the $(N \times 3)$ matrix with the constant term, the vectors of logarithms of investment rate and the logarithms of physical capital effective rates of depreciation, W the $(N \times N)$ spatial weight matrix, $\beta' = [\beta_0\ \beta_1\ \beta_2]$, $\theta' = [\theta_1\ \theta_2]$ and $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$ is the spatial autocorrelation coefficient.[8] ε is the $(N \times 1)$ vector of errors supposed identically and normally distributed so that $\varepsilon \sim N(0, \sigma^2 I)$.

[Table 1 around here]

[8] In practice, the spatially lagged constant is not included in WX, since there is an identification problem for row-standardized W (the spatial lag of a constant is the same as the original variable).

In the first column of table 1, I estimate the textbook Solow model. The coefficients of saving and population growth have the predicted signs. However, the coefficients are weakly significant and the effect of saving rate is lower than as expected. The overidentifying restriction is not rejected and the estimated capital share is close to 0.2 the lower bound of this value generally found. The Solow model is misspecified since it omits variables due to technological interdependence and physical capital externalities. Indeed, we can write the spatially augmented Solow model in the following matrix form:

$$y = \frac{\alpha}{1-\alpha} S + \frac{\phi}{1-\alpha}(I - \gamma W)^{-1} \ln k^* + (I - \gamma W)^{-1} \varepsilon \tag{17}$$

with S the $(N \times 1)$ vector of logarithms of investment rate divided by the effective rate of depreciation. Therefore the error term in the Solow model contains omitted information since we can rewrite it:

$$\varepsilon_{Solow} = \frac{\phi}{1-\alpha}(I - \gamma W)^{-1} k^* + (I - \gamma W)^{-1} \varepsilon \tag{18}$$

We also note the presence of spatial autocorrelation in the error term even if there is no physical capital externalities, and then the presence of technological interactions between all countries through the inverse spatial transformation $(I - \gamma W)^{-1}$.

In the second column of table 1, I estimate the spatially augmented Solow model with the maximum likelihood method.[9] Many aspects of the results support the model. First, all the coefficients have the predicted signs and the spatial autocorrelation coefficient, ρ, is highly positively significant. Second, the coefficients of saving rates of the region i and its neighboring regions j are significant. Third, the joint theoretical restriction $\beta_1 = -\beta_2$ and $\theta_2 = -\theta_1$ is not rejected since the $p-$value of the LR test is 0.572. Finally the α implied by the coefficients in the constrained regression is significantly close to one-third as expected. The coefficient γ, representing the strength of spatial externalities, is very strong since it is higher than 1. This result shows the importance of spatial externalities in the distribution of income in Europe. However, many aspects of the results seem do not support the model. Indeed, the implied value of $\alpha + \frac{\phi}{1-\gamma}$ is too high since its value is higher than 3 but it is not significant. Moreover, the ϕ estimated is negative with a $p-$value of 0.116 which is indicate there is not physical capital

[9] James LeSage provides a function to estimate this model in his Econometric Toolbox for Matlab (http://www.spatial-econometrics.com). The regularity conditions of the maximum likelihood estimators are described in Lee (2004).

externalities in the european regions. This result is convergent with the evidence against the importance of permanent within-industry knowledge spillovers for growth at the regional and urban level (see Glaeser et al. 1992). More specifically, we can test the absence of physical capital externalities represented by ϕ since $\phi = 0$ implies in the specification (15) the following expression:

$$\ln\left[\frac{Y_i}{L_i}\right] = \beta_0' + \beta_1' \ln s_i + \beta_2' \ln(n_i + g + \delta) + \theta_1' \sum_{j\neq i}^{N} w_{ij} \ln s_j + \theta_2' \sum_{j\neq i}^{N} w_{ij} \ln(n_j + g + \delta) + \gamma \sum_{j\neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \quad (19)$$

with $\beta_1' = -\beta_2' = \frac{\alpha}{1-\alpha}$, $\theta_2' = -\theta_1' = \frac{\alpha\gamma}{1-\alpha}$ hence $\theta_1' + \beta_1'\gamma = 0$ and $\theta_2' + \beta_2'\gamma = 0$. Specification (19) is the so-called constrained spatial Durbin model which is formally equivalent to a spatial error model written in matrix form:

$$y = X\beta' + \varepsilon_{Solow} \quad \text{and} \quad \varepsilon_{Solow} = \gamma W \varepsilon_{Solow} + \varepsilon \quad (20)$$

where $\beta' = [\beta_0' \; \beta_1' \; \beta_2']$ and ε_{Solow} is the same as before with $\phi = 0$. Hence, we have the textbook Solow model with spatial autocorrelation in the errors terms. We estimate the Spatial Error Model in the third column of the table 1. We note that the coefficients have the predicted signs and the spatial autocorrelation coefficient in error term, γ, is also highly positively significant. We can test the non-linear restrictions with the common factor test (Burridge, 1981). The LR value of the test is 1.883 and its $p-$value is close to 0.19, so we can't reject the non-linear restrictions. This direct test supports also the absence of physical capital externalities.

Finally, we should note that these regressions based on the methodology proposed by Mankiw et al. (1992), are valid only if the regions are at their steady states or if deviations from steady state are random. So, as already shown by Jones (1997) with international data, most of the regions in Europe have probably not reached their steady-state level. Then, in order to study more precisely the distribution of real income per worker in Europe, we must take into account out-of-steady-state dynamics with a spatial conditional convergence.

4.2 A spatial conditional convergence model

The spatial convergence model can't be estimate directly with equation (12). In this section, we suppose, with the results of the section (4.1), that there is no physical capital externalities ($\phi = 0$), which implies that the matrix J reduces to a diagonal matrix with the terms $-(1-\alpha)(n+g+\delta)$ on its diagonal.[10] As a result, the resolution is now identical to the traditional problem in the literature. Indeed, for each region $i = 1, ..., N$, the equation (6) can be rewrite for the income per worker[11]:

$$\frac{d \ln y_i(t)}{dt} = \frac{\mu}{1-\gamma} - (1-\alpha)(n+g+\delta)[\ln y_i(t) - \ln y_i^*] \tag{21}$$

The solution for $\ln y_i(t)$, subtracting $\ln y_i(0)$, the real income per worker at some initial date, from both sides, is:

$$\begin{aligned} \ln y_i(t) - \ln y_i(0) &= (1-e^{-\lambda t})\frac{\mu}{1-\gamma}\frac{1}{\lambda} - (1-e^{-\lambda t}) \ln y_i(0) \\ &+ (1-e^{-\lambda t}) \ln y_i^* \end{aligned} \tag{22}$$

The model predicts convergence since the growth of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady-state. Rewrite equation (22) in matrix form: $\ln y(t) - \ln y(0) = (1-e^{-\lambda t})[C - \ln y(0) + \ln y^*]$ where $\ln y(0)$ is the $(N \times 1)$ vector of the logarithms of initial level of real income per worker, $\ln y^*$ is the $(N \times 1)$ vector of the logarithms of real income per worker at steady-state, C is the $(N \times 1)$ vector of constant. Introducing equation (4) in matrix form: $\ln y^* = (I - \gamma W)^{-1}[\frac{1}{1-\alpha}\Omega + \frac{\alpha}{1-\alpha}S - \frac{\alpha\gamma}{1-\alpha}WS]$, where S is the $(N \times 1)$ vector of logarithms of saving rate divided by the effective rate of depreciation, premultiplying both sides by the inverse

[10] If the physical capital externalities are different to 0, Ertur and Koch (2005) proposes to simplify the system assuming that the gaps of economics in respect to their own steady states are proportionate. They can give a local version of the spatial $\beta-$convergence model display in this paper.

[11] I suppose also that the speed of convergence is identical for all regions as in the traditional literature about conditional convergence (Barro and Sala-i-Martin 1991, 1992, 1995, Mankiw et al. 1992). In fact, in the Solow growth model, each speed of convergence depends on each country because of the population rates of growth n_i in its expression. See Durlauf et al. (2001), Ertur et al. (2004) or Ertur and Koch (2005) for local version of the Solow growth model.

of $(I-\rho W)^{-1}$ and rearranging terms we obtain:

$$
\begin{aligned}
\ln y(t) - \ln y(0) &= (1-e^{-\lambda t})(C + \frac{1}{1-\alpha}\Omega) - (1-e^{-\lambda t})y(0) \\
&+ \gamma(1-e^{-\lambda t})Wy(0) + \frac{\alpha}{1-\alpha}(1-e^{-\lambda t})S \\
&- \frac{\alpha\gamma}{1-\alpha}(1-e^{-\lambda t})WS + \gamma W[\ln y(t) - \ln y(0)] \qquad (23)
\end{aligned}
$$

Finally, dividing by T in both sides, we can rewrite this equation for a region i:

$$
\begin{aligned}
\frac{\ln y_i(t) - \ln y_i(0)}{T} &= \beta_0 + \beta_1 \ln y_i(0) + \beta_2 \ln s_i + \beta_3 \ln(n_i + g + \delta) \\
&+ \theta_2 \sum_{j\neq i}^{N} w_{ij} \ln s_j + \theta_3 \sum_{j\neq i}^{N} w_{ij} \ln(n_j + g + \delta) \\
&+ \theta_1 \sum_{j\neq i}^{N} w_{ij} \ln y_j(0) + \gamma \sum_{j\neq i}^{N} w_{ij} \frac{\ln y_j(t) - \ln y_j(0)}{T} \\
&+ \varepsilon_i \qquad (24)
\end{aligned}
$$

where $\beta_0 = (1-e^{-\lambda T})(\frac{\mu}{1-\gamma}\frac{1}{\lambda} + \frac{1}{1-\alpha}\Omega(T))$ is a constant, $\beta_1 = -\frac{(1-e^{-\lambda T})}{T}$, $\beta_2 = -\beta_3 = \frac{(1-e^{-\lambda T})}{T}\frac{\alpha}{1-\alpha}$, $\theta_1 = \frac{(1-e^{-\lambda T})}{T}\gamma$, $\theta_3 = -\theta_2 = \frac{(1-e^{-\lambda T})}{T}\frac{\alpha\gamma}{1-\alpha}$. In matrix form, we have the constrained spatial Durbin model which is estimated in the same way as the model in the section (4.1).

We note that this empirical specification is very close to empirical studies in the recent growth literature using geographical data and applying the appropriate spatial econometric tools (see for example Ertur et al. (2005), Fingleton (1999) and Le Gallo et al. (2003b)). However, the model in this paper, is directly link to the theoretical model and the structural parameters can be now recovered.

[Table 2 around here]

In the first column of table 2, I estimate a model of unconditional convergence. This result shows that there is convergence between european regions since the coefficient on the initial level of income per worker is negative and strongly significative. Therefore, there is tendency for poor regions to grow faster on average than rich regions in Europe. Note that this result is different to the traditional result in the literature about the failure of income convergence in international cross-countries (De Long 1988, Romer 1987 and Mankiw et al. 1992). I test the convergence predictions of the textbook Solow model in the second column of table 2. I report regressions of growth rate over the period 1977 to 2000 on the logarithm of income per worker in 1977, controlling for investment rate and growth of working-age population. The coefficient on the initial level of income is also significantly negative; that is, there is strong evidence of convergence. The results support also the predicted signs of investment rate and working-age population growth rate. However, the speed of convergence associated with both estimations is close to 0.7% far below 2% usually found in the convergence literature (Barro and Sala-i-Martin 1995 for instance). The half-life is about 96 years which indicates that the process of convergence is indeed very weak.

[Table 3 around here]

The textbook Solow model is misspecified since it omits variables due to regional technological interdependence. Therefore, as in Section (4.1), the error terms of the Solow model contains omitted information and are spatially autocorrelated. In table 3, I estimate the spatially augmented Solow model. Many aspects of the results support this model. First, all the coefficients are significants and have the predicted signs. The spatial autocorrelation coefficient ρ is highly positively significant which shows the importance of the role played by regional technological interdependence on the convergence process. Second, the coefficient on the initial level of income is significantly negative, so there is strong evidence of convergence after controlling for those variables that the spatially augmented Solow model says determine the steady state. Third, the λ implied by the coefficient on the initial level of income is about 1.4% which is more closer to the value usually found about the speed of convergence in the literature. However, the common factor test is strongly reject since the LR value is 18.664 with a $p-$value of 0.000. The theoretical non-linear constrains are then reject by the data, so we don't conclude precisely about the hypothesis of the absence of physical capital externalities ($\phi = 0$). The Spatial Error Model implied by this hypothesis fits good the data since all the coefficients are significants and have the predicted signs and the implied λ is about 1.2%, a value less by those implied by the Spatial Durbin Model above.

5 Conclusion

In this paper, I developed a neoclassical growth model which explicitly takes into account technological interdependence between regions under the form of spatial externalities. The qualitative predictions of this spatially augmented Solow model provided us with a better understanding of the important role played by geographical location and neighborhood effects in the growth and convergence processes. In addition, the econometric model leads to estimates of structural parameters close to predicted values. The estimated capital share parameter is close to one third, but the physical capital externalities are not significant and we can conclued to absence of Marshallian externalities in European Regions. This result is close to those found in the literature as Glaeser et al. (1992) for instance. The strong value of technological parameter is convergent with the high spatial autocorrelation usually found in the regional science literature and shows also the important role played by technological interdependence in the economic growth and income distribution processes.

Our results are then important to better understand the phenomena of spatial autocorrelation generally found in the spatial distribution of income and in the regional economic growth and convergence. Moreover the empiric consequences show that the traditional econometrics results are misspecified, since they omit spatially autocorrelated errors and spatially autoregressive variable.

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Appendix

$$
\begin{aligned}
& \alpha + \frac{\phi}{1-\gamma} < 1 \\
\Leftrightarrow \quad & u_{ii} + \sum_{j \neq i}^{N} u_{ij} < 1 \\
\Leftrightarrow \quad & \alpha + \phi + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} + \phi \sum_{j \neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < 1 \\
\Leftrightarrow \quad & \phi \sum_{j \neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < (1-\alpha-\phi) - \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \\
\Leftrightarrow \quad & \sum_{j \neq i}^{N} \left| \phi \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} \right| < \left| -(1-\alpha-\phi) + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \right| \qquad \blacksquare
\end{aligned}
$$

Therefore, with the dominant negative diagonal theorem, the matrix J is d-stable and then the system is locally stable.

Table 1: Estimation results: Textbook Solow and spatially augmented Solow models

Model	TextBook Solow	Spatial aug. Solow (SDM)	Spatial aug. Solow (SEM)
Dep. var.	$\ln y_i(2000)$	$\ln y_i(2000)$	$\ln y_i(2000)$
Obs.	204	204	204
constant	10.256	1.628	10.239
	(0.000)	(0.198)	(0.000)
$\ln s_i$	0.292	0.303	0.262
	(0.074)	(0.038)	(0.067)
$\ln(n_i + 0.05)$	-0.135	-0.102	-0.077
	(0.566)	(0.569)	(0.666)
$W \ln s_j$	–	-0.504	–
		(0.059)	
$W \ln(n_j + 0.05)$	–	0.330	–
		(0.409)	
$W \ln y_j$ (SDM) / γ (SEM)	–	0.872	0.866
		(0.000)	(0.000)
Restricted regression			
constant	9.862	1.597	9.794
	(0.000)	(0.001)	(0.000)
$\ln s_i - \ln(n_i + 0.05)$	0.245	0.233	0.199
	(0.101)	(0.074)	(0.121)
$W[\ln s_j - \ln(n_j + 0.05)]$	–	-0.431	–
		(0.057)	
$W \ln y_j$	–	0.867	0.864
		(0.000)	(0.000)
Test of restriction	0.237 (Wald)	1.119 (LR)	0.953 (LR)
	(0.627)	(0.572)	(0.329)
Implied α	0.197	0.332	0.166
	(0.040)	(0.005)	(0.063)
Implied ϕ	–	-0.143	–
		(0.116)	
Implied γ	–	1.052	0.866
		(0.000)	(0.000)
$\alpha + \frac{\phi}{1-\gamma}$	–	3.071	–
		(0.923)	

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method. LR means likelihood ratio. I use, in the first column, the White heteroskedasticity consistent covariance matrix estimator for statistical inference in the OLS estimation.

Table 2: Estimation results: unconditional convergence and textbook Solow

Model	Unconditional conv.	TextBook Solow
Dep. var.	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$
Obs.	204	204
constant	0.085	0.073
	(0.000)	(0.045)
$\ln y_i(1960)$	-0.007	-0.007
	(0.000)	(0.000)
$\ln s_i$	–	0.019
		(0.000)
$\ln(n_i + 0.05)$	–	-0.013
		(0.001)
Implied λ	0.014	0.012
Half-life	51.34	57.28

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method. LR means likelihood ratio. I use the White heteroskedasticity consistent covariance matrix estimator for statistical inference in the OLS estimation.

Table 3: Estimation results: Textbook Solow and spatially augmented Solow models

Model	Spatial aug. Solow (SDM)	Spatial aug. Solow (SEM)
Dep. var.	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$
Obs.	204	204
constant	-0.001	0.114
	(0.979)	(0.000)
$\ln y_i(1960)$	-0.012	-0.011
	(0.000)	(0.000)
$\ln s_i$	0.031	0.028
	(0.000)	(0.000)
$\ln(n_i + 0.05)$	-0.019	-0.017
	(0.000)	(0.001)
$W \ln y_j(1960)$	0.010	–
	(0.000)	
$W \ln s_j$	-0.041	–
	(0.000)	
$W \ln(n_j + 0.05)$	0.015	–
	(0.165)	
$W\left(\frac{\ln y_j(1995)-\ln y_j(1960)}{35}\right)$ (SDM) / γ (SEM)	0.447	0.664
	(0.000)	(0.000)
Implied λ	0.014	0.012
Half-life	51.34	57.28

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method. LR means likelihood ratio.