Growth, Technological Interdependence and Spatial Externalities: Theory and Evidence

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Abstract

This paper presents a theoretical growth model which explicitly takes into account technological interdependence among economies and examines the impact of neighborhood effects. Technological interdependence is assumed to operate through spatial externalities. The magnitude of the physical capital externalities at steady state, which is not usually identified in the literature, is estimated using a spatial econometric specification. Spatial externalities are found to be significant. This spatially augmented Solow model yields a conditional convergence equation which is characterized by parameter heterogeneity. A locally linear spatial autoregressive specification is then estimated providing a convergence speed estimate for each country of the sample.

KEYWORDS: Conditional convergence, technological interdependence, spatial externalities, spatial autocorrelation, parameter heterogeneity, locally linear estimation

JEL: C14, C31, O4

1 Introduction

Why have some countries grown rich while others have remained poor? This is a recurrent question in the literature on theoretical and empirical economic growth. One of the traditional stylized facts about growth over the last fifty years is that national growth rates appear to depend critically on the growth rates and income levels of other countries, rather than just on any one country's own domestic investment rates in physical and human capital. For example, Easterly and Levine (2001) present as a stylized fact the concentration of economic activity at different scales: world, countries, regions, cities. More recently, Klenow and Rodriguez-Clare (2005) present four main facts reflecting world-wide interdependence, which could be explained by cross-country externalities. First, the growth slowdown that began in the mid-1970s was a world-wide phenomenon. It hit both rich and poor economies on every continent. Second, richer countries grew much more slowly from 1950 to around 1980, although richer economies invested at higher rates in physical and human capital. Third, differences in national investment rates are far more persistent than differences in national growth rates. Finally, countries with high investment rates tend to have high levels of income rather than high growth rates. Klenow and Rodriguez-Clare (2005) argue that these facts are evidence for large international spillovers playing an important role in the long-run growth process.

Knowledge accumulated in one country depends on knowledge accumulated in other countries. These spatial externalities involve technological interdependence among countries. Therefore, in this paper we argue that a model needs to include this global interdependence phenomenon in order to explain development and growth. Several models of economic growth emphasize the importance of international spillovers as a major engine of technological progress. These international spillovers result from foreign knowledge through international trade and foreign direct investment (Coe and Helpman 1995, Eaton and Kortum 1996, Caves 1996), or technology transfers (Barro and Sala-i-Martin 1997, Howitt 2000) or human capital externalities (Lucas 1988, 1993). In addition, Keller (2002) suggests that the international diffusion of technology is geographically localized, in the sense that the productivity effects of R&D decline with the geographic distance between countries.

Moreover, in the recent literature, several papers provide empirical evidence of spatial effects, spatial autocorrelation and heterogeneity, on growth. As noted by DeLong and Summers (1991, p.487: "it is difficult to believe that Belgian and Dutch or US and Canadian economic growth would ever significantly diverge, or that substantial productivity gaps would appear within Scandinavia". Mankiw (1995) points out that multiple regression in the standard framework treats each country as if it were an independent observation. Temple (1999), in his survey of the new growth evidence, draws attention to error correlation and regional spillovers though he interprets these effects as mainly reflecting an omitted variable problem. In empirical papers, Conley and Ligon (2002), Ertur et al. (2005) and Moreno and Trehan (1997) use geographic and economic distance to underline the impact of cross-country spillovers on growth processes.

In theoretical work, international spillovers have been studied mostly in the framework of endogenous growth models (Aghion and Howitt 1992; Grossman and Helpman 1991; Rivera-Batiz and Romer 1991; Howitt, 2000). Nevertheless, in this paper we consider the more tractable neoclassical growth model (Solow 1956) as augmented for example by Mankiw et al. (1992) and we propose an estimate of the magnitude of world-wide technological interdependence. More precisely, this paper presents an augmented Solow model that includes spatial externalities between countries in order to take account of the neighborhood effects on growth and convergence processes. Thus, we consider a growth model including both physical capital externalities as suggested by the Frankel-Arrow-Romer model (Arrow 1962, Frankel 1962 and Romer 1986) and spatial externalities in knowledge to model technological interdependence. In Section 2, we suppose that technical progress depends on the stock of physical capital per worker, which is complementary with the stock of knowledge in the home country as in Romer (1986). It also depends on the stock of knowledge in other countries which affects the technical progress of the home country. The intensity of this spillover effect is assumed to be related to some concept of socioeconomic or institutional proximity, which we capture by exogenous geographical proximity. Our model provides an equation for the steady state income level as well as a conditional convergence equation characterized by parameter heterogeneity. Therefore, in Section 3, after presenting the database and the spatial weight matrix which is used to model the spatial connections between all the countries in the sample, we estimate these equations and test the qualitative and quantitative predictions of the model.

In Section 4, we estimate the effects of investment rate, population growth and location on real income per worker at steady state using a spatial autoregressive specification. This estimation can be used to assess the values of the structural parameters in the model. First, we estimate the share of physical capital (α) to be close to one-third as expected. Indeed the estimated value of the capital share of GDP in the textbook Solow model is overestimated (about 0.7). Two approaches are suggested in the literature to explain this value: first, as proposed by Mankiw, Romer and Weil (1992), human capital should be taken into consideration together with physical capital to achieve the commonly accepted value of one-third for the capital share with a specification of the form $Y = AK^{1/3}H^{1/3}L^{1/3}$. This first approach has been largely developed in the theoretical as well as the empirical literature. Second, as suggested by Romer (1986, 1987) among others, another way to raise the capital share from one-third to two-thirds is to argue that there are positive externalities to physical capital (ϕ). Using time series and cross-section regressions, he supports the claim that output takes the form $Y = K^{\alpha+\phi}L^{1-\alpha-\phi}$ with a value for $\alpha + \phi$ that is comprised in [0.7, 1] (Romer 1987). However, he is unable to identify and hence estimate the value of physical capital externalities (ϕ) in the model he develops. In contrast, we show in this paper that in our model, we can indeed identify the parameter associated with physical capital externalities at steady state and estimate it. We find evidence in favor of physical capital externalities but these externalities are not strong enough to generate endogenous growth. Finally, we assess the effect of technological interdependence in growth processes by estimating the parameter describing spatial externalities, which is also significant. Therefore, in our opinion, taking into account technological interdependence is fundamental to understanding differences between income levels and growth rates in a world-wide economy.

In Section 5, we estimate a spatial version of the conditional convergence model. In fact, several empirical papers have found strong evidence of convergence between economies after controlling for differences in steady states. Mankiw, Romer and Weil (1992) show that the neoclassical growth model with exogenous technological progress and decreasing returns for physical capital explains much of the difference in cross-country per capita growth rates. This empirical evidence for conditional β-convergence is also confirmed by several other cross-country empirical studies (Barro and Sala-i-Martin 1992, 2004). However, we show in this paper that spatial autocorrelation often detected in empirical cross-country growth regressions has to be explained at the theoretical level and included in the structural model. In addition, Durlauf and Johnson (1995) have directly tested and rejected the hypothesis that the coefficients in these cross-country regressions are the same in different subsets of the sample of countries, highlighting the heterogeneity problem. Moreover, Durlauf et al. (2001) account for country-specific heterogeneity in the Solow growth model using varying coefficients. The model we propose takes into account both of these problems simultaneously. Therefore, we first estimate, as a benchmark, a homogenous version of our spatially augmented conditional convergence model which yields a convergence rate close to 2% as generally found in the literature. We show that the technological interdependence generated by spatial externalities is important in explaining the conditional convergence process. Finally, we estimate a spatial heterogeneous version of the local convergence model using the spatial autoregressive local estimation method developed by LeSage and Pace (2004).

2 Model

2.1 Technology and spatial externalities

In this section, we develop a growth model with Arrow-Romer externalities and spatial externalities, which implies international technological interdependence in a world with N countries denoted by $i = 1, ..., N$.

Let us consider an aggregate Cobb-Douglas production function for country i at time t exhibiting constant returns to scale in labor and reproducible physical capital:

$$Y_i(t) = A_i(t) K_i^{\alpha}(t) L_i^{1-\alpha}(t) \tag{1}$$

with the standard notations: $Y_i(t)$ the output, $K_i(t)$ the level of reproducible physical capital, $L_i(t)$ the level of labor, and $A_i(t)$ the aggregate level of technology:

$$A_i(t) = \Omega(t) k_i^{\phi}(t) \prod_{j \neq i}^{N} A_j^{\gamma w_{ij}}(t) \tag{2}$$

The function describing the aggregate level of technology $A_i(t)$ of any country i depends on three terms. First, as in the Solow model (Solow 1956, Swan 1956), we suppose that some proportion of technological progress is exogenous and identical in all countries: $\Omega(t) = \Omega(0)e^{\mu t}$ where μ is its constant rate of growth. Second, we suppose that each country's aggregate level of technology increases with the aggregate level of physical capital per worker $k_i(t) = K_i(t)/L_i(t)$ available in that country.[1] The parameter ϕ, with $0 < \phi < 1$, describes the strength of home externalities generated by physical capital accumulation. Therefore,

[1]We suppose that all knowledge is embodied in physical capital per worker and not in the level of capital in order to avoid scale effects (Jones, 1995).

we follow Arrow's (1962) and Romer's (1986) treatment of knowledge spillover from capital investment and we assume that each unit of capital investment not only increases the stock of physical capital but also increases the level of technology for all firms in the economy through knowledge spillover. However, there is no clear reason to constrain these externalities within the borders of the economy. In fact, we can suppose that the external effect of knowledge embodied in capital in place in one country extends across its borders but does so with diminished intensity because of friction generated by socioeconomic and institutional dissimilarities captured by exogenous geographic distance or border effects for instance. This idea is modeled by the third term in equation (2). The particular functional form we assume for this term in a country i, is a geometrically weighted average of the stock of knowledge of its neighbors denoted by j. The degree of international technological interdependence generated by the level of spatial externalities is described by γ, with $0 < \gamma < 1$. This parameter is assumed identical for each country but the net effect of these spatial externalities on the level of productivity of the firms in a country i depends on the relative connectivity between this country and its neighbors. We represent the connectivity between a country i and all the countries belonging to its neighborhood by the exogenous friction terms w_{ij}, for $j = 1, ..., N$ and $j \neq i$. We assume that these terms are non negative, non stochastic and finite; we have $0 \leq w_{ij} \leq 1$ and $w_{ij} = 0$ if $i = j$. We also assume that $\sum_{j \neq i}^{N} w_{ij} = 1$ for $i = 1, ..., N$.[2] The more a given country i is connected to its neighbors, the higher w_{ij} is, and the more country i benefits from spatial externalities.

[2] This hypothesis allows us to assume a relative connectivity among all countries in order to underline the importance of neighborhood effects for economic growth. Moreover, it allows us to avoid scale effects and subsequent explosive growth.

This international technological interdependence implies that countries cannot be analyzed in isolation but must be analyzed as an interdependent system. Therefore, rewrite function (2) in matrix form:

$$A = \Omega + \phi k + \gamma W A \tag{3}$$

with A the $(N \times 1)$ vector of the logarithms of the level of technology, k the $(N \times 1)$ vector of the logarithms of the aggregate level of physical capital per worker, and W the $(N \times N)$ Markov-matrix with friction terms w_{ij}. We can resolve (3) for A, if $\gamma \neq 0$ and if $1/\gamma$ is not an eigenvalue of W:[3]

$$A = (I - \gamma W)^{-1}\Omega + \phi (I - \gamma W)^{-1} k \tag{4}$$

we can develop (4), if $|\gamma| < 1$, and regroup terms to obtain:

$$A = \frac{1}{1-\gamma}\Omega + \phi k + \phi \sum_{r=1}^{\infty} \gamma^r W^{(r)} k \tag{5}$$

where $W^{(r)}$ is the matrix W to the power of r. For a country i, we have:

$$A_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^{\phi}(t) \prod_{j=1}^{N} k_j^{\phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)}}(t) \tag{6}$$

The level of technology in a country i depends on its own level of physical capital per worker and on the level of physical capital per worker in its neighborhood. Replacing (6) in the production function (1) written per worker, we

[3]Actually $(I - \gamma W)^{-1}$ exists if and only if $|I - \gamma W| \neq 0$. This condition is equivalent to: $|\gamma||W - (1/\gamma)I| \neq 0$ where $|\gamma| \neq 0$ and $|W - (1/\gamma)I| \neq 0$.

have finally:

$$y_i(t) = \Omega^{\frac{1}{1-\gamma}}(t)k_i^{u_{ii}}(t)\prod_{j\neq i}^{N} k_j^{u_{ij}}(t) \tag{7}$$

with: $u_{ii} = \alpha + \phi\left(1 + \sum_{r=1}^{\infty}\gamma^r w_{ii}^{(r)}\right)$ and $u_{ij} = \phi\sum_{r=1}^{\infty}\gamma^r w_{ij}^{(r)}$. The terms $w_{ij}^{(r)}$ are the elements of row i and column j of the matrix W to the power of r, and $y_i(t) = Y_i(t)/L_i(t)$ the level of output per worker.

This model implies spatial heterogeneity in the parameters of the production function. However, we can note that if there are no physical capital externalities, that is $\phi = 0$, we have $u_{ii} = \alpha$ and $u_{ij} = 0$, and then the production function is written in the usual form.

Finally, we can evaluate the social elasticity of income per worker in a country i with respect to all physical capital. In fact, from equation (7), it can be seen that when country i increases its own stock of physical capital per worker, it obtains a social return of u_{ii}, whereas this return increases to $u_{ii} + \sum_{j\neq i}^{N} u_{ij} = \alpha + \frac{\phi}{1-\gamma}$ if all countries simultaneously increase their stocks of physical capital per worker.[4] In order to warrant the local convergence and then avoid explosive or endogenous growth, we suppose that there is decreasing social return: $\alpha + \frac{\phi}{1-\gamma} < 1$.[5] This hypothesis is tested in section 4.2.

2.2 Capital accumulation and steady state

As in the textbook Solow model, we assume that a constant fraction of output s_i is saved and that labor grows exogenously at the rate n_i for a country i. We suppose also a constant and identical annual rate of depreciation of physical

[4]See appendix 1 for the proof.
[5]See appendix 3 for the proof.

capital for all countries, denoted by δ. The evolution of output per worker in country i is governed by the fundamental dynamic equation of Solow:

$$\dot{k}_i(t) = s_i y_i(t) - (n_i + \delta) k_i(t) \tag{8}$$

where the dot over a variable represents its derivative with respect to time. Since the production function per worker is characterized by decreasing returns, equation (8) implies that the physical capital-output ratio of country i, for $i = 1, ..., N$, is constant and converges to a balanced growth rate defined by $\dot{k}_i(t)/k_i(t) = g$, or: $[k_i/y_i]^* = s_i/(n_i + g + \delta)$, or in other words:[6]

$$k_i^* = \Omega^{\frac{1}{(1-\gamma)(1-u_{ii})}}(t) \left(\frac{s_i}{n_i + g + \delta} \right)^{\frac{1}{1-u_{ii}}} \prod_{j \neq i}^{N} k_j^{*\frac{u_{ij}}{1-u_{ii}}}(t) \tag{9}$$

As the production technology is characterized by externalities across countries, we can observe how the physical capital per worker at steady state depends on the usual technological and preference parameters but also on the level of physical capital per worker in neighboring countries. The influence of the spillover effect increases with the externalities generated by the physical capital accumulation, ϕ, and the coefficient γ that measures the strength of technological interdependence.

In order to determine the equation describing the real income per worker of country i at steady-state, we rewrite the production function in matrix form: $y = A + \alpha k$, and substitute A by its expression in equation (4) to obtain:

$$y = (I - \gamma W)^{-1} \Omega + \alpha k + \phi (I - \gamma W)^{-1} k \tag{10}$$

[6]The balanced growth rate is $g = \frac{\mu}{(1-\alpha)(1-\gamma)-\phi}$

premultiplying both sides by $(I - \gamma W)$, we have:

$$y = \Omega + (\alpha + \phi)k - \alpha\gamma W k + \gamma W y \tag{11}$$

Rewrite this equation for economy i:

$$\ln y_i^*(t) = \ln \Omega(t) + (\alpha+\phi)\ln k_i^*(t) - \alpha\gamma \sum_{j\neq i}^{N} w_{ij} \ln k_j^*(t) + \gamma \sum_{j\neq i}^{N} w_{ij} \ln y_j^*(t) \tag{12}$$

Finally, introducing the equation of capital-output ratio at steady-state in logarithms for $i = 1, ..., N$ in equation (12), we obtain the real income per worker of country i at steady-state:[7]

$$\begin{aligned} \ln y_i^*(t) &= \frac{1}{1-\alpha-\phi}\ln \Omega(t) + \frac{\alpha+\phi}{1-\alpha-\phi}\ln s_i - \frac{\alpha+\phi}{1-\alpha-\phi}\ln(n_i + g + \delta) \\ &- \frac{\alpha\gamma}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln s_j + \frac{\alpha\gamma}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln(n_j + g + \delta) \\ &+ \frac{\gamma(1-\alpha)}{1-\alpha-\phi}\sum_{j\neq i}^{N} w_{ij}\ln y_j^*(t) \end{aligned} \tag{13}$$

This spatially augmented Solow model has the same qualitative predictions as the textbook Solow model about the influence of the domestic saving rate and the domestic population growth rate on the real income per worker of a country i at steady-state. First, the real income per worker at steady state for a country i depends positively on its own saving rate and negatively on its own population growth rate. Second, it can also be shown that the real income per worker for a country i depends positively on the saving rates of neighboring

[7]Note that when $\gamma = 0$, we have the model developed by Romer (1986) with $\alpha + \phi < 1$ and when $\gamma = 0$ and $\phi = 0$, we have the Solow model.

countries and negatively on their population growth rates. In fact, although the sign of the coefficient of the saving rates of neighboring countries is negative, each of those saving rates ($\ln s_j$) positively influences its own real income per worker at steady state ($\ln y_j^*(t)$) which in turn positively influences the real income per worker at steady state for country i through spatial externalities and global technological interdependence. The net effect is indeed positive as can also be shown by computing the elasticity of income per worker in country i with respect to its own rate of saving ξ_s^i and with respect to the rates of saving of its neighbors ξ_s^j . We then obtain respectively:[8]

$$\xi_s^i = \frac{\alpha + \phi}{1 - \alpha - \phi} + \frac{\phi}{(1-\alpha)(1-\alpha-\phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left(\frac{\gamma(1-\alpha)}{1-\alpha-\phi} \right)^r \tag{14}$$

and:

$$\xi_s^j = \frac{\phi}{(1-\alpha)(1-\alpha-\phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left(\frac{\gamma(1-\alpha)}{1-\alpha-\phi} \right)^r \tag{15}$$

These elasticities help us to better understand the effects of an increase in the saving rate in a country i or in one of its neighbors j on its income per worker at steady state. First, we note that an increase in the saving rate in a country i leads to a higher impact on the real income per worker at steady state than in the textbook Solow model because of technological interdependence modeled as a spatial multiplier effect representing the knowledge diffusion. Furthermore, an increase in the saving rate of a neighboring country j positively influences the real income per worker at steady state in country i.

We can also compute the elasticity of income per worker with respect to the depreciation rate for country i denoted by ξ_n^i, and for neighboring countries j,

[8] See appendix 2 for details.

denoted ξ_n^j:

$$\xi_n^i = -\frac{\alpha+\phi}{1-\alpha-\phi} - \frac{\phi}{(1-\alpha)(1-\alpha-\phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left(\frac{\gamma(1-\alpha)}{1-\alpha-\phi}\right)^r \tag{16}$$

and:

$$\xi_n^j = -\frac{\phi}{(1-\alpha)(1-\alpha-\phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left(\frac{\gamma(1-\alpha)}{1-\alpha-\phi}\right)^r \tag{17}$$

We will test these qualitative and quantitative predictions of the spatially augmented Solow model in section 4.2.

2.3 Conditional Convergence

Like the textbook Solow model, our model predicts that income per worker in a given country converges to that country's steady state value. Rewriting the fundamental dynamic equation of Solow (8) including the production function (7), we obtain:

$$\frac{\dot{k}_i(t)}{k_i(t)} = s_i \Omega^{\frac{1}{1-\gamma}}(t) k_i^{-(1-u_{ii})}(t) \prod_{j\neq i}^{N} k_j^{u_{ij}}(t) - (n_i+\delta) \tag{18}$$

The main element behind the convergence result in this model is also diminishing returns to reproducible capital. In fact, $\partial(\dot{k}_i(t)/k_i(t))/\partial k_i(t) < 0$ since $u_{ii} < 1$. When a country increases its physical capital per worker, the rate of growth decreases and converges to its own steady state. However, an increase in physical capital per worker in a neighboring country j increases the firms' productivity in country i because of the technological interdependence. We have: $\partial(\dot{k}_i(t)/k_i(t))/\partial k_j(t) > 0$ since $u_{ij} > 0$. Physical capital externalities and technological interdependence only slow down the decrease of marginal productivity of physical capital, therefore the convergence result is still valid under the hypothesis $\alpha + \frac{\phi}{1-\gamma} < 1$, in contrast with endogenous growth models, where marginal productivity of physical capital is constant.

In addition, our model makes quantitative predictions about the speed of convergence to steady state. As in the literature, the transitional dynamics can be quantified by using a log linearization of equation (18) around the steady state, for $i = 1, ..., N$:

$$\begin{aligned} \frac{d \ln k_i(t)}{dt} &= -(1-u_{ii})(n_i+g+\delta)\left[\ln k_i(t) - \ln k_i^*\right] \\ &+ \sum_{j \neq i}^{N} u_{ij}(n_i+g+\delta)\left[\ln k_j(t) - \ln k_j^*\right] \end{aligned} \quad (19)$$

We obtain a system of differential linear equations whose resolution is too complicated to obtain clear predictions. However, considering the following relations between the gaps of countries with respect to their own steady state:

$$\ln k_i(t) - \ln k_i^* = \Phi_j \left[\ln k_j(t) - \ln k_j^*\right] \quad (20)$$

$$\ln y_i(t) - \ln y_i^* = \Theta_j \left[\ln y_j(t) - \ln y_j^*\right] \quad (21)$$

the speed of convergence is given by:[9]

$$\frac{d \ln y_i(t)}{dt} = \frac{\mu}{1-\gamma} - \lambda_i \left[\ln y_i(t) - \ln y_i^*\right] \quad (22)$$

with:

$$\lambda_i = \frac{\sum_{j=1}^{N} u_{ij} \frac{1}{\Phi_j}(n_j+g+\delta)}{\sum_{j=1}^{N} u_{ij} \frac{1}{\Phi_j}} - \sum_{j=1}^{N} u_{ij} \frac{1}{\Theta_j}(n_j+g+\delta) \quad (23)$$

[9] See appendix 4 for details.

These hypotheses postulate that the gap of country i relative to its own steady state is proportional to the corresponding gap for country j. Therefore, if $\Theta_j = 1$, countries i and j lie at the same distance from their steady states. If $\Theta_j > 1$ (resp. $\Theta_j < 1$) then country i is farther from (resp. closer) its own steady state than country j. The relative gap between countries in relation to their steady states affects the speed of convergence. In fact, $\partial\lambda_i/\partial\Theta_j = u_{ij}(n_j + g + \delta)/\Theta_j^2 > 0$, and the speed of convergence is high if country i is far from its own steady state. Moreover, the speed of convergence is high if country j is close to its own steady state. So, there is a strong form of heterogeneity in our model since the speed of convergence of country i is both a function of the parameters w_{ij} representing friction and a function of the distance of the neighboring countries from their own steady states. When there are no physical capital externalities ($\phi = 0$), the heterogeneity of the speed of convergence reduces to that of the textbook Solow model: $\lambda_i = -(1-\alpha)(n_i + g + \delta)$. Therefore, we have the same connection between physical capital externalities and heterogeneity as the one we obtained with the production function.

The solution for $\ln y_i(t)$, subtracting $\ln y_i(0)$, the real income per worker at some initial date, from both sides, is:

$$\begin{aligned} \ln y_i(t) - \ln y_i(0) &= (1 - e^{-\lambda_i t})\frac{\mu}{1-\gamma}\frac{1}{\lambda_i} - (1 - e^{-\lambda_i t})\ln y_i(0) \\ &+ (1 - e^{-\lambda_i t})\ln y_i^* \qquad (24) \end{aligned}$$

The model predicts convergence since the growth of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady state.

We rewrite equation (24) in matrix form: $G = DC - Dy(0) + Dy^*$ where G is the $(N \times 1)$ vector of growth rates of real income per worker, $y(0)$ is the $(N \times 1)$ vector of the logarithms of the initial level of real income per worker, y^* is the $(N \times 1)$ vector of the logarithms of real income per worker at steady state, C is the $(N \times 1)$ vector of constants and D is the $(N \times N)$ diagonal-matrix with $(1 - e^{-\lambda_i t})$ terms on the main diagonal. Introducing equation (13) in matrix form: $y^* = (I - \rho W)^{-1} \left[\frac{1}{1-\alpha-\phi}\Omega + \frac{\alpha+\phi}{1-\alpha-\phi}S - \frac{\alpha\gamma}{1-\alpha-\phi}WS \right]$, where $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$ and S is the $(N \times 1)$ vector of the logarithms of the saving rate divided by the effective rate of depreciation, premultiplying both sides by the inverse of $D(I - \rho W)^{-1}$ and rearranging terms we obtain:

$$\begin{aligned} G &= D(C + \frac{1}{1-\alpha-\phi}\Omega) - Dy(0) + \rho DWy(0) + \frac{\alpha+\phi}{1-\alpha-\phi}DS \\ &- \frac{\alpha\gamma}{1-\alpha-\phi}DWS + \rho DWD^{-1}G \end{aligned} \quad (25)$$

Finally, we can rewrite this equation for country i:

$$\begin{aligned} \ln y_i(t) &- \ln y_i(0) = \Delta_i - (1 - e^{-\lambda_i t}) \ln y_i(0) \\ &+ (1 - e^{-\lambda_i t})\frac{\alpha+\phi}{1-\alpha-\phi} \ln s_i - (1 - e^{-\lambda_i t})\frac{\alpha+\phi}{1-\alpha-\phi} \ln(n_i + g + \delta) \\ &+ (1 - e^{-\lambda_i t})\frac{\gamma(1-\alpha)}{1-\alpha-\phi} \sum_{j \neq i}^{N} w_{ij} \ln y_j(0) \\ &- (1 - e^{-\lambda_i t})\frac{\alpha\gamma}{1-\alpha-\phi} \sum_{j \neq i}^{N} w_{ij} \ln s_j \\ &+ (1 - e^{-\lambda_i t})\frac{\alpha\gamma}{1-\alpha-\phi} \sum_{j \neq i}^{N} w_{ij} \ln(n_j + g + \delta) \\ &+ \frac{\gamma(1-\alpha)}{1-\alpha-\phi}(1 - e^{-\lambda_i t}) \sum_{j \neq i}^{N} \frac{1}{(1 - e^{-\lambda_j t})} w_{ij}[\ln y_j(t) - \ln y_j(0)] \end{aligned} \quad (26)$$

with Δ_i the constant equal to: $\Delta_i = (1 - e^{-\lambda_i t})(\frac{\mu}{1-\gamma}\frac{1}{\lambda_i} + \frac{1}{1-\alpha-\phi}\Omega)$. The growth rate of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady state. More specifically, the growth rate of real income per worker depends positively on its own saving rate and negatively on its own population growth rate. Moreover, it depends also, in the same direction, on the same variables in the neighboring countries because of technological interdependence. We can observe that the growth rate is higher the larger the initial level of income per worker and the higher the growth rate in neighboring countries. Finally, the last term of the equation (26) shows that the rate of growth of a country i depends on the rate of growth of its neighboring countries weighted by their speed of convergence and by the friction terms. In section 5, we test the predictions of the spatially augmented Solow model. We then show how technological interdependence may influence growth and conditional convergence.

3 Data

Following the literature on empirical growth, we draw our basic data from the Heston, Summers and Aten (2002) Penn World Tables (PWT version 6.1), which contain information on real income, investment and population (among many other variables) for a large number of countries. In this paper, we use a sample of 91 countries over the period 1960-1995. These countries are those of the Mankiw et al. (1992) non-oil sample, for which PWT 6.1 provide data. We mesure n as the average growth of the working-age population (ages 15 to 64). For this, we have computed the number of workers like Caselli (2004):

$RGDPCH \times POP/RGDPW$, where $RGDPCH$ is real GDP per capita computed by the chain method, $RGDPW$ is real-chain GDP per worker, and POP is the total population. Real income per worker is measured by the real GDP computed by the chain method, divided by the number of workers. Finally, the saving rate s is measured as the average share of gross investment in GDP as in Mankiw et al. (1992).

The Markov-matrix W defined in equation (3) corresponds to the so-called spatial weight matrix commonly used in spatial econometrics to model spatial interdependence between regions or countries (Anselin 1988; Anselin and Bera 1998; Anselin 2001). More precisely, each country is connected to a set of neighboring countries by means of a purely spatial pattern introduced exogenously in W. Elements w_{ii} on the main diagonal are set to zero by convention whereas elements w_{ij} indicate the way country i is spatially connected to country j. In order to normalize the outside influence upon each country, the weight matrix is standardized such that the elements of a row sum up to one. For the variable x, this transformation means that the expression Wx, called the spatial lag variable, is simply the weighted average of the neighboring observations. Lee (2004) presents some technical properties for the W matrix.

Various matrices are considered in the literature: a simple binary contiguity matrix, a binary spatial weight matrix with a distance-based critical cut-off, above which spatial interactions are assumed negligible and more sophisticated generalized distance-based spatial weight matrices with or without a critical cut-off. The notion of distance is quite general and different functional forms based on distance decay can be used (e.g. inverse distance, inverse squared distance, negative exponential etc.). The critical cut-off may be the same for all countries or may be defined to be specific to each country leading in this case, for example, to k-nearest neighbors weight matrices when the critical cut-off for each country is determined so that each country has the same number of neighbors.

It is important to stress that the friction terms w_{ij} should be exogenous to the model to avoid the identification problems raised by Manski (1993) in the social sciences. This is why we consider pure geographical distance, more precisely great-circle distance between capitals, which is indeed strictly exogenous. Geographical distance has also been considered among others by Eaton and Kortum (1996), Klenow and Rodriguez-Clare (2005)[10] and Moreno and Trehan (1997). The functional forms we consider are simply the inverse of squared distance, which can be interpreted as reflecting a gravity function, and the negative exponential of squared distance to check for the robustness of the results.

The general term of the first matrix $W1$ is defined as follows in standardized form $[w1_{ij}]$:

$$w1_{ij} = w1_{ij}^* / \sum_j w1_{ij}^* \quad \text{with} \quad w1_{ij}^* = \begin{cases} 0 \ \text{ if } \quad i = j \\ d_{ij}^{-2} \quad \text{otherwise} \end{cases} \tag{27}$$

[10] Klenow and Rodriguez-Clare (2005, p. 28-29) suggest that use of pure geographical distance could capture trade and FDI related spillovers.

The general term of the second matrix $W2$ is defined as follows in standardized form $[w2_{ij}]$:

$$w2_{ij} = w2^*_{ij} / \sum_j w2^*_{ij} \quad \text{with} \quad w2^*_{ij} = \begin{cases} 0 \ \text{ if } \ i = j \\ e^{-2d_{ij}} \quad \text{otherwise} \end{cases} \tag{28}$$

where d_{ij} is the great-circle distance between country capitals.[11]

4 Impact of saving, population growth and neighborhood on real income

4.1 Specification

In this section, we follow Mankiw et al. (1992) in order to evaluate the impact of saving, population growth, and location on real income. Taking equation (13), we find that the real income per worker along the balanced growth path,

[11] The great-circle distance is the shortest distance between any two points on the surface of a sphere measured along a path on the surface of the sphere (as opposed to going through the sphere's interior). It is computed using the equation:

$$d_{ij} = radius \times \cos^{-1}[\cos|long_i - long_j| \cos lat_i \cos lat_j + \sin lat_i \sin lat_j]$$

where radius is the Earth's radius, lat and long are respectively latitude and longitude for i and j.

at a given time ($t = 0$ for simplicity) is:

$$\ln\left[\frac{Y_i}{L_i}\right] = \beta_0 + \beta_1 \ln s_i + \beta_2 \ln(n_i + g + \delta) + \theta_1 \sum_{j\neq i}^{N} w_{ij} \ln s_j + \theta_2 \sum_{j\neq i}^{N} w_{ij} \ln(n_j + g + \delta) + \rho \sum_{j\neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \quad (29)$$

where $\frac{1}{1-\alpha-\phi} \ln \Omega(0) = \beta_0 + \varepsilon_i$, for $i = 1, ..., N$, with β_0 a constant and ε_i a country-specific shock since the term $\Omega(0)$ reflects not just technology but also resource endowments, climate, etc. and so it may differ across countries. We suppose also that $g + \delta = 0.05$ as is common in the literature since Mankiw et al. (1992) and Romer (1989). We have finally the following theoretical constraints between coefficients: $\beta_1 = -\beta_2 = \frac{\alpha+\phi}{1-\alpha-\phi}$ and $\theta_2 = -\theta_1 = \frac{\alpha\gamma}{1-\alpha-\phi}$. Thus equation (29) is our basic econometric specification in this section. Rewriting this equation in matrix form, we have:

$$y = X\beta + WX\theta + \rho Wy + \varepsilon \quad (30)$$

where y is the $(N \times 1)$ vector of the logarithms of real income per worker, X the $(N \times 3)$ matrix of the explanatory variables, including the constant term, the vector of the logarithms of the investment rate and the vector of the logarithms of the physical capital effective rate of depreciation. W is the row standardized $(N \times N)$ spatial weight matrix, WX is the $(N \times 2)$ matrix of the spatially lagged exogenous variables[12] and Wy the endogenous

[12]The spatially lagged constant is not included in WX, since there is an identification problem for row-standardized W: the spatial lag of a constant is the constant itself.

spatial lag variable. $\beta' = [\beta_0, \beta_1, \beta_2]$, $\theta' = [\theta_1, \theta_2]$ and $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$ is the spatial autoregressive parameter. ε is the $(N \times 1)$ vector of independently and identically distributed errors with mean zero and variance σ^2. In the spatial econometrics literature, this kind of specification, including the spatial lags of exogenous variables in addition to the lag of the endogenous variable, is referred to as the spatial Durbin model (SDM). The model with the endogenous spatial lag variable and the explanatory variables only is referred to as the mixed regressive, spatial autoregressive model (SAR).[13]

For ease of exposition, equation (30) may also be written as follows:

$$y = \widetilde{X}b + \rho W y + \varepsilon \tag{31}$$

with $\widetilde{X} = [X \quad WX]$ and $b = (\beta', \theta')'$. If $\rho \neq 0$ and if $1/\rho$ is not an eigenvalue of W, $(I - \rho W)$ is nonsingular and we have, in reduced form:

$$y = (I - \rho W)^{-1}\widetilde{X}b + (I - \rho W)^{-1}\varepsilon \tag{32}$$

Since for row-standardized spatial weight matrices $|\rho| < 1$ and $w_{ij} < 1$, the inverse matrix in equation (32) can be expanded into an infinite series as:

$$(I - \rho W)^{-1} = (I + \rho W + \rho^2 W^2 + ...) \tag{33}$$

The reduced form has two important implications. First, in conditional mean, real income per worker in a location i will not only be affected by the investment rate and the physical capital effective rate of depreciation in i, but

[13]see Anselin 1988; Anselin and Bera 1998; Anselin, 2001.

also by those in all other locations through the inverse spatial transformation $(I - \rho W)^{-1}$. This is the so-called spatial multiplier effect or global interaction effect. Second, a random shock in a specific location i does not only affect the real income per worker in i, but also has an impact on the real income per worker in all other locations through the same inverse spatial transformation. This is the so-called spatial diffusion process of random shocks.

The variance-covariance matrix for y is easily seen to be equal to:

$$Var(y) = \sigma^2 (I - \rho W)^{-1} (I - \rho W')^{-1} \tag{34}$$

The structure of this variance-covariance matrix is such that every location is correlated with every other location in the system, but closer location more so. It is also interesting to note that the diagonal elements in equation (34), the variance at each location, are related to the neighborhood structure and therefore are not constant, leading to heteroskedasticity even though the initial process (30) is not heteroskedastic.

It also follows from the reduced form (32) that the spatially lagged variable Wy is correlated with the error term since:

$$E(Wy\varepsilon') = \sigma^2 W (I - \rho W)^{-1} \neq 0 \tag{35}$$

Therefore OLS estimators will be biased and inconsistent. The simultaneity embedded in the Wy term must be explicitly accounted for in a maximum likelihood estimation framework as first outlined by Ord (1975).[14] More re-

[14] In addition to the maximum likelihood method, the method of instrumental variables (Anselin 1988, Kelejian and Prucha 1998, Lee 2003) may also be applied to estimate SAR models.

cently, Lee (2004) presents a comprehensive investigation of the asymptotic properties of the maximum likelihood estimators of SAR models.

Under the hypothesis of normality of the error term, the log-likelihood function for the SAR model (31) is given by:

$$\begin{aligned}\ln L(\beta', \rho, \sigma^2) &= -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) + \ln|I - \rho W| \\ &\quad -\frac{1}{2\sigma^2}\left[(I-\rho W)y - \widetilde{X}b\right]'\left[(I-\rho W)y - \widetilde{X}b\right] \qquad (36)\end{aligned}$$

An important aspect of this log-likelihood function is the Jacobian of the transformation, which is the determinant of the $(N \times N)$ full matrix $I - \rho W$ for our model. This could complicate the computation of the ML estimators which involves the repeated evaluation of this determinant. However Ord (1975) suggested that it can be expressed as a function of the eigenvalues ω_i of the spatial weight matrix as:

$$|I - \rho W| = \prod_{i=1}^{N}(1 - \rho\omega_i) \quad \Longrightarrow \quad \ln|I - \rho W| = \sum_{i=1}^{N}\ln(1 - \rho\omega_i) \qquad (37)$$

This expression simplifies considerably the computations since the eigenvalues of W only have to be computed once at the outset of the numerical optimization procedure.

¿From the usual first-order conditions, the maximum likelihood estimators of β and σ^2, given ρ, are obtained as:

$$\hat{\beta}_{ML}(\rho) = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'(I - \rho W)y \qquad (38)$$

$$\hat{\sigma}^2_{ML}(\rho) = \frac{1}{N}\left[(I-\rho W)y - \widetilde{X}\hat{\beta}_{ML}(\rho)\right]'\left[(I-\rho W)y - \widetilde{X}\hat{\beta}_{ML}(\rho)\right] \qquad (39)$$

Note that, for convenience, $\hat{\beta}_{ML}(\rho) = \hat{\beta}_O - \rho\hat{\beta}_L$ where $\hat{\beta}_O = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'y$ and $\hat{\beta}_L = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'Wy$. Define $\hat{e}_O = y - \widetilde{X}\hat{\beta}_O$ and $\hat{e}_L = y - \widetilde{X}\hat{\beta}_L$, it can be then easily seen that $\hat{\sigma}^2_{ML}(\rho) = \left[\frac{(\hat{e}_O - \rho\hat{e}_L)'(\hat{e}_O - \rho\hat{e}_L)}{N}\right]$.

Substitution of (38) and (39) in the log-likelihood function (36) yields a concentrated log-likelihood as a non-linear function of a single parameter ρ:

$$\begin{aligned} \ln L(\rho) \quad &= \quad -\frac{N}{2}\left[\ln(2\pi) + 1\right] + \sum_{i=1}^{N} \ln(I - \rho\omega_i) \\ &\quad -\frac{N}{2}\ln\left[\frac{(\hat{e}_O - \rho\hat{e}_L)'(\hat{e}_O - \rho\hat{e}_L)}{N}\right] \end{aligned} \tag{40}$$

where $\hat{e}_O$ and $\hat{e}_L$ are the estimated residuals in a regression of y on X and Wy on X, respectively. A maximum likelihood estimate for ρ is obtained from a numerical optimization of the concentrated log-likelihood function (40).[15] Under the regularity conditions described for instance in Lee (2004), it can be shown that the maximum likelihood estimators have the usual asymptotic properties, including consistency, normality, and asymptotic efficiency. The asymptotic variance-covariance matrix follows as the inverse of the information matrix, defining $W_A = W(I - \rho W)^{-1}$ to simplify notation, we have:

AsyVar$[b', \rho, \sigma^2] =$

$$\begin{bmatrix} \frac{1}{\sigma^2}\widetilde{X}'\widetilde{X} & \frac{1}{\sigma^2}(\widetilde{X}'W_A\tilde{X}b)' & 0 \\ \frac{1}{\sigma^2}\widetilde{X}'W_A\tilde{X}b & tr\left[(W_A + W_A')W_A\right] + \frac{1}{\sigma^2}(W_A\widetilde{X}b)'(W_A\widetilde{X}b) & \frac{1}{\sigma^2}trW_A \\ 0 & \frac{1}{\sigma^2}trW_A & \frac{N}{2\sigma^4} \end{bmatrix}^{-1} \tag{41}$$

[15]The quasi-maximum likelihood estimators of the SAR model can also be considered if the disturbance in the model are not truly normally distributed (Lee 2004).

4.2 Results

In the first column of table 1, we estimate the textbook Solow model by OLS. Our results for its qualitative predictions are essentially identical to those of Mankiw et al. (Table 1, p. 414 of their paper), since the coefficients on saving and population growth have the predicted signs and are significant. But, as also underlined by Bernanke et al. (2003) with the recent vintage of PWT, the overidentifying restriction is rejected (the p-value is 0.038). The estimated capital share remains close to 0.6 as in Mankiw et al. (1992). It is therefore too high. In addition, Moran's I test (Cliff and Ord, 1981) against spatial autocorrelation in the error term strongly rejects the null hypothesis whatever the spatial weight matrix used.

We claim that the textbook Solow model is misspecified since it omits variables due to technological interdependence and physical capital externalities. In fact, the econometric specification of our theoretical model is, in matrix form:

$$y = \frac{\alpha}{1-\alpha}S + \frac{\phi}{1-\alpha}(I-\gamma W)^{-1}k^* + (I-\gamma W)^{-1}\varepsilon \tag{42}$$

with S the $(N \times 1)$ vector of logarithms of investment rate divided by the effective rate of depreciation. Therefore the error term in the Solow model contains omitted information since we can rewrite it:

$$\varepsilon_{Solow} = \frac{\phi}{1-\alpha}(I-\gamma W)^{-1}k^* + (I-\gamma W)^{-1}\varepsilon \tag{43}$$

We also note the presence of spatial autocorrelation in the error term even if there are no physical capital externalities (i.e. $\phi = 0$), and then the presence of technological interactions between all countries through the inverse spatial transformation $(I - \gamma W)^{-1}$. Furthermore, it is straightforward to show that OLS lead to biased estimators if the endogenous spatial lag variable is omitted as in the textbook Solow model.

[Table 1 around here]

In the subsequent columns of table 1, we estimate the spatially augmented Solow model for the two spatial weight matrices $W1$ and $W2$ using maximum likelihood.[16] Many aspects of the results support our model. First, all the coefficients have the predicted signs and the spatial autocorrelation coefficient, ρ, is positive and significant. Second, the joint theoretical restriction $\beta_1 = -\beta_2$ and $\theta_2 = -\theta_1$ is not rejected since the p-value of the LR-test is 0.455 for the $W1$ matrix and 0.311 for the $W2$ matrix. Third, the α implied by the coefficient in the constrained regression is very close to one-third for the both matrices. The ϕ estimate is about 0.15 to 0.18 and remains significant (p-values are respectively 0.08 and 0.12).

More specifically, we can test the absence of physical capital externalities represented by ϕ. In fact, if $\phi = 0$ in the specification (29), we have:

$$\begin{aligned} \ln\left[\frac{Y_i}{L_i}\right] &= \beta_0' + \beta_1' \ln s_i + \beta_2' \ln(n_i + g + \delta) \\ &+ \theta_1' \sum_{j \neq i}^{N} w_{ij} \ln s_j + \theta_2' \sum_{j \neq i}^{N} w_{ij} \ln(n_j + g + \delta) \\ &+ \gamma \sum_{j \neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \end{aligned} \tag{44}$$

[16] James LeSage provides a function for estimating the spatial Durbin model in his Econometrics Toolbox for Matlab (http://www.spatial-econometrics.com).

with $\beta'_1 = -\beta'_2 = \frac{\alpha}{1-\alpha}$, $\theta'_2 = -\theta'_1 = \frac{\alpha\gamma}{1-\alpha}$ hence $\theta'_1 + \beta'_1\gamma = 0$ and $\theta'_2 + \beta'_2\gamma = 0$. Specification (44) is the so-called constrained spatial Durbin model, which is formally equivalent to a spatial autoregressive error model written in matrix form:

$$y = X\beta' + \varepsilon_{Solow} \qquad \text{and} \qquad \varepsilon_{Solow} = \gamma W \varepsilon_{Solow} + \varepsilon \tag{45}$$

where $\beta' = [\beta'_0 \ \beta'_1 \ \beta'_2]$ and ε_{Solow} is the same as before with $\phi = 0$. Hence, we have the textbook Solow model with spatial autocorrelation in the error term. Estimation results by maximum likelihood using $W1$ and $W2$ are presented in table 2.[17] We can test the non-linear restrictions with the common factor test (Burridge, 1981). We reject these restrictions and then the null hypothesis $\phi = 0$ and we conclude that there are some physical capital externalities.

The γ estimate is higher than 0.5 indicating the importance of technological interdependence between countries and the importance of neighborhood in determining real income. However, these externalities are not strong enough to generate endogenous growth since the value of $\alpha + \frac{\phi}{1-\gamma}$ is below 1 and close to 0.6 or 0.7. We obtain lower results than those obtained by Romer (1987) about the importance of physical capital externalities and social return since he finds an elasticity of output with respect to physical capital close to unity.

[Table 2 around here]

A last result of our model is of interest. Indeed, it is well known that the neoclassical model fails to predict the large differences in income observed in

[17]see Anselin and Bera (1998) for more details on the maximum likelihood estimation method applied to this kind of models.

the real world. The calibrations of Mankiw (1995) indicate that the Solow model, with reasonable differences in rates of saving and population growth, can explain incomes that vary by a multiple of slightly more than two. However, there is much more disparity in international living standards than the neoclassical model predicts since they vary by a multiple of more than ten. These calculations have been made with an evaluation of the elasticities of real income per worker with respect to the saving rate and to the effective rate of depreciation which are approximately 0.5 and -0.5. Mankiw (1995) notes that we can obtain better predicted real income per worker differences with higher elasticities. Our model predicts that the saving rate and population growth have greater effects on real income per worker because of physical capital externalities and technological interdependence.

In order to compute these elasticities of real income per worker at steady state with respect to the saving rate and the effective rate of depreciation, we can rewrite equations (14 and 15) in matrix form:[18]

$$\Xi = \beta_1 I + (\beta_1 \rho + \theta_1) W (I - \rho W)^{-1} \tag{46}$$

Therefore, from estimations reported in table 1, we obtain a (91×91) matrix Ξ with direct elasticities on the main diagonal and off-diagonal terms representing cross-elasticities. In a column j, we have the effects of an increase of the saving rate s_j of the country j on all countries. Of course, because of the w_{ij} terms, the effect is greater for closer countries. In a row i, we have the effects of an increase in the saving rate of each country in the neighborhood of

[18] We focus here on the elasticities of income with regard to the saving rate. The elasticities of income with regard to the effective depreciation rate are symmetric.

country i on its real income per worker. We note also that the sum of each line is identical for all countries. This property, deriving from the Markov proprety of W, means that an identical increase of the saving rate in all countries will have the same effect on their real income per worker at steady state.

On average, the elasticity of real income per worker relative to the saving rate is about 0.9 for the $W1$ matrix and 0.84 for the $W2$ matrix. In the same way, on average, the elasticity of real income per worker relative to the effective rate of depreciation is about -1.65 for the $W1$ matrix and -1.69 for the $W2$ matrix. We also have complete results for cross-elasticities indicating effects of saving rates or population growth rates of neighboring countries on real income per worker of the country under study.[19] Therefore, these values of elasticities provide a much better explanation of the differences between countries' real income per worker. In fact, physical capital externalities, technological interdependence, and more generally neighborhood effects, explain these income inequalities between countries since they imply higher elasticities.

5 Impact of saving, population growth and neighborhood on growth

We now assess the predictions for conditional convergence of our spatially augmented Solow model in two polar cases. First, like Mankiw et al. (1992) and Barro and Sala-i-Martin (1992), we suppose that the speed of convergence is identical for all countries and we refer to this case as the homegenous model. Second, we estimate a model with complete parameter heterogeneity and we

[19] All results are available from the authors upon request.

refer to this case as the heterogenous model.

5.1 Homogenous model

In this section, we follow Mankiw et al. (1992) and Barro and Sala-i-Martin (1992, 2004) in order to estimate equation (26): we first assume that the speed of convergence is homogenous and so identical for all countries: $\lambda_i = \lambda$ for $i = 1, ..., N$. Rewrite equation (26), dividing both sides by T, in the following form:

$$\begin{aligned} \frac{[\ln y_i(t) - \ln y_i(0)]}{T} &= \beta_0 + \beta_1 \ln y_i(0) + \beta_2 \ln s_i + \beta_3 \ln(n_i + g + \delta) \\ &+ \theta_1 \sum_{j \neq i}^{N} w_{ij} \ln y_j(0) + \theta_2 \sum_{j \neq i}^{N} w_{ij} \ln s_j \\ &+ \theta_3 \sum_{j \neq i}^{N} w_{ij} \ln(n_j + g + \delta) \\ &+ \rho \sum_{j \neq i}^{N} w_{ij} \frac{[\ln y_j(t) - \ln y_j(0)]}{T} + \varepsilon_i \qquad (47) \end{aligned}$$

where β_0 is a constant, $\beta_1 = -\frac{(1-e^{-\lambda T})}{T}$, $\beta_2 = -\beta_3 = \frac{(1-e^{-\lambda T})}{T}\frac{\alpha+\phi}{1-\alpha-\phi}$, $\theta_1 = \frac{(1-e^{-\lambda T})}{T}\frac{\gamma(1-\alpha)}{1-\alpha-\phi}$, $\theta_3 = -\theta_2 = \frac{(1-e^{-\lambda T})}{T}\frac{\alpha\gamma}{1-\alpha-\phi}$ and $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$. In matrix form, we have also a non-constrained spatial Durbin model which is estimated in the same way as the model in the section 4.2.

In the first column of table 3, we estimate a model of unconditional convergence. This result is identical to that reported by many previous authors about the failure of income to converge (De Long, 1988, Romer, 1987 and Mankiw et al. 1992). The coefficient on the initial level of income per worker is slightly positive and non significant. Therefore, there is no tendency for poor countries to grow faster on average than rich countries.

[Table 3 around here]

We test the convergence predictions of the textbook Solow model in the second column of table 3. We report regressions of growth rates over the period 1960 to 1995 on the logarithms of income per worker in 1960, controlling for investment rates and growth rates of working-age population. The coefficient on the initial level of income is now significant and negative; that is, there is strong evidence of convergence. The results also support the predicted signs of investment rates and working-age population growth rates. However, it is well-known in the literature that the implied value of λ, the parameter governing the speed of convergence is much smaller than the prediction of the textbook Solow model or the 2% per year found by Barro and Sala-i-Martin (2004). Indeed, our results give a value of $\lambda = 0.008$, which implies a half-life of about 91 years.

Once again, we claim that the textbook Solow model is misspecified since it omits variables due to technological interdependence and physical capital externalities. Therefore, as in Section 4.2, the error terms of the Solow model contain omitted information and are spatially autocorrelated as also indicated by Moran's I tests whatever the spatial weight matrix considered.

In table 4, we estimate the conditional convergence equation implied by our spatially augmented Solow model for the two spatial weight matrices $W1$ and $W2$. Many aspects of the results support this model. First, all the coefficients are significant and have the predicted signs. The spatial autocorrelation coefficient ρ is positive and strongly significant, which shows the importance of the role played by technological interdependence on the growth of countries. Second, the coefficient on the initial level of income is negative and significant, so there is strong evidence of convergence after controlling for those variables that the spatially augmented Solow model says determine the steady state. Third, the λ implied by the coefficient on the initial level of income is about 1.5% to 1.7% which is closer to the value usually found for the speed of convergence in the literature.

[Table 4 around here]

Finally, in table 5, we test the absence of physical capital externalities since $\phi = 0$ implies a spatial Durbin model in constrained form and then a spatial autoregressive error model. Using the same approach as in Section 4.2, we now strongly reject the null hypothesis $\phi = 0$ (p-values are close to 0.01) and we conclude that there are indeed physical capital externalities.

[Table 5 around here]

5.2 Heterogenous model

In recent papers, Durlauf (2000, 2001) and Brock and Durlauf (2001) draw attention to the assumption of parameter homogeneity imposed in cross-section growth regressions. Indeed, it is unlikely to assume that the parameters describing growth are identical across countries. Moreover, evidence of parameter heterogeneity has been found using different statistical methodologies such as in Canova (2004), Desdoigts (1999), Durlauf and Johnson (1995) and Durlauf et al. (2001). Each of these studies suggests that the assumption of a single linear statistical growth model applying to all countries is incorrect.

¿From the econometric methodology perspective, Islam (1995), Lee, Pesaran and Smith (1997) or Evans (1998) have suggested the use of panel data to address this problem, but this approach is of limited use in empirical growth contexts, because variation in the time dimension is typically small. Some variables as for example political regime do not vary by nature over high frequencies and some other variables are simply not measured over such high frequencies. Moreover high frequency data will contain business cycle factors that are presumably irrelevant for long run output movements. The use of long run averages in cross sectional analysis has still a powerful justification for identifying growth as opposed to cyclical factors. Durlauf and Quah (1999) underline also that it might appear to to be a proliferation of free parameters not directly motivated by economic theory.

The empirical methodology we propose takes into account the heterogeneity embodied in our spatially augmented Solow model. Reconsider equation (26), dividing both sides by T:

$$\begin{aligned} \frac{[\ln y_i(t) - \ln y_i(0)]}{T} &= \beta_{0i} + \beta_{1i} \ln y_i(0) + \beta_{2i} \ln s_i + \beta_{3i} \ln(n_i + g + \delta) \\ &+ \theta_{1i} \sum_{j \neq i}^{N} w_{ij} \ln y_j(0) + \theta_{2i} \sum_{j \neq i}^{N} w_{ij} \ln s_j \\ &+ \theta_{3i} \sum_{j \neq i}^{N} w_{ij} \ln(n_j + g + \delta) \\ &+ \rho_i \sum_{j \neq i}^{N} w_{ij} \frac{[\ln y_j(t) - \ln y_j(0)]}{T} + \varepsilon_i \qquad (48) \end{aligned}$$

with $\beta_{0i} = \frac{(1-e^{-\lambda_i T})}{T}(\frac{g}{1-\gamma}\frac{1}{\lambda_i}+\frac{1}{1-\alpha-\phi}\Omega)$, $\beta_{1i} = -\frac{(1-e^{-\lambda_i T})}{T}$, $\beta_{2i} = -\beta_{3i} = \frac{(1-e^{-\lambda_i T})}{T}\frac{\alpha+\phi}{1-\alpha-\phi}$, $\theta_{1i} = \frac{(1-e^{-\lambda_i T})}{T}\frac{\gamma(1-\alpha)}{1-\alpha-\phi}$, $\theta_{3i} = -\theta_{2i} = \frac{(1-e^{-\lambda_i T})}{T}\frac{\alpha\gamma}{1-\alpha-\phi}$ and $\rho_i = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}\frac{(1-e^{-\lambda_i T})}{\Gamma_i}$. The term Γ_i is a scale parameter reflecting the effects of the speeds of convergence in the neighboring countries. To accommodate both spatial dependence and heterogeneity, we produce estimates using N-models, where N represents the number of cross-sectional sample observations, using the locally linear spatial autoregressive model in (48). The original specification was proposed by LeSage and Pace (2004) and labeled spatial autoregressive local estimation (SALE). This specification is used in Ertur et al. (2004) for example in the regional convergence context in Europe. We consider an extended version of this specification here as we also include spatially lagged exogenous variables and label it the local SDM model:

$$U(i)y = U(i)X\beta_i + U(i)WX\theta_i + \rho_i U(i)Wy + U(i)\varepsilon \tag{49}$$

where $U(i)$ represents an $N \times N$ diagonal matrix containing distance-based weights for observation i that assign weights of one to the m nearest neighbors to observation i and weights of zero to all other observations. This results in the product $U(i)y$ representing an $m \times 1$ sub-sample of observed GDP growth rates associated with the m observations nearest in location to observation i (using great-circle distance). Similarly, the product $U(i)X$ extracts a sub-sample of explanatory variable information based on m nearest neighbors and so on. The local SDM model assumes $\varepsilon_i \sim N(0, \sigma_i^2 U(i) I_N)$. The model is estimated by the recursive spatial maximum likelihood approach developed by Pace and LeSage (2004).

The scalar parameter ρ_i measures the influence of the variable, $U(i)Wy$ on $U(i)y$. We note that as $m \rightarrow N$, $U(i) \rightarrow I_N$ and these estimates approach the global estimates based on all N observations that would arise from the global SDM model. The local SDM model in the context of convergence analysis means that each region converges to its own steady state at its own rate (represented by the parameter λ_i). Therefore, heterogeneity in both the level of steady states and transitional growth rates toward this steady state is allowed. Estimation results are presented in Figures 1 to 8. Complete results are displayed in Table 6. Countries are ordered by continent and increasing latitude in each continent. The solid line in these figures display the corresponding parameters estimated in our spatially augmented Solow model and the dashed lines display the corresponding parameters estimated in the textbook Solow model.

[Figures 1 to 4 around here]

We note strong evidence for parameter heterogeneity like Durlauf et al. (2001). This heterogeneity is furthermore linked to the location of the observations and is spatial by nature. The parameters for non spatially lagged variables all have the predicted signs. First in Figure 2, we note that the speed of convergence is high for European countries (especially for Belgium, Netherlands, France), and for the USA, Canada and central American countries (Jamaica, Trinidad and Tobago, Panama, etc.). However the speed of convergence is low for some South American countries and most of African and Asian countries. We note that it is very low for Japan and the Republic of Korea, countries known for their high growth rates. However, this may be because that the countries in their neighborhood are farther away from their steady states since the speed of convergence is positively linked to that gap. Second, in Figure 3, the estimates of the saving rate are the highest for Asian countries, a result which is consistent with the findings of Young (1995). Peru in South America and some African countries also present high estimates for the saving rate. In Figure 4, we see that there is no particular pattern for the estimates of the population growth rates.

[Figures 5 to 8 around here]

Estimates of the spatially lagged saving have the predicted sign for all countries except for Mexico, which could be a local outlier, as well as Japan (Figure 5). The estimates of the spatially lagged population growth rate are relatively stable except for South America, Australia and New-Zealand (Figure 6). The impact of the spatially lagged initial income level is strong in Africa and Europe while it is weaker for Asian countries (especially for Japan) and South American countries (Figure 7). The estimates of the lagged growth rate are positive for all countries, they are high for Asian countries and low for countries belonging to America (Figure 8). Therefore, in our model, there is strong evidence for local interdependence.

Further research will have to treat the potential outliers by using robust Bayesian estimation methods for spatial models as proposed in LeSage (1997) and extended to local models in Ertur et al. (2004).

6 Conclusion

In this paper, we develop a growth model which models technological interdependence between countries using spatial externalities. Actually, the stock of knowledge in one country produces externalities that may cross national borders and spill over into other countries with an intensity which decreases with distance. We refer simply in this paper to pure geographical distance. Its exogeneity is largely admitted and therefore represents its main advantage. However, a general distance concept related to socioeconomic or institutional proximity could also be considered.

Our results have several implications: first, countries cannot be treated as spatially independent observations and growth models should explicitly take into account spatial interactions because of technological interdependence. The predictions of our spatially augmented Solow model provide us with a better understanding of the important role played by geographical location and neighborhood effects in international growth and convergence processes. Second, our theoretical result shows that the textbook Solow model is misspecified since variables representing these effects are omitted.

Our estimation results support our model. All the estimated coefficients are significant with the predicted sign. The spatial autocorrelation coefficient is also positive and highly significant. In addition, our econometric model leads to estimates of structural parameters close to predicted values. The estimated capital share parameter is close to 1/3, the estimated parameter for spatial externalities is close to 1/2 and shows the importance of technological interactions in the economic growth process as well as in the world income distribution. Estimation of physical capital externalities shows that knowledge accumulation in the form of learning by doing also plays an important role in the economic growth process. Actually, we show that these externalities imply parameter heterogeneity in the conditional convergence equation. Finally, the spatial autoregressive local estimation method developed by LeSage and Pace (2004) allows estimation of local parameters reflecting the implied spatial heterogeneity.

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Appendix 1

$$
\begin{aligned}
u_{ii} + \sum_{j \neq i}^{N} u_{ij} &= \alpha + \phi \left(1 + \sum_{r=1}^{\infty} \gamma^{i} \sum_{j=i}^{N} w_{ij}^{(r)} \right) \\
&= \alpha + \phi \left(1 + \gamma \sum_{j=i}^{N} w_{ij} + \gamma^{2} \sum_{j=i}^{N} w_{ij}^{(2)} + ... \right) \\
&= \alpha + \phi (1 + \gamma + \gamma^{2} + ...) \\
&= \alpha + \frac{\phi}{1-\gamma} \qquad (50)
\end{aligned}
$$

We can do this because of the Markov property of the W matrix. Indeed, the powers of the W matrix are also Markov matrices and then: $\sum_{j=i}^{N} w_{ij} = \sum_{j=i}^{N} w_{ij}^{(2)} = ... = 1$ for $i = 1, ..., N$.

Appendix 2: Elasticities

Take equation (16) in matrix form:

$$y = \frac{1}{1-\alpha-\phi}\Omega + \frac{\alpha+\phi}{1-\alpha-\phi}S - \frac{\alpha\gamma}{1-\alpha-\phi}WS + \frac{(1-\alpha)\gamma}{1-\alpha-\phi}Wy \qquad (51)$$

where S is the $(N \times 1)$ vector of logarithms of saving rates divided by the effective rate of depreciation. Subtracting $\frac{(1-\alpha)\gamma}{1-\alpha-\phi}Wy$ from both sides, and pre-multiplying both sides by $(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W)^{-1}$, we obtain:

$$\begin{aligned} y &= \frac{1}{1-\alpha-\phi}\left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\Omega \\ &+ \left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right)S \qquad (52) \end{aligned}$$

Deriving this expression in respect to the vector S, we obtain the expression of elasticities in matrix form:

$$\begin{aligned} \Xi &= \left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right) \\ &= \left(I + \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W + \left(\frac{(1-\alpha)\gamma}{1-\alpha-\phi}\right)^2 W^2 + ...\right)\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right) \\ &= \frac{\alpha+\phi}{1-\alpha-\phi}I + \left(\frac{\phi}{(1-\alpha)(1-\alpha-\phi)}\right)\sum_{r=1}^{\infty} W^r \left(\frac{(1-\alpha)\gamma}{1-\alpha-\phi}\right)^r \qquad (53) \end{aligned}$$

Finally, we can rewrite these expressions for each country i and we obtain the expressions in the main text.

Appendix 3: Local Convergence

In order to study the local stability of the system, rewrite equation (22) in matrix form:

$$\dot{\chi}(t) = J\chi(t) \tag{54}$$

where $\chi(t)$ is the $(N \times 1)$ vector of terms $[\ln k_i(t) - \ln k_i^*]$ and J is the Jacobian matrix of the linearized system in the vicinity of the steady state:

$$J = -(1-\alpha-\phi)diag(n_i+g+\delta) + \phi diag(n_i+g+\delta)(I-\gamma W)^{-1} \tag{55}$$

with $diag(n+g+\delta)$ the diagonal matrix with the general term $(n_i+g+\delta)$. We will show that the hypothesis $\alpha + \frac{\phi}{1-\gamma} < 1$ implies the following relation for all rows j of the Jacobian matrix J:

$$|J_{ii}| > \sum_{j\neq i}^{N} |J_{ij}| \quad \text{for all} \;\; i = 1, ..., N. \tag{56}$$

Proof:

$$\begin{aligned}
& \alpha + \frac{\phi}{1-\gamma} < 1 \\
\Leftrightarrow \;\; & u_{ii} + \sum_{j\neq i}^{N} u_{ij} < 1 \\
\Leftrightarrow \;\; & \alpha + \phi + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} + \phi \sum_{j\neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < 1 \\
\Leftrightarrow \;\; & \phi \sum_{j\neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < (1-\alpha-\phi) - \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \\
\Leftrightarrow \;\; & \sum_{j\neq i}^{N} \left| \phi \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} \right| < \left| -(1-\alpha-\phi) + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \right| \qquad \blacksquare
\end{aligned} \tag{57}$$

Therefore, with the dominant negative diagonal theorem, the matrix J is d-stable and then the system is locally stable.

Appendix 4: Convergence Speed

Introducing equation (22), for $i = 1, ..., N$, in the production function (7) rewriting it in the following form: $\frac{d \ln y_i(t)}{dt} = \frac{\mu}{1-\gamma} + u_{ii}\frac{d \ln k_i(t)}{dt} + \sum_{j \neq i}^{N} u_{ij}\frac{d \ln k_j(t)}{dt}$, we obtain:

$$\begin{aligned} \frac{d \ln y_i(t)}{dt} &= \frac{\mu}{1-\gamma} + u_{ii}(n_i + g + \delta)([\ln y_i(t) - \ln y_i^*] - [\ln k_i(t) - \ln k_i^*]) \\ &+ \sum_{j \neq i}^{N} u_{ij}(n_j + g + \delta)([\ln y_j(t) - \ln y_j^*] - [\ln k_j(t) - \ln k_j^*]) \end{aligned} \quad (58)$$

Taking the following relation:

$$\sum_{j \neq i}^{N} u_{ij}(n_j+g+\delta)[\ln k_j(t)-\ln k_j^*] = \Lambda_i \left[u_{ii}[\ln k_i(t) - \ln k_i^*] + \sum_{j \neq i}^{N} u_{ij}[\ln k_j(t) - \ln k_j^*] \right] \quad (59)$$

we obtain, with hypothesis (23), the expression of Λ_i:

$$\Lambda_i = \frac{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}(n_j + g + \delta)}{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}} \quad (60)$$

and then:

$$\begin{aligned} \frac{d \ln y_i(t)}{dt} &= \frac{\mu}{1-\gamma} + \sum_{j \neq i}^{N} u_{ij}(n_j + g + \delta)[\ln y_j(t) - \ln y_j^*] \\ &- \Lambda_i[\ln y_i(t) - \ln y_i^*] \\ &= \frac{\mu}{1-\gamma} - \lambda_i[\ln y_i(t) - \ln y_i^*] \end{aligned} \quad (61)$$

with hypothesis (24). We obtain finally the speed of convergence:

$$\lambda_i = \frac{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}(n_j + g + \delta)}{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}} - \sum_{j=1}^{N} u_{ij}\frac{1}{\Theta_j}(n_j + g + \delta) \quad (62)$$

Table 1: Estimation results: Textbook Solow and spatially augmented Solow models

Model	Textbook Solow	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\ln y_i(1995)$	$\ln y_i(1995)$	$\ln y_i(1995)$
Obs. / Weight matrix	91	91 / ($W1$)	91 / ($W2$)
constant	4.651	0.988	0.530
	(0.010)	(0.602)	(0.778)
$\ln s_i$	1.276	0.825	0.792
	(0.000)	(0.000)	(0.000)
$\ln(n_i + 0.05)$	−2.709	−1.498	−1.451
	(0.000)	(0.008)	(0.009)
$W \ln s_j$	–	−0.322	−0.372
		(0.079)	(0.024)
$W \ln(n_j + 0.05)$	–	0.571	0.137
		(0.501)	(0.863)
$W \ln y_j$	–	0.740	0.658
		(0.000)	(0.000)
Moran's I test ($W1$)	0.427	–	–
	(0.000)		
Moran's I test ($W2$)	0.456	–	–
	(0.000)		
Restricted regression			
constant	8.375	2.060	2.908
	(0.000)	(0.000)	(0.000)
$\ln s_i - \ln(n_i + 0.05)$	1.379	0.841	0.818
	(0.000)	(0.000)	(0.000)
$W[\ln s_j - \ln(n_j + 0.05)]$	–	−0.284	−0.276
		(0.107)	(0.088)
$W \ln y_j$	–	0.742	0.648
		(0.000)	(0.000)
Moran's I test ($W1$)	0.427	–	–
	(0.000)		
Moran's I test ($W2$)	0.456	–	–
	(0.000)		
Test of restriction	4.427 (Wald)	1.576 (LR)	2.338 (LR)
	(0.038)	(0.455)	(0.311)
Implied α	0.580	0.276	0.299
	(0.000)	(0.016)	(0.031)
Implied ϕ	–	0.180	0.151
		(0.080)	(0.120)
Implied γ	–	0.557	0.508
		(0.000)	(0.000)
$\alpha + \frac{\phi}{1-\gamma}$	–	0.683	0.606
		(0.008)	(0.025)

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method. LR means likelihood ratio.

Table 2: Spatial Autoregressive Error Model and non linear restrictions tests

Model	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\ln y_i(1995)$	$\ln y_i(1995)$
Obs. / Weight matrix	91 / ($W1$)	91 / ($W2$)
constant	6.483	6.708
	(0.000)	(0.000)
$\ln s_i$	0.826	0.803
	(0.000)	(0.000)
$\ln(n_i + 0.05)$	-1.692	-1.551
	(0.002)	(0.004)
γ	0.829	0.738
	(0.000)	(0.000)
Common factor test (LR)	5.927	4.216
	(0.052)	(0.121)
Restricted regression		
constant	8.788	8.690
	(0.000)	(0.000)
$\ln s_i - \ln(n_i + 0.05)$	0.841	0.809
	(0.000)	(0.000)
γ	0.831	0.748
	(0.000)	(0.000)
Test of restriction	2.342	1.846
	(0.126)	(0.174)
Implied α	0.457	0.447
	(0.000)	(0.000)
Common factor test (LR)	6.693	3.723
	(0.010)	(0.054)

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method. LR means likelihood ratio.

Table 3: Unconditional and conditional convergence models

Model	Uncon. conv.	Textbook Solow
Dep. var.	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$
Obs. / Weight matrix	91	91
const.	-0.006	0.030
	(0.718)	(0.359)
$\ln y_i(1960)$	0.002	-0.007
	(0.197)	(0.000)
$\ln s_i$	–	0.021
		(0.000)
$\ln(n_i + 0.05)$	–	-0.032
		(0.008)
Implied λ	-0.002	0.008
		(0.001)
Half-life	–	91.20
Moran's I test ($W1$)	0.229	0.230
	(0.000)	(0.000)
Moran's I test ($W2$)	0.356	0.264
	(0.000)	(0.000)

Notes: p-values are in parentheses; p-value for the implied parameter is computed using the delta method.

Table 4: Conditional convergence in the spatially augmented homogenous Solow model

Model	Spatial aug. Solow	Spatial aug. Solow
Dep. var.	$\frac{\ln y_i(1995)-\ln y_i(1960)}{35}$	$\frac{\ln y_i(1995)-\ln y_i(1960)}{35}$
Obs. / Weight matrix	91 / $(W1)$	91 / $(W2)$
const.	0.008	0.015
	(0.858)	(0.738)
$\ln y_i(1960)$	-0.013	-0.012
	(0.000)	(0.000)
$\ln s_i$	0.018	0.018
	(0.000)	(0.000)
$\ln(n_i + 0.05)$	-0.035	-0.033
	(0.005)	(0.008)
$W \ln y_j(1960)$	0.014	0.010
	(0.000)	(0.002)
$W \ln s_j$	-0.010	-0.007
	(0.029)	(0.102)
$W \ln(n_j + 0.05)$	0.032	0.021
	(0.086)	(0.237)
$W\left(\frac{\ln y_j(1995)-\ln y_j(1960)}{35}\right)$	0.485	0.423
	(0.000)	(0.000)
Implied λ	0.017	0.015
	(0.000)	(0.000)
Half-life	40.30	46.52

Notes: p-values are in parentheses; p-values for the implied parameters are computed using the delta method.

Table 5: Conditional convergence with spatially autocorrelated errors and non linear restrictions tests

Model	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\frac{\ln y_i(1995)-\ln y_i(1960)}{35}$	$\frac{\ln y_i(1995)-\ln y_i(1960)}{35}$
Obs. / Weight matrix	91 / $(W1)$	91 / $(W2)$
constant	0.033	0.027
	(0.349)	(0.437)
$\ln y_i(1960)$	-0.010	-0.008
	(0.000)	(0.000)
$\ln s_i$	0.020	0.019
	(0.000)	(0.000)
$\ln(n_i + 0.05)$	-0.041	-0.038
	(0.001)	(0.002)
γ	0.531	0.444
	(0.000)	(0.000)
Common factor test (LR)	10.943	6.432
	(0.012)	(0.011)
Implied λ	0.012	0.010
	(0.000)	(0.002)
Half-life	59.162	70.874

Notes: p-values are in parentheses. p-values for the implied parameters are computed using the delta method. LR means likelihood ratio.

Table 6: Conditional convergence for the spatially augmented heterogenous Solow model

country	*constant*	*speed*	*lns*	*lnngd*	*WlnY0*	*Wlns*	*Wlnngd*	*rho*
Mozambique	0,007	1,641	0,016	-0,027	0,020	-0,015	0,050	0,520
South Africa	0,009	1,810	0,017	-0,025	0,020	-0,015	0,046	0,490
Botswana	0,009	1,246	0,016	-0,026	0,017	-0,015	0,047	0,520
Mauritius	0,014	1,723	0,018	-0,019	0,019	-0,014	0,038	0,510
Madagascar	0,020	1,480	0,018	-0,031	0,018	-0,016	0,055	0,500
Zimbabwe	-0,001	1,573	0,017	-0,026	0,020	-0,018	0,048	0,560
Zambia	-0,003	1,411	0,017	-0,029	0,018	-0,016	0,045	0,480
Malawi	0,002	1,665	0,017	-0,027	0,020	-0,016	0,048	0,520
Angola	-0,043	1,388	0,015	-0,027	0,019	-0,018	0,037	0,420
Tanzania	0,012	2,078	0,020	-0,029	0,020	-0,019	0,049	0,600
Congo, Dem, Rep,	-0,016	1,428	0,015	-0,026	0,018	-0,014	0,036	0,430
Congo, Republic of	-0,066	1,443	0,014	-0,026	0,020	-0,020	0,033	0,550
Burundi	-0,016	1,542	0,017	-0,026	0,020	-0,019	0,046	0,560
Rwanda	0,000	1,567	0,017	-0,025	0,021	-0,018	0,051	0,530
Kenya	-0,007	1,508	0,017	-0,025	0,020	-0,018	0,046	0,520
Uganda	0,002	1,465	0,019	-0,031	0,019	-0,019	0,055	0,550
Cameroon	-0,065	1,438	0,015	-0,025	0,021	-0,021	0,033	0,460
Central African Republic	-0,067	1,368	0,015	-0,031	0,020	-0,020	0,037	0,440
Cote d'Ivoire	-0,068	1,482	0,015	-0,031	0,021	-0,019	0,036	0,370
Ghana	-0,066	1,448	0,015	-0,031	0,020	-0,019	0,035	0,370
Togo	-0,056	1,582	0,015	-0,028	0,021	-0,019	0,035	0,400
Nigeria	-0,058	1,570	0,015	-0,028	0,021	-0,019	0,034	0,410
Benin	-0,057	1,588	0,015	-0,029	0,021	-0,019	0,036	0,410
Sierra Leone	-0,070	1,691	0,014	-0,018	0,021	-0,016	0,018	0,340
Ethiopia	-0,008	1,555	0,017	-0,025	0,020	-0,018	0,046	0,520
Chad	-0,069	1,411	0,015	-0,030	0,021	-0,021	0,036	0,450
Burkina Faso	-0,070	1,628	0,015	-0,029	0,021	-0,020	0,032	0,400
Mali	-0,077	1,644	0,014	-0,018	0,023	-0,020	0,025	0,370
Niger	-0,073	1,596	0,015	-0,029	0,021	-0,019	0,031	0,410
Senegal	-0,098	1,630	0,012	-0,022	0,023	-0,018	0,021	0,340
Mauritania	-0,095	1,681	0,012	-0,021	0,023	-0,017	0,020	0,330
Egypt	-0,004	1,487	0,018	-0,035	0,018	-0,016	0,050	0,500
Morocco	-0,073	1,782	0,014	-0,020	0,022	-0,016	0,020	0,370
Tunisia	-0,059	1,708	0,014	-0,025	0,022	-0,018	0,033	0,440
Uruguay	-0,202	1,132	0,014	-0,036	0,014	-0,012	-0,031	0,060
Argentina	-0,214	1,071	0,014	-0,029	0,014	-0,014	-0,040	0,070
Chile	-0,151	1,339	0,013	-0,020	0,016	-0,012	-0,025	0,110
Paraguay	-0,146	1,459	0,013	-0,026	0,016	-0,013	-0,017	0,120
Bolivia	-0,101	1,782	0,018	-0,031	0,018	-0,015	0,002	0,240
Brazil	-0,150	1,478	0,013	-0,023	0,017	-0,012	-0,021	0,110
Peru	-0,067	1,933	0,019	-0,030	0,018	-0,014	0,012	0,260
Ecuador	-0,082	2,111	0,018	-0,035	0,020	-0,016	0,017	0,330
Colombia	-0,085	2,357	0,017	-0,037	0,021	-0,016	0,019	0,380

country	$constant$	$speed$	lns	$lnngd$	$WlnY0$	$Wlns$	$Wlnngd$	rho
Panama	-0,010	2,514	0,017	-0,033	0,022	-0,008	0,035	0,250
Costa Rica	-0,018	2,323	0,017	-0,036	0,021	-0,009	0,035	0,260
Venezuela	-0,012	2,415	0,017	-0,033	0,021	-0,008	0,034	0,250
Trinidad Tobago	-0,013	2,543	0,017	-0,032	0,021	-0,008	0,032	0,270
Nicaragua	-0,016	2,212	0,016	-0,034	0,021	-0,009	0,038	0,270
El Salvador	-0,010	2,408	0,018	-0,034	0,020	-0,008	0,033	0,250
Honduras	-0,025	2,446	0,017	-0,034	0,022	-0,010	0,035	0,280
Guatemala	-0,008	2,304	0,017	-0,035	0,020	-0,007	0,035	0,240
Jamaica	0,002	2,712	0,018	-0,038	0,022	-0,009	0,045	0,300
Dominican Republic	-0,013	2,471	0,017	-0,034	0,021	-0,008	0,035	0,260
Mexico	0,105	2,097	0,014	-0,030	0,011	0,007	0,039	0,470
USA	-0,013	2,725	0,017	-0,033	0,022	-0,008	0,032	0,270
Canada	0,007	2,456	0,018	-0,038	0,020	-0,008	0,040	0,320
New Zealand	-0,125	1,499	0,020	-0,042	0,011	-0,009	-0,020	0,380
Australia	-0,042	1,584	0,019	-0,025	0,013	-0,011	0,003	0,490
Papua New Guinea	0,028	1,649	0,020	-0,038	0,015	-0,015	0,048	0,510
Indonesia	0,011	1,519	0,019	-0,024	0,018	-0,018	0,045	0,540
Singapore	0,005	1,644	0,018	-0,033	0,019	-0,017	0,051	0,530
Malaysia	-0,006	1,627	0,019	-0,027	0,019	-0,019	0,044	0,550
Sri Lanka	0,016	1,474	0,019	-0,025	0,018	-0,018	0,049	0,540
Thailand	0,010	1,600	0,019	-0,027	0,019	-0,018	0,047	0,530
Philippines	0,013	1,637	0,019	-0,024	0,019	-0,018	0,045	0,560
Hong Kong	-0,008	1,662	0,018	-0,022	0,020	-0,020	0,043	0,560
Bangladesh	0,012	1,555	0,019	-0,025	0,019	-0,018	0,048	0,540
Nepal	0,010	1,564	0,019	-0,025	0,018	-0,018	0,045	0,550
India	0,013	1,518	0,019	-0,025	0,019	-0,017	0,048	0,540
Israel	0,008	1,417	0,018	-0,035	0,019	-0,018	0,060	0,530
Jordan	0,000	1,507	0,017	-0,034	0,019	-0,016	0,053	0,520
Syria	0,001	1,506	0,017	-0,035	0,018	-0,016	0,053	0,500
Pakistan	0,014	1,483	0,019	-0,027	0,019	-0,017	0,051	0,540
Japan	0,038	0,924	0,013	-0,028	0,006	-0,001	0,025	0,590
Korea, Republic of	0,012	1,332	0,018	-0,034	0,016	-0,016	0,051	0,540
Greece	-0,055	1,585	0,016	-0,031	0,020	-0,019	0,037	0,470
Portugal	-0,071	2,348	0,011	-0,022	0,024	-0,011	0,017	0,270
Turkey	-0,017	1,613	0,018	-0,031	0,018	-0,016	0,039	0,500
Spain	-0,075	1,909	0,012	-0,020	0,023	-0,016	0,022	0,380
Italy	-0,060	1,741	0,014	-0,021	0,022	-0,018	0,030	0,450
Switzerland	-0,078	2,363	0,012	-0,025	0,024	-0,012	0,019	0,330
Austria	-0,050	1,974	0,014	-0,023	0,023	-0,016	0,031	0,490
France	-0,078	2,443	0,011	-0,026	0,025	-0,012	0,021	0,330
Belgium	-0,064	2,612	0,013	-0,023	0,024	-0,011	0,018	0,380
United Kingdom	-0,075	1,902	0,011	-0,030	0,021	-0,010	0,019	0,310
Netherlands	-0,066	2,578	0,013	-0,020	0,024	-0,011	0,015	0,380
Ireland	-0,082	2,052	0,011	-0,030	0,022	-0,010	0,018	0,300
Denmark	-0,054	2,311	0,013	-0,024	0,022	-0,010	0,018	0,410
Sweden	-0,024	1,922	0,015	-0,031	0,019	-0,011	0,031	0,490
Norway	-0,030	1,979	0,015	-0,034	0,019	-0,010	0,031	0,470
Finland	-0,009	2,008	0,016	-0,034	0,020	-0,012	0,042	0,490

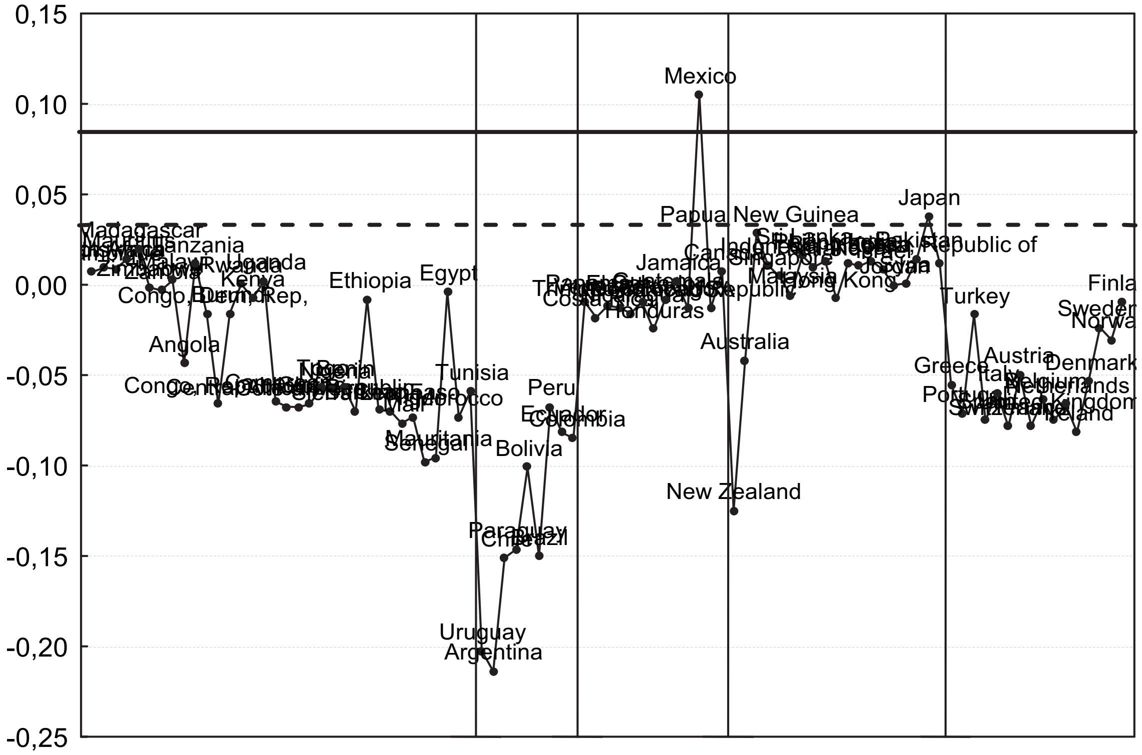

Figure 1: Distribution of constant estimates

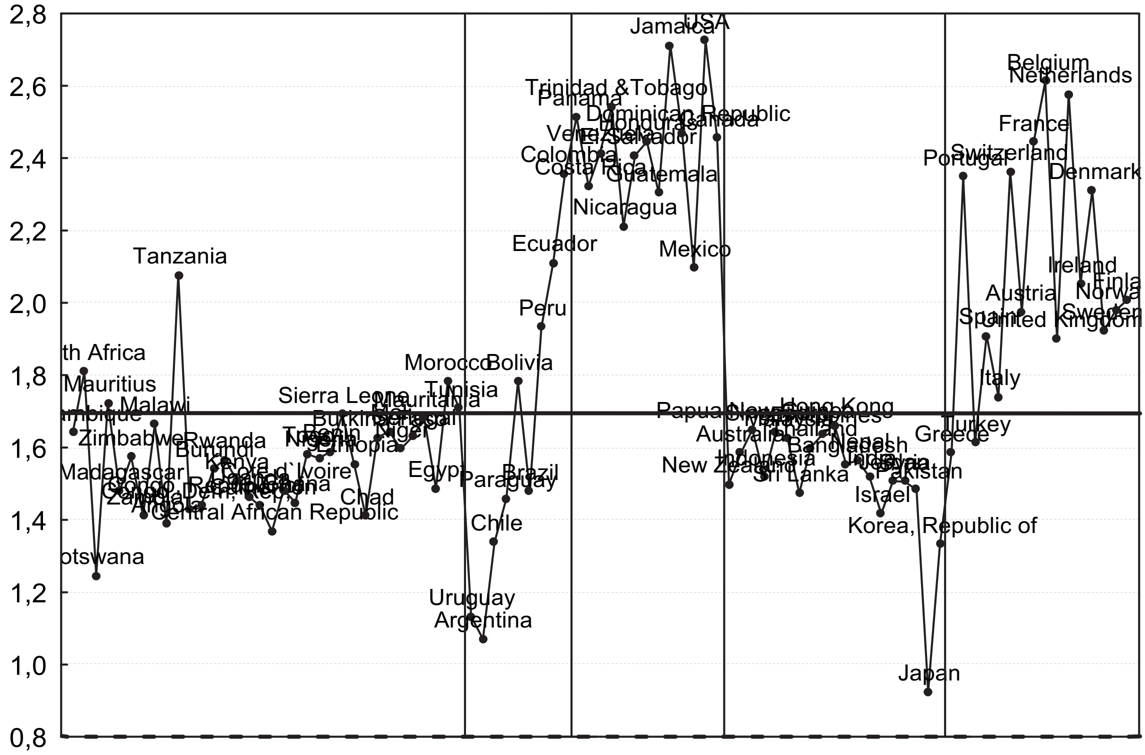

Figure 2: Distribution of convergence speed estimates

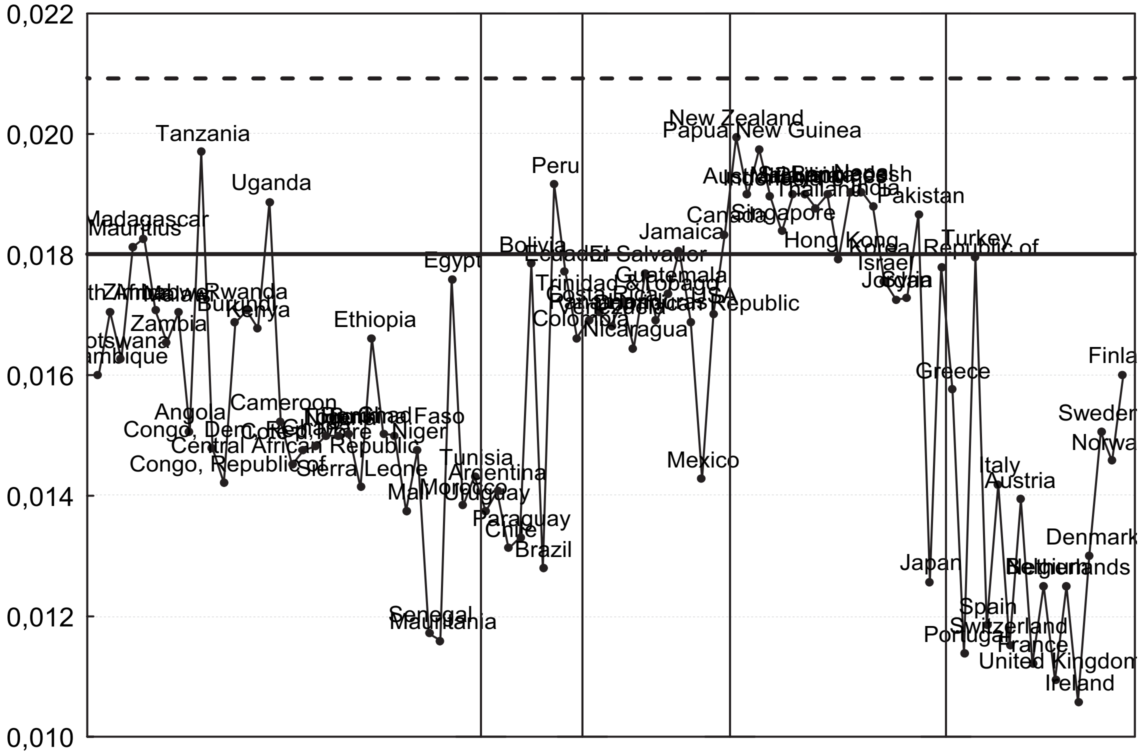

Figure 3: Distribution of saving rate estimates

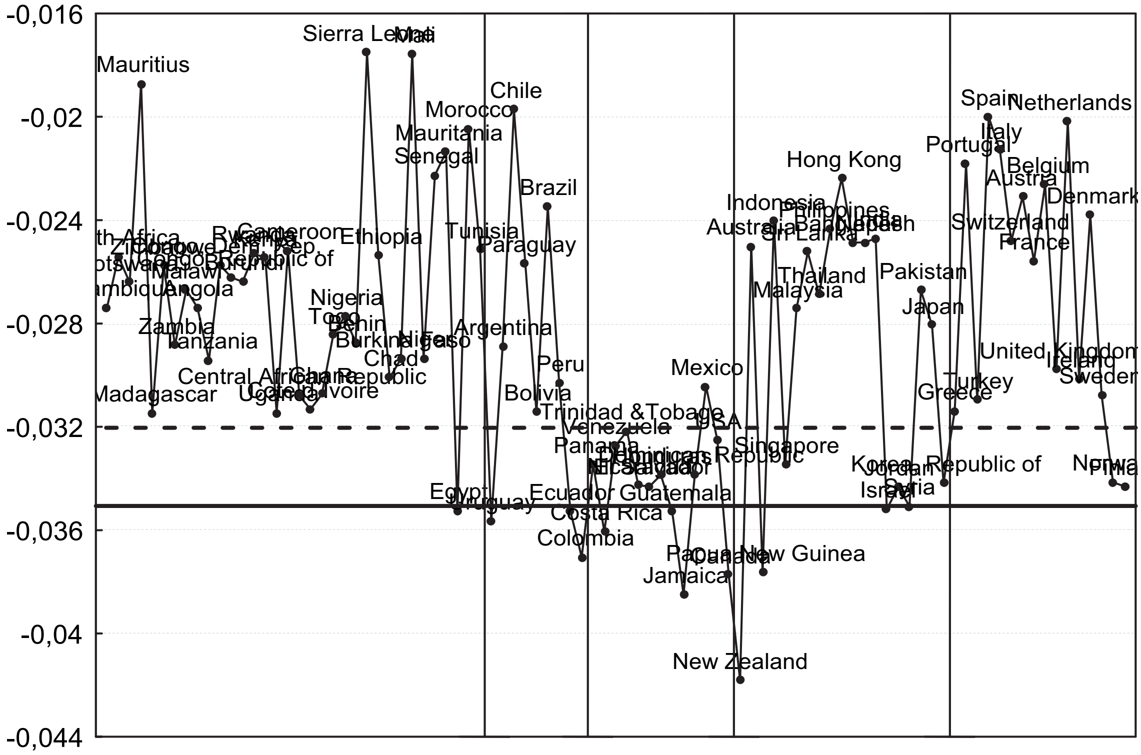

Figure 4: Distribution of population growth rate estimates

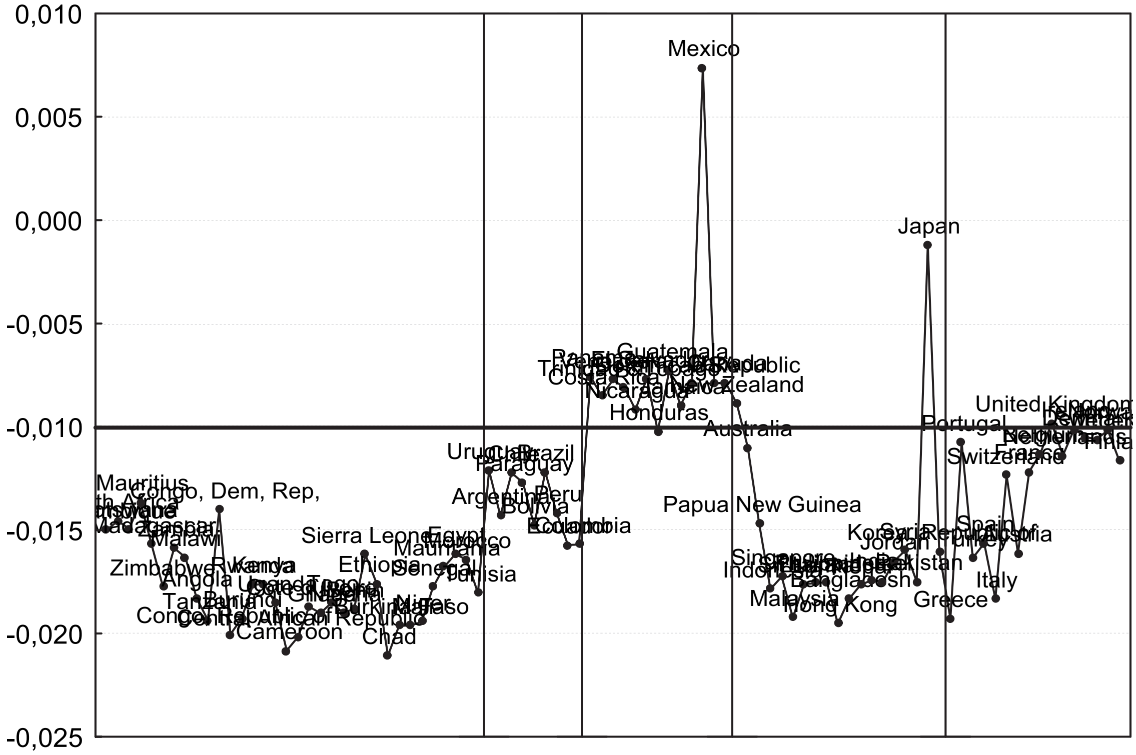

Figure 5: Distribution of lagged saving rate estimates

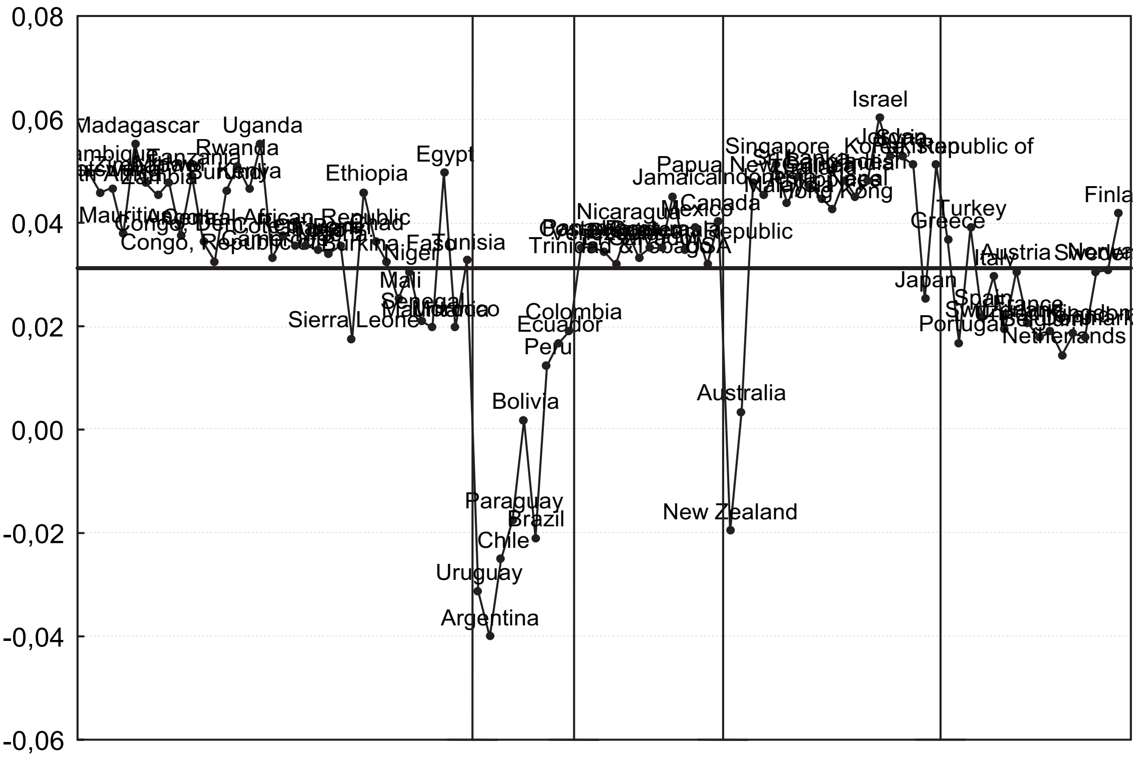

Figure 6: Distribution of lagged population growth rate estimates

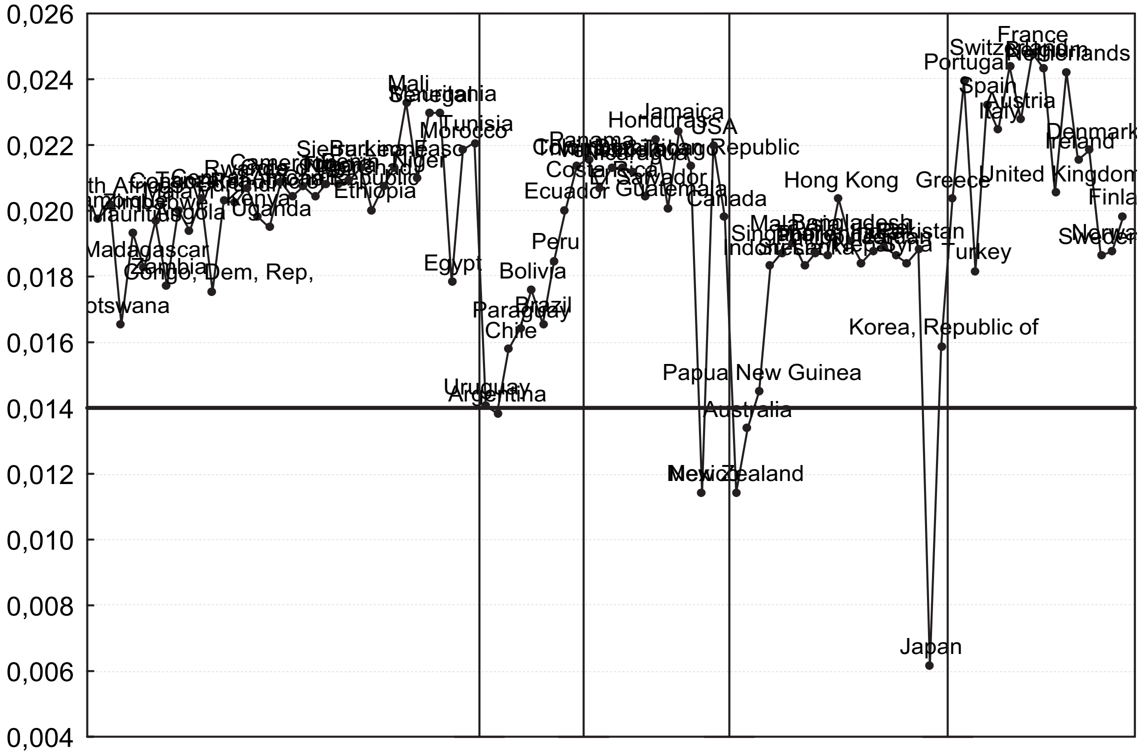

Figure 7: Distribution of the lagged initial income level estimates

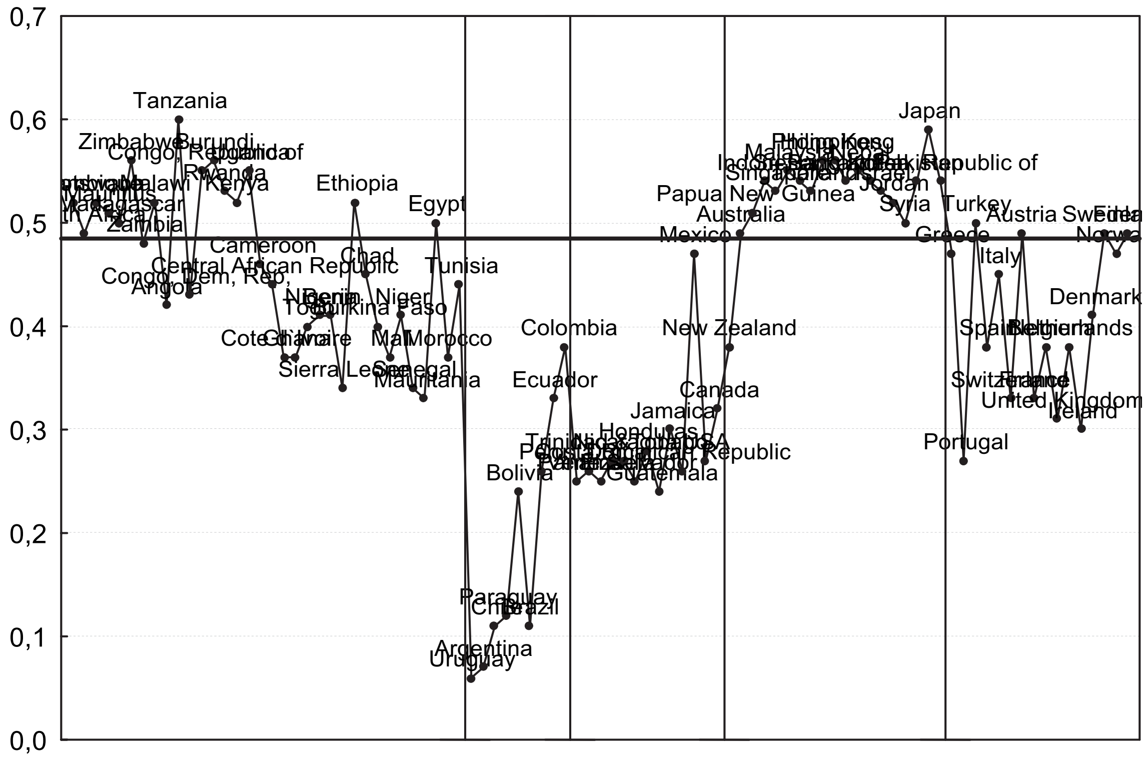

Figure 8: Distribution of lagged growth rate estimates