Asymmetric trade and agglomeration

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Abstract

We extend the quadratic utility approach by Ottaviano et al. [19] and by Behrens [1] to the range of intermediate values of transport costs in order to investigate how asymmetric trade affects the regional distribution of economic activities. Asymmetry in trade is an endogenous result of price competition and transport costs: depending on both the spatial distribution of firms and the value of transport costs, only firms located in one of the two regions can profitably access the foreign market, which gives rise to more complex patterns of trade. We show that the unilateral breaking of autarky gives rise to strong agglomeration forces which lead to the absorption of the smaller regions' industry. Further, the number of equilibria increases once the structure of trade is endogenously accounted for.

Résumé

Nous étendons le modèle quadratique de Ottaviano et al. [19] et Behrens [1] aux valeurs intermédiaires de coûts de transport afin d'examiner la manière dont le commerce asymétrique affecte la répartition régionale de l'activité économique. L'asymétrie dans le commerce est un résultat endogène de la compétition en prix et de l'existence de coûts de transport. Suivant la répartition spatiale des firmes et les valeurs des coûts de transport, seules les firmes installées dans l'une des deux régions peuvent accéder aux marchés extérieurs, ce qui résulte en des structures d'échanges plus complexes. Nous montrons que l'abandon unilatéral de l'autarcie donne naissance à de fortes forces d'agglomération, lesquelles mènent à l'absorption de l'industrie de la petite région. De plus, le nombre d'équilibres augmente dès que la structure des échanges devient endogène.

Keywords : unilateral trade, asymmetry, agglomeration, geography

JEL Classification : R11, R12, O18

1. Introduction

In recent years, an increasing number of authors in international trade theory and spatial economics have emphasized that, in order to understand the workings of trade and agglomeration in a spatial context, we need to move away from traditional trade theory and have to consider *models in which production factors are mobile*. While the traditional theory of international trade assumes that production factors are immobile whereas goods are mobile, new trade theory and new economic geography (see Krugman [14] or Wong [21]) have started to take into account the fact that production factors move between regions and countries in response to economic opportunities. That this mobility of factors can have a strong qualitative impact on equilibria is known in international trade theory in which, as argued by Wong [21], it is *"shown that international trade in goods only, international capital movement only, and international trade in goods plus capital movement generally lead to different world equilibria"*. It is less known though that, as argued by Behrens [1], the mobility of goods plays an equally important role in shaping the space-economy: *different assumptions concerning the mobility of output also lead to different world equilibria*. This aspect is further developed in the present paper.

As shown by Behrens [1], agglomeration can arise even if there is no trade between regions. Although this result is interesting by itself, we implicitly assumed that all goods were *nontradeable for all spatial distributions of firms between the two regions*, a scenario we refered to as inconditional autarky. This particular assumption is not more satisfying than the one used in the original model by Ottaviano et al. [19], in which *all goods are assumed to be tradeable*. The distinction between tradeable and nontradeable is both artificial and blurry, since most goods do not systematically fall in either of those two categories. In the present paper, we complete the basic model by investigating the case in which transport costs take intermediate values. We show that in this case, all goods are only *potentially tradeable* so that the precise structure of trade and the regional availability of goods *depends on both the value of trade costs and the spatial distribution of economic activities*. We believe this case is empirically important, because both tradeables and nontradeables coexist in each economy (due to the rise of the service industry, nontradeables or imperfectly tradeables play an ever increasing role in modern economies). We also believe this case is theoretically the most interesting one because, instead of assuming in an ad hoc way that goods are either tradeable or nontradeable, *we show how economic forces endogenously determine whether certain goods will be traded or not*.

Before investigating those issues more closely, we ask a question of fact. *Are asymmetric structures of trade of empirical relevance or is the assumption of bilateral intraindustry trade between all couples of regions satisfying?* First, one should notice that empirically most regions are specialized in the production and export of a restricted range of products to a restricted range of other regions. As pointed out by Mori et al. [16] for the japanese case, *"Of the 125 three-digit manufacturing industries, 38 (resp. 16) have positive employment in less than 50% (resp. 25%) of the metro areas"*. Even the most diversified japanese city Tokyo has 'only' positive employment in 122 of those 125 industries. As further pointed out by Head and Mayer [10] for the case of the EU, while *"the monopolistic competition model predicts positive amounts of trade between each set of partners in each industry* [. . .] *Greece, Ireland and Portugal have zeros for more than 5 percent of their industry-partner trade flows"*. Therefore, the assumption of systematic bilateral trade is questionable in case of developed countries. Second, the systematic assumption of bilateral trade is also far from being satisfying for developing countries, which ususally have a very different import–export structure than industrialized countries and in which interregional trade is often highly asymmetric (the cases of China and Brazil seem to be rather obvious ones). [1] *Unilateral trade relations between regions and even countries seem hence to play an important role for certain industrial sectors and should therefore not be neglected from both an empirical and a theoretical point of view*.

Why do we observe asymmetric patterns of interregional or international trade in

[1] Unfortunately, data on interregional trade flows are almost impossible to obtain for most countries, especially developing ones. Even in case of the OECD countries, only the US and Canada seem to have reliable and rather exhaustive statistics.

certain industries? It is known that unilateral trade can be either a *spontaneous market outcome* (driven by a subtle interplay of price competition and transportation costs), a result of *tariffs and barriers to trade* (as was for a long time the case for the japanese market), a result of *political factors* (like restrictions on the mobility of goods and workers in China) or simply another illustration of *comparative advantages* as developed in traditional international trade theory. In this paper, we focus on the spontaneous market outcome only. No matter which factors are at the origin of unilateral trade, we show that the existence of one-way access to certain regions introduces significant distortions into the space-economy by modifying firms' *market potential*. This point has been emphasized in the recent literature on *transportation hubs*, in which one explains how economies of transport density create endogenous asymmetries in transport costs and market access, which leads to agglomeration of economic activities at certain hub locations (refer to Konishi [12] and to Mori and Nishikimi [15] for further details).

We believe the case of asymmetric unilateral trade is an important one, both in order to better understand the workings of real space-economies and in order to fill an intellectual gap in the literature. [2] We therefore extend, in the present paper, the framework developed by Ottaviano et al. [19] and Behrens [1] to the case of unilateral trade between regions. To our knowledge, this is the first attempt at describing the workings of a spatial economy in the presence of *an endogenously determined structure of trade* (see, however, Mori and Nishikimi [15] for an alternative approach). We show that *as soon as symmetry breaks and regions open themselves to trade, the larger region absorbs the industry of the smaller one*, which confirms the claim Kaldor (1970) made more than thirty years ago. Asymmetry in the patterns of trade leads to asymmetry in the distribution of firms, which further exacerbates the former. This circular aspect is of fundamental importance and drives most of the results developed in this paper. We further show that as soon as one endogenously accounts for the structure of trade,

[2] Fujita et al. [6] develop a model in which an urban hierarchy in the spirit of Christaller emerges as the result of a process of spatial self-organization in the presence of an exogenously growing population. They show that if we start with a monocentric configuration, the highest order good will only be produced in the central city as the economy develops. Whether this can be interpreted in terms of unilateral trade is not quite clear to us.

the number of potential equilibria becomes larger. The set of equilibria now changes qualitatively with the structure of trade, which renders the investigation quite complex.

The remainder of this paper is organized as follows:

In Section 2, we briefly recall the basic model developed by Behrens [1]. We explain in detail the mechanism of the endogenous determination of trade patterns and show that interregional trade is first unilateral and then bilateral as the level of transport costs steadily decreases. In Section 3, we extend the previous developments to the case of unilateral trade and derive the equilibrium conditions in a two region setting. In Section 4, we establish our main results which state that *when unilateral trade becomes feasible, the larger region always absorbs at least a part of the industrial sector of the smaller one*. Furthermore, while the larger region benefits from this commercial expansion, it can be detrimental to the smaller region. *The joint benefits from trade are shown to be not necessarily positive*. In Section 5, we discuss the welfare issue in the three possible core-periphery structures under autarky, unilateral trade and bilateral trade. Finally, Section 6 offers some preliminary conclusions and points towards future research directions.

2. The conditional levels of trade costs

Let us briefly recall the model developed by Ottaviano et al. [19] and extended by Behrens [1]. We consider an economy with two regions H and F and two production factors: geographically mobile manufacturing workers which produce a differentiated good under monopolistic competition and increasing returns to scale and immobile agricultural workers which produce a homogeneous good (the numéraire) under constant returns to scale and perfect competition. We denote by L the mass of mobile and by A the mass of immobile factor. The immobile factor is evenly split between the two regions, while $\lambda \in [0, 1]$ denotes the share of mobile factor located in region H. Finally, N denotes the mass of varieties (resp. of firms) in the economy. The differentiated good can be transported at no cost in the interior of each region, while shipping a unit of M-good between the two regions requires $\tau > 0$ units of the numéraire. Because there are no intraregional transportation costs for M-goods each firm sells its output at least in the market it has chosen to locate in (refer to Behrens [1] for more details). All agents are assumed to be homogeneous and have the same quadratic utility. As shown by Behrens [1], in such a setting the *extended (or perceived) demand functions* are given by

$$x^\star(i) = \big[a - (b + cN)p(i) + cP(i)\big]^+, \tag{1}$$

where a, b and c are positive coefficients, given by

$$a = \frac{\alpha}{\epsilon + N\gamma}, \quad b = \frac{1}{\epsilon + N\gamma} \quad \text{and} \quad c = \frac{\gamma}{\epsilon(\epsilon + N\gamma)} \tag{2}$$

and where $\epsilon := \beta - \gamma$ is a parameter related to the degree of substitutability between varieties. Using (1) and the symmetry between firms, the demand a firm located in region $r = H, F$ faces in region $s = H, F$ is given by

$$x^\star_{rs}(p_{rs}) = \big[a - (b + cN)p_{rs} + cP_s\big]^+, \tag{3}$$

where p_{rs} is the price charged by a firm located in region r when selling in market s. Hence, the profit of a representative firm located in region $r = H, F$ is given by

$$\pi_r = p_{rr}x^\star_{rr}(p_{rr})\left(\frac{A}{2} + \phi n_r\right) + (p_{rs} - \tau)x^\star_{rs}(p_{rs})\left(\frac{A}{2} + \phi n_s\right) - \phi w_r \tag{4}$$

where $x^\star_{rr}$ and $x^\star_{rs}$ are given by (3), w_r is the wage rate in region r and ϕ is fixed labor requirement in terms of mobile production factor L. In accord with empirical evidence, firm's use spatial discriminatory pricing and set a particular price for each market (see, e.g., Greenhut [9] and Head and Mayer [10]). Maximizing (4) yields

$$p^\star_{rr} = \frac{a + cP_r}{2(b + cN)} = \tfrac{1}{2}\tilde{p}_r \tag{5}$$

on the home market and

$$p^\star_{rs} = \begin{cases} \frac{1}{2}(\tilde{p}_s + \tau) & \text{if} \quad \tau < \tilde{p}_s \\ \tilde{p}_s & \text{if} \quad \tau \geq \tilde{p}_s \end{cases} \tag{6}$$

on the foreign market, where $\tilde{p}_r$ is the *cut-off price* in region r. The regional price-indices are given by

$$P_r = n_r p^{\star}_{rr}(P_r) + n_s p^{\star}_{sr}(P_r), \quad r \neq s \tag{7}$$

where $n_H = \lambda L/\phi$ and $n_F = (1-\lambda)L/\phi$ are the mass of firms in each region when labor markets clear. Solving for the equilibrium prices, we obtain

$$p^{\star}_{rr} = \begin{cases} \dfrac{1}{2}\dfrac{2a + cn_s\tau}{2b + cN} & \text{if} \quad \tau < \tilde{p}_r \\ \dfrac{a}{2b + cn_r} & \text{if} \quad \tau \geq \tilde{p}_r \end{cases} \tag{8}$$

on the home markets,

$$p^{\star}_{sr} = \begin{cases} p^{\star}_{rr} + \dfrac{\tau}{2} & \text{if} \quad \tau < \tilde{p}_r \\ 2p^{\star}_{rr} & \text{if} \quad \tau \geq \tilde{p}_r \end{cases} \tag{9}$$

on the foreign markets and

$$P^{\star}_r = \begin{cases} \dfrac{aN + n_s(b + cN)\tau}{2b + cN} & \text{if} \quad \tau < \tilde{p}_r \\ \dfrac{a(N + n_s)}{2b + cn_r} & \text{if} \quad \tau \geq \tilde{p}_r \end{cases} \tag{10}$$

as the regional equilibrium price indices.

In Behrens [1], we have investigated the equilibrium configurations of the spatial economy when τ is so large that no trade occurs no matter how firms are distributed accross regions. As shown, this is the case if and only if

$$\tau \geq \tau_a := \frac{a}{b} = \alpha. \tag{11}$$

Ottaviano et al. [19] have focused on the polar case in which

$$\tau \leq \tau_{trade} := \frac{2a\phi}{2b\phi + cL}, \tag{12}$$

i.e. trade costs are sufficiently low so that bilateral trade occurs for all possible firm distributions. In between these two extreme cases lies a *range of intermediate values of transport costs for which only firms located in one of the two regions are able to* *profitably access the foreign market*. Hence, intraindustry trade is unilateral and this asymmetry endogenously appears in the model.

While the inconditional autarky case and the inconditional bilateral trade case are characterized by the fact that there is either no possiblility to supply the foreign regions or the possibility to supply both foreign regions simultaneously, the *conditional unilateral trade case* is more involved. It is especially characterized by the existence of two different critical values of trade costs, which we denote by τ_F and τ_H. The threshold τ_F (resp. τ_H) corresponds to *the maximum value of trade costs τ such that firms located in H (resp. F) can profitably supply market F (resp. H)*. Of course, these two values need not and generally will not be the same. Hence, this approach captures the empirical fact that transport costs incurred by firms are not pairwise symmetric between different countries and regions and that these asymmetries are partly an endogenous result of the existing distribution of economic activities.

The expressions of τ_H and τ_F can be derived from the conditions on τ in the optimal pricing decisions of firms. We know from (8) and (10) that a firm located in H can only be profitably active in market F if

$$\tau \leq \tau_F(\lambda) := \frac{a + cP_F^\star(\lambda)}{b + cN} = \frac{2a\phi}{2b\phi + c(1-\lambda)L} = \frac{2\alpha\epsilon}{2\epsilon + \gamma(1-\lambda)N}. \qquad (13)$$

Because $0 \leq \lambda \leq 1$, expression (13) shows that

$$\tau_F(\lambda) \in [\tau_{trade}\ ,\ \tau_a] := \left[\frac{2a\phi}{2b\phi + cL}\ ,\ \frac{a}{b}\right].$$

A similar reasoning shows that firms in region F will be active in H if and only if

$$\tau \leq \tau_H(\lambda) := \frac{a + cP_H^\star(\lambda)}{b + cN} = \frac{2a\phi}{2b\phi + \lambda cL} = \frac{2\alpha\epsilon}{2\epsilon + \gamma\lambda L}. \qquad (14)$$

It is important to understand right from the beginning that *the value of trade costs τ_F (resp. τ_H) is a function of the spatial distribution λ of firms*, which was not the case in the inconditional configurations. Expression (13) shows that the larger the share of firms in region H, and hence the closer λ to 1, the higher the value of τ_F. This amounts to saying that *the more firms are located in region H and the less in region F, the higher the maximal value of trade costs such that firms in H can profitably enter market F*. This result requires some explanation. First, if there are only a few firms in market F, local price competition in that market's differentiated industry is relatively mild. Therefore, firms in F charge higher equilibrium prices which leads to a larger value of the price-index in that region. This in turn implies that firms located in region H can enter the foreign market F, charging both a relatively high price and capturing a share of local demand. The higher the price firms in H can charge in market F, the higher the level of trade costs they can absorb while making profits in that market. Therefore, it is the *local monopoly profits of firms in region F that are a sufficient condition for firms in region H to be able to profitably operate in the foreign market*. Consider now the reverse situation. If firms are relatively numerous in region H, price competition in that region is fierce and local equilibrium prices are therefore relatively low. Hence, firms located in F must charge sufficiently low prices in order to capture a share of local demand when trying to penetrate market H. If, at the same time, the level of trade costs is relatively high, firms in F need to charge a sufficiently high price that allows them to absorb (part of) these costs. As a result, firms in F cannot profitably enter region H. If they, on the one hand, charge a high price in order to cover trade costs, they do not capture a share of the market. If they, on the other hand, charge a low price in order to capture a share of the market, they cannot cover trade costs. Either way round, their net profit in market H is negative so that it is not profitable to export to that market.

One can see from (13) and (14) that

$$\frac{\tau_F(\lambda)}{\tau_H(\lambda)} \geq 1 \quad \Leftrightarrow \quad \lambda \geq \frac{1}{2} \tag{15}$$

and that both τ_F and τ_H are monotonic, convex functions of λ on the interval $[0, 1]$. Expression (15) shows that *the larger region has a sort of endogenous comparative advantage in terms of profitably exporting to the smaller region*. This effect is quite different from the traditional *home market effect* (refer to Krugman [13] and Ottaviano and Thisse [18]) which asserts that, as neatly stated by Head and Ries [11], *"home de-*

mand plays a crucial role in explaining export performance". In our model, segmented markets imply that the degree of price competition can vary greatly between regions, hence creating sufficient incentives or disincentives for firms to have an export activity. Condition (15) partly explains why american and japanese authorities were worried about the Single European Act. Even if external tariff and non-tariff barriers (τ in our model) of the EU remained constant, a more integrated and unified Single European Market, by reducing market-segmentation and increasing price competition between regions, could render more difficult the access of producers operating outside the EU. Some empirical results confirming partly those effects have been established by Head and Mayer [10] and by Winters and Chang [20].

Using (13) and (14), one can check that the two levels of trade costs τ_F and τ_H are equal for $\lambda = 1/2$. In that case

$$\tilde{\tau} := \tau_F(1/2) = \tau_H(1/2) = \frac{2a\phi}{2b\phi + \frac{1}{2}cL} = \frac{4\alpha\epsilon}{4\epsilon + \gamma N}. \tag{16}$$

The value $\tilde{\tau}$ plays an important role in determining which structures of trade we have to take into consideration. If $\tau \geq \tilde{\tau}$, we can have either autarky or unilateral trade. If $\tau \leq \tilde{\tau}$, we can have either unilateral or bilateral trade.

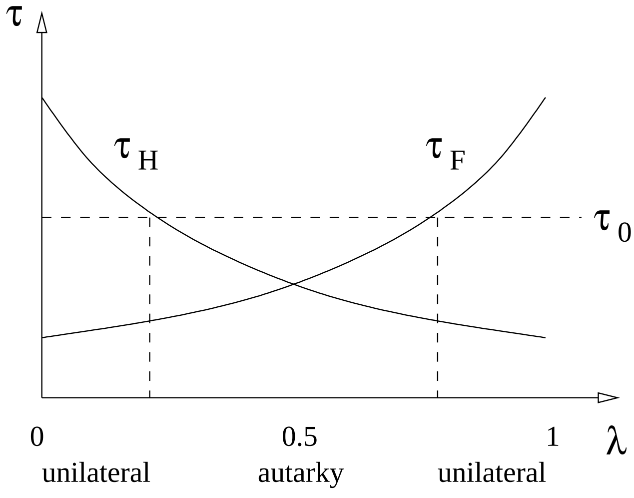

Figure 1: Conditional values of trade costs τ_H and τ_F in the intermediate cases

Figure 1 illustrates the patterns of trade for different values of transport costs τ. Consider a given and fixed value of transport costs $\tau_0 \in [\tilde{\tau}, \tau_a]$ and ask the following question: Under which conditions do we observe unilateral trade? It can be seen from Figure 1 that for $\tau \in [\tilde{\tau}, \tau_a]$ there can never be bilateral trade since there are no values of λ such that $\tau_F(\lambda) \geq \tau$ and $\tau_H(\lambda) \geq \tau$ simultaneously hold. Hence, there is either autarky or unilateral trade, *depending on the spatial distribution of firms*. We have autarky for all spatial distributions λ such that the two conditions $\tau_H(\lambda) \leq \tau$ and $\tau_F(\lambda) \leq \tau$ simultaneously hold (i.e. no region can profitably export to the other one *given the actual distribution of firms*). In Figure 1, this autarky region is centered around $\lambda = 1/2$ and delimited by the dashed vertical lines. Rewriting the condition for autarky, using (13) and (14), yields

$$\frac{2\phi(a - b\tau)}{\tau c L} \leq \lambda \leq 1 - \frac{2\phi(a - b\tau)}{\tau c L}, \tag{17}$$

which can of course only be met if $\tau \geq \tilde{\tau}$. For all values of λ outside of this interval, the pattern of trade is unilateral, with the larger region exporting to the smaller one.

Finally, it is important to note how the possibility for unilateral trade evolves as τ decreases from a high initial value. Consider a *given* autarky equilibrium with $\tau > \tau_a$ and assume that the value of transport costs decreases. Once $\tau \in [\tilde{\tau}, \tau_a]$, there is the possibility to observe unilateral trade for *certain spatial configurations* λ. The precise value of trade costs τ from which on we effectively observe a passage to unilateral trade strongly depends on initial conditions (do we start from a dispersed, partially agglomerated or fully agglomerated autarky equilibrium?). Hence, "history matters" a lot in this case. [3] Suppose, for example, that we start from a fully agglomerated initial equilibrium with $\lambda = 1$. As soon as $\tau = \tau_a$, conditions (11) and (17) show that the interval for regional autarky is given by $0 \leq \lambda \leq 1$, which still holds since we consider an agglomerated initial equilibrium. Should τ continue to decrease strictly below τ_a, the new interval for regional autarky is given by $0 < \lambda < 1$ so that the actual agglomerated configuration with $\lambda = 1$ is no longer compatible with autarky.

[3] Although mostly associated with Krugman [13], the term "history matters" can already be found in Eaton and Lipsey [3].

Firms located in region H can, due to the decrease in transportation costs, profitably export to the foreign market F. One can further see that, as τ continues to decrease towards $\tilde{\tau}$, the interval of spatial distributions λ compatible with autarky gets smaller and smaller. In the limit, when $\tau = \tilde{\tau}$ the 'interval' reduces to $\lambda = 1/2$ so that only the dispersed configuration remains compatible with regional autarky. As transport costs continue to decrease below $\tilde{\tau}$, but remain higher than τ_{trade}, the passage to bilateral trade becomes possible. Depending on the spatial distribution of economic activities, we have either unilateral or bilateral trade (note from Figure 1 that autarky is no longer a feasible option, since the conditions $\tau_F(\lambda) \geq \tau$ and $\tau_H(\lambda) \geq \tau$ never simultaneously hold). When do we observe bilateral trade? Analogously to the previous case, for this to be possible, the conditions $\tau \leq \tau_F(\lambda)$ and $\tau \leq \tau_H(\lambda)$ must simultaneously hold. Rewriting these conditions yields

$$1 - \frac{2\phi(a - b\tau)}{\tau c L} \leq \lambda \leq \frac{2\phi(a - b\tau)}{\tau c L}, \tag{18}$$

which is possible if and only if $\tau \leq \tilde{\tau}$. It is easy to check that in case $\tau = \tilde{\tau}$, condition (18) reduces to $\lambda = 1/2$, so that bilateral trade is possible if and only if we start from a dispersed configuration. As τ continues to decrease, the interval of values of λ compatible with bilateral trade gets larger until finally the condition $0 \leq \lambda \leq 1$ is met as $\tau = \tau_{trade}$. Hence, as soon as $\tau \leq \tau_{trade}$, we are back to inconditional bilateral trade as developed by Ottaviano et al. [19].

Let us close this section with two remarks. First, it is interesting to notice that the passage from autarky to unilateral trade and the passage from unilateral to bilateral trade are sort of 'asymmetric'. On the one hand, the stronger the initial differences in terms of λ between the two regions, the sooner the passage to unilateral trade but the later the passage to bilateral trade. On the other hand, the more symmetric the regions in terms of λ, the later the passage to unilateral trade but the sooner the passage to bilateral trade. Hence, it seems that *symmetry (resp. asymmetry) in the spatial distribution of economic activities leads to symmetry (resp. asymmetry) in the patterns of trade*. The extreme scenario is the one in which we start with full agglomeration $\lambda = 1$. In that particular case, unilateral trade becomes feasible as soon as τ hits τ_a, but bilateral trade only becomes feasible when τ hits τ_{trade}. If we start, on the contrary, with a dispersed configuration $\lambda = 1/2$, we never observe unilateral trade. There will be autarky until $\tau \leq \tilde{\tau}$, from which point we directly switch to bilateral trade. Hence, asymmetry in the spatial distribution of firms, which creates endogenous comparative advantages, is needed for unilateral trade to possibly emerge. Second, as one can see from (16), the more differentiated the varieties (the smaller the value of c), the higher $\tilde{\tau}$. In the limit case in which varieties are independent (hence as $c \to 0$), the three critical levels of transport costs τ_a, $\tilde{\tau}$ and τ_{trade} coincide. Therefore, *more product differentiation leads to an earlier passage from autarky to unilateral (resp. bilateral) trade*.

As shown in this section, unilateral trade can emerge as the joint consequence of decreasing transport costs and asymmetric firm distributions. Each region is hence characterized by a particular critical level of trade costs, above which foreign firms cannot be active in it.

3. Extending the model to unilateral trade

We develop in this section the expressions we need in order to establish the indirect utility differential in the case of conditional unilateral trade. In what follows, we assume for simplicity, without loss of generality, that H is the large region i.e. that $\lambda > 1/2$. [4] Hence, as one can see from (15) we have $\tau_F > \tau_H$. Assume further that $\tilde{\tau} \leq \tau \leq \tau_F$ so that firms operating in the large region H can be active in the foreign market F, while firms operating in the small region F cannot be active in the foreign market H. Using expressions (5) and (6), local prices are given by

$$p^{\star}_{FF} = \frac{a + cP_F}{2(b + cN)} \quad \text{and} \quad p^{\star}_{HH} = \frac{a + cP_H}{2(b + cN)}, \tag{19}$$

while export prices are given by

$$p^{\star}_{HF} = \frac{a + cP_F + (b + cN)\tau}{2(b + cN)} \quad \text{and} \quad p^{\star}_{FH} = \frac{a + cP_H}{b + cN}. \tag{20}$$

[4] We can disregard the case in which we start from a symmetric distribution λ=1/2. As mentioned in the previous section, we never observe unilateral trade in that situation. Hence, Behrens [1] and Ottaviano et al. [19] completely cover this special case.

It is important to notice the *asymmetry* between $p^\star_{HF}$ and $p^\star_{FH}$, which is due to the unilateral trade pattern under which firms in H export to F but not the other way round. Using expressions (7), (19) and (20), we can solve for the equilibrium prices. Those are given by

$$p^\star_{HH} = \frac{a}{2b + c\lambda N}, \quad p^\star_{FH} = \frac{2a}{2b + c\lambda N} \tag{21}$$

and

$$p^\star_{FF} = \frac{1}{2}\frac{2a + \tau c\lambda N}{2b + cN}, \quad p^\star_{HF} = \frac{1}{2}\frac{2a + \tau\left[2b + c(1+\lambda)N\right]}{2b + cN}. \tag{22}$$

Note that because firms in region F do not export to region H, the prices $p^\star_{HH}$ and $p^\star_{FH}$ given by (21) are *naturally independent of the transport cost parameter* τ. This does of course not hold for $p^\star_{FF}$ and $p^\star_{HF}$, as can be seen from (22). While it is easy to understand that the prices firms located in H set in market F do depend on τ, it is more difficult to see a priori why prices firms in F set in their home market also depend on τ. This is due to an *indirect price competition effect*. Due to the substitutability between varieties, firms in F need to adapt their local prices to the prices of imports, which depend themselves on τ. Therefore, *local prices of all varieties sold in region F depend on the value of transport costs, no matter whether they are produced locally or imported*. Finally, the aggregate price indices in both regions are given by

$$P^\star_H = \frac{a(2-\lambda)N}{2b + c\lambda N} \quad \text{and} \quad P^\star_F = \frac{aN + (b + cN)\tau\lambda N}{2b + cN}. \tag{23}$$

Expressions (3), (21), (22) and (23) allow us to derive the individual equilibrium demands of agents, which are given by

$$x^\star_{HH} = \frac{a(b + cN)}{2b + c\lambda N}, \quad x^\star_{FH} = 0 \tag{24}$$

in region H and by

$$x^\star_{FF} = \frac{(b + cN)(2a + c\tau\lambda N)}{2(2b + cN)}$$

and

$$x^\star_{HF} = \frac{(b + cN)\left[2a - \tau\left(c(1-\lambda)N + 2b\right)\right]}{2(2b + cN)} \tag{25}$$

in region F. Replacing a, b and c by their expressions (2), we finally obtain

$$x^{\star}_{HH} = \frac{\alpha}{2\epsilon + \gamma\lambda N}, \quad x^{\star}_{FH} = 0 \tag{26}$$

and

$$x^{\star}_{HF} = \frac{2\epsilon(\alpha - \tau) - \tau\gamma(1-\lambda)N}{2\epsilon(2\epsilon + \gamma N)}, \quad x^{\star}_{FF} = \frac{2\epsilon\alpha + \gamma\tau\lambda N}{2\epsilon(2\epsilon + \gamma N)} \tag{27}$$

in terms of the model's primitives. Note that the export demand $x^{\star}_{HF}$ is always positive, because $\tau \leq \tau_F(\lambda)$ is assumed to hold. Finally, the expressions of the equilibrium prices (21) and (22) in terms of the model's primitives are given by

$$p^{\star}_{HH} = \epsilon x^{\star}_{HH}, \;\; p^{\star}_{FF} = \epsilon x^{\star}_{FF} \tag{28}$$

and

$$p^{\star}_{FH} = 2p^{\star}_{HH}, \;\; p^{\star}_{HF} = \frac{2(\alpha+\tau)\epsilon + \tau\gamma(\lambda+1)N}{2[2\epsilon + \gamma N]}. \tag{29}$$

In order to establish the expressions of the indirect utilities in regions H and F, we need the expressions of the equilibrium wages $w^{\star}_H$ and $w^{\star}_F$ in the two regions. These wages are determined by a bargaining process in the monopolistically competitive industry, in which firms compete for workers by offering higher wages until no additional firm can profitably enter the market. All profits are hence absorbed by the wage bill, which is synonymous with zero profits for firms in the M-industry. The equilibrium wages can be obtained by substituting (26), (27), (28) and (29) into (4) and equating the resulting expression to zero, which yields

$$w^{\star}_F = \left(\frac{A}{2} + (1-\lambda)L\right)\frac{[2\alpha\epsilon + \gamma\tau\lambda N]^2}{4\phi\epsilon[2\epsilon + \gamma N]^2} \tag{30}$$

and

$$w^{\star}_H = \left(\frac{A}{2} + \lambda L\right)\frac{\alpha^2\epsilon}{\phi[2\epsilon + \gamma\lambda N]^2} + \left(\frac{A}{2} + (1-\lambda)L\right)\frac{[2\epsilon(\alpha-\tau) - \tau\gamma(1-\lambda)N]^2}{4\phi\epsilon[2\epsilon + \gamma N]^2}. \tag{31}$$

Let us turn next to the indirect utilities in regions H and F. Since neither prices nor wages are symmetric anymore, the indirect utilities will not be symmetric either. Using the quadratic utility as given by Ottaviano et al. [19], the indirect utility is given by

$$V^\star = \alpha \int_0^N x^\star(i)\mathrm{d}i - \frac{\epsilon}{2}\int_0^N x^\star(i)^2 \mathrm{d}i$$
$$- \frac{\gamma}{2}\left(\int_0^N x^\star(i)\mathrm{d}i\right)^2 + \left(w^\star + \phi_0 - \int_0^N p^\star(i)x^\star(i)\mathrm{d}i\right). \quad (32)$$

Using the symmetry between firms and the relations $N = n_H + n_F$, $n_H = \lambda L/\phi$ and $n_F = (1-\lambda)L/\phi$, the indirect utility in region H can be expressed as (recall that $x^\star_{FH} = 0$ since there are no trade flows from F to H)

$$V^\star_H = n_H x^\star_{HH}\left(\alpha - \frac{\epsilon}{2}x^\star_{HH} - \frac{\gamma}{2}n_H x^\star_{HH} - p^\star_{HH}\right) + w^\star_H + \phi_0.$$

Using (24), (28) and (31), straightforward but cumbersome calculus shows that the indirect utility in region H is given by

$$V^\star_H(\lambda) = \tfrac{1}{2}\left(\frac{\alpha}{2\epsilon + \lambda L\phi^{-1}\gamma}\right)^2\left[3\frac{\lambda L}{\phi}\epsilon + \lambda^2\gamma\frac{L^2}{\phi^2} + A\frac{\epsilon}{\phi}\right] + \phi_0$$
$$+ \left(\frac{A}{2} + (1-\lambda)L\right)\frac{[2\epsilon(\alpha-\tau) - (1-\lambda)\tau\gamma\phi^{-1}L]^2}{4\phi\epsilon[2\epsilon + \gamma\phi^{-1}L]^2}\,. \quad (33)$$

As one can see from (33), the indirect utility in region H under unilateral trade is given by the sum of two terms. The first term is the indirect utility under autarky (refer to Behrens [1]), to which we add a positive second term corresponding to the benefits of increased market size for the local export industry. This positive effect is due to the fact that *exploitation of scale economies lowers average production costs and increases nominal wages, while prices stay constant in region H*. Hence, real wages in the industrial sector rise. As for region F, the indirect utility is given by (recall that $x^\star_{HF} > 0$ since there are positive trade flows from H to F)

$$V^\star_F = \alpha\left(n_H x^\star_{HF} + n_F x^\star_{FF}\right) - \frac{\epsilon}{2}\left(n_H x^{\star 2}_{HF} + n_F x^{\star 2}_{FF}\right)$$
$$- \frac{\gamma}{2}\left(n_H x^\star_{HF} + n_F x^\star_{FF}\right)^2 + w^\star_F + \phi_0 - n_H p^\star_{HF}x^\star_{HF} - n_F p^\star_{FF}x^\star_{FF},$$

which can be rewritten as the sum of three terms (dropping the constant ϕ_0). Therefore

$$V_F^\star = A_1(\lambda) + A_2(\lambda) + A_3(\lambda) \tag{34}$$

where

$$A_1(\lambda) := n_H x_{HF}^\star \left(\alpha - \frac{\epsilon}{2} x_{HF}^\star - \frac{\gamma}{2} n_H x_{HF}^\star - p_{HF}^\star \right)$$

and

$$A_2(\lambda) := n_F x_{FF}^\star \left(\alpha - \frac{\epsilon}{2} x_{FF}^\star - \frac{\gamma}{2} n_F x_{FF}^\star - p_{FF}^\star \right)$$

and

$$A_3(\lambda) := w_F^\star - \gamma \big(n_H x_{HF}^\star \big) \big(n_F x_{FF}^\star \big).$$

Substitution of (27), (28), (29) and (30) into (34) yields, after some rather long rearrangements, the following expressions for A_1, A_2 and A_3:

$$\begin{aligned} A_1(\lambda) = \lambda N \frac{2\epsilon(\alpha - \tau) - \tau\gamma(1-\lambda)N}{4\epsilon[2\epsilon + \gamma N]^2} \Bigg[& 2\alpha(\epsilon + \gamma N) - \tau[2\epsilon \\ & + \gamma(\lambda + 1)N] - \frac{1}{2}(\epsilon + \gamma\lambda N) \frac{2\epsilon(\alpha - \tau) - \tau\gamma(1-\lambda)N}{\epsilon} \Bigg] \end{aligned} \tag{35}$$

$$\begin{aligned} A_2(\lambda) = (1-\lambda)N \frac{2\alpha\epsilon + \gamma\tau\lambda N}{4\epsilon[2\epsilon + \gamma N]^2} \Bigg[& 2\alpha[2\epsilon + \gamma N] \\ & - \frac{1}{2} \frac{2\alpha\epsilon + \gamma\tau\lambda N}{\epsilon} [3\epsilon + \gamma(1-\lambda)N] \Bigg] \end{aligned} \tag{36}$$

$$\begin{aligned} A_3(\lambda) = \frac{2\alpha\epsilon + \gamma\tau\lambda N}{4\epsilon[2\epsilon + \gamma N]^2} \Bigg[& \left(\frac{A}{2\phi} + (1-\lambda)N \right) [2\alpha\epsilon + \gamma\tau\lambda N] \\ & - \gamma\lambda(1-\lambda)N^2 \frac{2\epsilon(\alpha - \tau) - \tau\gamma(1-\lambda)N}{\epsilon} \Bigg] \cdot \end{aligned} \tag{37}$$

Finally, the indirect utility differential is defined as

$$\Delta V^\star(\lambda) := V_H^\star(\lambda) - V_F^\star(\lambda). \tag{38}$$

An equilibrium arises at $\lambda = 0$ if expression (38) is negative, an equilibrium arises at $\lambda = 1$ if this expression is positive and an interior equilibrium arises at $0 < \lambda < 1$ if this expression is equal to zero. The two fully agglomerated equilibria are always stable if they exist, while an interior equilibrium is stable if and only if the slope of the indirect utility differential is negative in a neighborhood of the equilibrium.

As one can already see from (33), (35), (36) and (37), the analytical expression of the indirect utility differential in the case of asymmetric trade turns out to be a relatively complex function of λ. This is further exacerbated by the fact that, for a given and fixed value of transport costs τ, the patterns of trade (autarky, unilateral or bilateral) depend on the spatial distribution λ of firms. Therefore, *the indirect utility differential has an analytical expression that changes structurally with the spatial distribution of firms as we pass from autarky to unilateral trade*. As argued in Section 2, for some values of λ, given by (17), we have autarky, for the other values we have unilateral trade. Because we assume that H is the large region so that $\lambda > 1/2$, we can restrict ourselves to $\lambda \in [1/2, 1]$. Nevertheless, a fundamental technical difficulty could appear in the model. *Since $\Delta V^\star$ is defined by two different functions on two different domains, nothing ensures a priori that $\Delta V^\star$ remains a continuous function of* λ. Hence, the first natural questions to investigate is the continuity and the differentiability of the global indirect utility differential in the case of unilateral trade.

As shown by Behrens [1], in case of autarky the indirect utility differential is given by

$$\Delta V^\star_{aut}(\lambda) := V^\star_{H\,aut}(\lambda) - V^\star_{F\,aut}(\lambda),$$

where

$$V^\star_{H\,aut}(\lambda) = \tfrac{1}{2}\left(\frac{\alpha}{2\epsilon + \gamma\lambda L\phi^{-1}}\right)^2 \left[3\frac{\lambda L}{\phi}\epsilon + \lambda^2\gamma\frac{L^2}{\phi^2} + A\frac{\epsilon}{\phi}\right] + \phi_0 \qquad (39)$$

and

$$V^{\star}_{F aut}(\lambda) = \tfrac{1}{2}\left(\frac{\alpha}{2\epsilon + \gamma(1-\lambda)L\phi^{-1}}\right)^2 \times \left[3\frac{(1-\lambda)L}{\phi}\epsilon + (1-\lambda)^2\gamma\frac{L^2}{\phi^2} + A\frac{\epsilon}{\phi}\right] + \phi_0. \quad (40)$$

In case of unilateral trade, the indirect utility differential is given by the difference between expressions (33) and (34). Fortunately, the structural change in the indirect utility differential is located such that our function remains continuous with respect to λ. Hence, pathological cases in which an equilibrium might not exist can be excluded (see Ginsburgh et al. [8]). This is summarized by the following proposition.

Proposition 3.1 (CONTINUITY OF THE INDIRECT UTILITY DIFFERENTIAL)

Assume that $\lambda \geq 1/2$ so that H is the large region. For all $\tilde{\tau} \leq \tau \leq \tau_a$, the global indirect utility differential $\Delta V^{\star}$ is a continuous function of λ on the interval $[1/2, 1]$.

PROOF. Clearly, both $p^{\star}_{rs}$ and $x^{\star}_{rs}$ are all continuous functions of λ (refer to (1), (5) and (6)). Because the integral operator preserves continuity, we conclude that the indirect utility differential, given by the difference of two continuous functions $V^{\star}_r$, $r = H, F$ is a continuous function of the firm distribution λ. □

Yet, while the indirect utility differential *remains a continuous function of λ, it is no longer a differentiable one*. This is easy to check and due to the fact that the function $\Delta V^{\star}$ depends on the extended demand functions, given by (1), which are themselves non-differentiable functions. This non-differentiability requires us to be careful when studying the stability of equilibria, since the sign of the 'derivative' at the equilibrium eventually no longer provides an adequat information concerning stability. As one can already guess, the interesting cases will be precisely the ones in which the non-differentiability occurs for the current equilibrium value $\lambda^{\star}$.

4. Equilibria under unilateral trade

As shown by Behrens [1], the case of inconditional autarky is neither subject to multiple equilibria nor to catastrophic bifurcations. Those results no longer hold in the case of conditional unilateral trade, where transitions can be catastrophic and where the number of equilibria increases. Hence, the analysis gets more involved because *initial conditions and history play an important role*. In order to tackle this issue, we use a 'standard' approach of new economic geography and appeal to an artifical history (refer to Fujita and Krugman [4] and Fujita et al. [6]). We assume we start from an initial equilibrium configuration in which transport costs are so high that we are in the inconditional autarky case (i.e. $\tau \geq \tau_a$) and we examine how the opening to trade of region F, due to a decrease in transport costs, modifies the existing spatial equilibrium. As shown by Behrens [1], there are three possible types of equilibria under autarky: full agglomeration if A lies below the sustain point A_s, dispersion if A lies above the break point A_b and partial agglomeration in between. Further, the interior equilibria are given by

$$\lambda^{\pm} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 4\epsilon\phi \frac{3\epsilon\phi + \gamma(L-A)}{L^2\gamma^2} - \frac{A}{L}} \tag{41}$$

when $A_s < A < A_b$. As argued in Section 2, some asymmetry between regions is needed in order for unilateral trade to be feasible for at least some spatial distributions λ. This rules out the case of initial dispersion and leaves us with two possible scenarii: that of full and that of partial initial agglomeration. In what follows, the subscripts aut, uni and bil refer to variables and expressions under autarky, unilateral and bilateral patterns of trade respectively.

Let us begin by investigating how the unilateral opening to trade of region F affects the spatial equilibrium if we start from a fully agglomerated autarky equilibrium with $\lambda = 1$. Evaluating expressions (34) (using (35), (36) and (37)) and (40) at $\lambda = 1$ yields

$$V^{\star}_{F\,aut}\Big|_{\lambda=1} = \frac{A\alpha^2}{8\phi\epsilon} \tag{42}$$

and

$$V^{\star}_{F\,uni}\Big|_{\lambda=1} = \frac{1}{2[2\epsilon+\gamma N]^2}\left[N(\alpha-\tau)^2(\epsilon+\gamma N) + \frac{A}{4\phi\epsilon}\big[2\alpha\epsilon+\tau\gamma N\big]^2\right]. \quad (43)$$

Evaluating the difference of (42) and (43) yields

$$V^{\star}_{F\,uni}\Big|_{\lambda=1} - V^{\star}_{F\,aut}\Big|_{\lambda=1} = \frac{N(\alpha-\tau)}{2[2\epsilon+\gamma N]^2}\left[(\alpha-\tau)\epsilon+\gamma N) - \frac{A\gamma}{4\phi\epsilon}\big[4\alpha\epsilon+\gamma N(\alpha+\tau)\big]\right], \quad (44)$$

which (recall that $\tau < \alpha$) is positive if and only if

$$(\alpha-\tau)(\epsilon+\gamma N) \geq \frac{A\gamma}{4\phi\epsilon}\big[4\alpha\epsilon+\gamma N(\alpha+\tau)\big]. \quad (45)$$

Several remarks are in order. [5] As one can see from (45), the unilateral opening to trade of region F is never profitable to agents in that region if τ remains close to $\tau_a = \alpha$. Therefore, *if transport costs remain close to τ_a, unilateral trade does not allow to increase the utility of agents in region F, even if goods are highly differentiated* (ϵ is large). Roughly speaking, the terms of trade get worse for region F. This suggests that 'partial liberalization' and suppression of only a small part of tariffs and barriers to trade can have a detrimental effect on the opening region, because prices of import varieties remain high while foreign competition degrades local market conditions. Hence, *transport costs, tariffs and barriers to trade must decrease sufficiently in order for unilateral trade to possibly have a positive impact on the welfare in the smaller region*. This result shows that, as Duncan et al. [2] have pointed out, "proximity matters" so that positive impacts of unilateral opening to trade are more likely if the industrialized regions are not too far away from the peripheral ones. If τ decreases

[5] Recall that the indirect utility is given for mobile M-workers only. Hence, because there is no intersectoral mobility, the utility of immobile A-workers can behave quite differently, especially since the agricultural wage does not depend on considerations of competition in the M-industry. Note also that in case λ=1, the indirect utility in the periphery corresponds to the maximal utility a firm could offer to its workers by relocating to the peripheral region.

sufficiently below τ_a, condition (45) is more likely to hold if ϵ is large i.e. if goods are bad substitutes and the impact on consumer utility is hence strong. This shows that complementary patterns of trade (characterized by low elasticities of substitution) are more likely to lead to benefits from liberalization than similar patterns of trade (characterized by high elasticities of substitution). In this case, *the positive effect on the demand side in region F, going through preference for variety due to a larger array of available goods, can possibly offset the negative effect on the production side, consisting in stronger price competition, profit erosion and wage cuts*. Stronger scale economies (hence larger values of ϕ) make condition (45) also more likely to hold. The important point to note is that the opening of the small region to imports from the larger one is *not automatically synonymous with increasing utility in the small region. Only if the terms of trade are not too detrimental to the smaller region can the agents of that region benefit from this unilateral opening*.

Using (33) and (39), it is readily verified for region H that

$$V^{\star}_{H\,uni}\Big|_{\lambda=1} - V^{\star}_{H\,aut}\Big|_{\lambda=1} = \frac{A\epsilon(\alpha-\tau)^2}{2\phi[2\epsilon+\gamma N]^2} \geq 0, \qquad (46)$$

so that a unilateral export activity is always profitable to agents in that region. This is due to the fact that only a positive effect on wages is at work in region H. We can hence conclude that a *sufficient condition* for a fully agglomerated initial autarky equilibrium with $\lambda = 1$ to remain stable during the passage to unilateral trade is given by the reverse of condition (45). Note that this condition is a rather special one, because it states that the stability of the initial equilibrium is guaranteed by a deterioration of the indirect utility in region F. Therefore, this condition is not compatible with a simultaneous increase of the utility levels in both regions (which would be more in line with what one would expect from trade). Note also that it is a priori unclear if the joint benefits from the passage to unilateral trade are positive. *We can therefore not exclude the situation in which trade is globally detrimental to the two regions since benefits to region H are not sufficient in order to compensate for losses to region F*. In order for the (individual) net benefits from unilateral trade to be positive, we need to know when the condition

$$\left(V^{\star}_{Huni}\Big|_{\lambda=1} - V^{\star}_{Haut}\Big|_{\lambda=1}\right) + \left(V^{\star}_{Funi}\Big|_{\lambda=1} - V^{\star}_{Faut}\Big|_{\lambda=1}\right) \geq 0 \qquad (47)$$

holds. Using (44) and (46), condition (47) is equivalent to

$$A\epsilon(\alpha-\tau) + L\left[(\alpha-\tau)\big(\epsilon+\gamma N\big) - A\gamma\frac{L}{4\phi\epsilon}\big(4\alpha\epsilon+\gamma N(\alpha+\tau)\big)\right] \geq 0. \qquad (48)$$

Because $\tau < \alpha$, inspection of condition (48) reveals that it never holds if goods are perfect substitutes (when $\epsilon \to 0$) and that it always holds if goods are independent (when $\gamma \to 0$ and hence $\epsilon \to \beta$). It is also easily seen that the stronger the economies of scale, the more likely that trade will be globally beneficial. When scale economies are large (formally when $\phi \to +\infty$) trade will always be globally beneficial; when there are no economies of scale (formally when $\phi \to 0$) trade will never be globally beneficial. We may hence conclude that the opening of a region to exports from another is a) beneficial to the exporting region, b) not necessarily beneficial to the importing region and c) not necessarily jointly beneficial to the two regions. *Only if goods are sufficiently differentiated and scale economies strong enough can unilateral trade have a positive impact on the regions' joint level of welfare*. In that case, even if trade is detrimental to the importing region, there is the possibility for a welfare maximizing planner to use lump-sum transfers in order to compensate the importing region for the losses so that trade is jointly beneficial to all involved parties.

We now establish a *necessary and sufficient condition* under which full agglomeration is an equilibrium under unilateral trade.

Proposition 4.1 (UNILATERAL TRADE AND FULL AGGLOMERATION)

Assume that $\lambda > 1/2$ so that H is the large region. Consider the case in which $\tilde{\tau} \leq \tau \leq \tau_a$ so that unilateral trade can occur for certain spatial distributions λ. Then full agglomeration $\lambda = 1$ is a spatial equilibrium if and only if

$$3\alpha^2\phi N\epsilon + \alpha^2\phi\gamma N^2 + A\alpha^2\epsilon - A\frac{[2\alpha\epsilon+\gamma\tau N]^2}{4\epsilon} \geq (\alpha-\tau)^2\big[\epsilon(\phi N - A) + \phi\gamma N^2\big].$$

PROOF. Evaluating expressions (33) and (34) (using (35), (36) and (37)) at $\lambda = 1$, one gets the indirect utility differential

$$\Delta V^\star \Big|_{\lambda=1} = \frac{1}{2\phi[2\epsilon + \gamma N]^2} \Big[3\alpha^2 \phi N \epsilon + \alpha^2 \phi \gamma N^2 + A\alpha^2 \epsilon + A\epsilon(\alpha - \tau)^2 - \phi N(\alpha - \tau)^2 (\epsilon + \gamma N) - A\frac{[2\alpha\epsilon + \gamma\tau N]^2}{4\epsilon} \Big],$$

which is positive if and only if the term in square brackets is positive. This expression is easily rewritten as

$$3\alpha^2 \phi N \epsilon + \alpha^2 \phi \gamma N^2 + A\alpha^2 \epsilon - A\frac{[2\alpha\epsilon + \gamma\tau N]^2}{4\epsilon} \geq (\alpha - \tau)^2 \left[\epsilon(\phi N - A) + \phi\gamma N^2 \right] \quad (49)$$

which is precisely the condition of Proposition 4.1. Since the indirect utility differential is positive at $\lambda = 1$ if condition (49) holds, we conclude that in this case full agglomeration is always a stable spatial equilibrium under unilateral trade. □

As one can see from condition (49), at least some product differentiation is needed for it to possibly hold. If varieties are independent, (49) always holds so that full agglomeration is a spatial equilibrium in the unilateral trade case. Further, the next proposition shows that *condition* (49) *automatically holds if we start from a completely agglomerated equilibrium in autarky.*

Proposition 4.2 (STABILITY OF INITIAL AGGLOMERATION)

Assume that $\lambda > 1/2$ so that H is the large region. Consider the case in which $\tilde{\tau} \leq \tau \leq \tau_a$ so that unilateral trade can occur for certain spatial distributions λ and suppose that $0 \leq A \leq A_s$ (i.e. we have a completely agglomerated equilibrium in autarky). Then full agglomeration $\lambda = 1$ is always a spatial equilibrium for unilateral trade.

PROOF. Due to its length, the proof is relegated to Appendix A. □

Proposition 4.2 roughly states the following. If we consider an initial configuration in which full agglomeration is a stable spatial equilibrium for autarky, full agglomeration remains a stable spatial equilibrium for unilateral trade. As we have argued previously in this section, the opening of the small region F to imports from region H is always beneficial to H and eventually beneficial to F. Proposition 4.2 states that *even if unilateral trade is beneficial to the importing region, it will never be beneficial enough to attract firms from the larger region to the smaller one*. Hence, a fully agglomerated autarky equilibrium cannot be broken by the sole passage to unilateral trade, which highlights again the fact that spatial equilibria can be *highly robust to even significant modifications in transportation costs*. This is due to the fact that the structural change in the patterns of trade, by creating asymmetries in market access, gives rise to new agglomeration forces that were not active in the case of autarky. Therefore, the decrease in transport costs and the emergence of trade (which could a priori have worked against agglomeration in this scenario) give rise to new agglomeration forces via the passage to unilateral trade, which locks-in the initial equilibrium configuration. One should finally note that Proposition 4.2 only holds for $0 \leq A \leq A_s$. Examples for which full agglomeration is not an equilibrium configuration for unilateral trade when $A > A_s$ are easy to construct.

In the remainder of this section, we examine how a *partially* agglomerated initial equilibrium evolves when autarky is broken by the passage to unilateral trade. This issue, concerning the impacts of trade liberalization on the spatial distribution of economic activities and on regional inequalities, has always been and still is a main point of interest both from a theoretical and a policy point of view. These last decades, several large-scale projects of trade liberalization like e.g. the EU, NAFTA and APEC, have raised renewed interest in those questions. Of course, those projects and the resulting trading blocks are usually characterized by bilateral trade. We nevertheless remind the reader that *bilateral trade liberalization*, consisting in an a priori symmetric decrease of trade costs between two regions, can lead to unilateral patterns of trade. Especially, as explained in Section 2, symmetric liberalization only leads to bilateral trade if a) the reduction in trade costs is large enough and b) the initial distribution of economic activities is not too asymmetric between the two regions. Both of those points strike us as being empirically fairly implausible.

Does a decrease in transport costs, and the eventual opening to trade, lead to regional convergence or divergence? Will the Single Market Program of the EU favor dispersion of economic activities among member states or could we observe an additional polarization of the space-economy? Although there is still no consensus on those issues (despite the results established in new economic geography), the fear of regional divergence has always been on economists' minds. Kaldor wrote in 1970: *"When trade is opened up between them, the region with the more developed industry will be able to supply the need of the agricultural area of the other region on more favourable terms: with the result that the industrial center of the second region will lose its market and will tend to be eliminated"* (as quoted in Ottaviano and Thisse [17]). There are many examples of regions or nations that used, or still use, barriers to trade in order to protect their nascent industry against outside competition from a larger neighbor. Even in todays world of 'free trade', trade is often restricted in complex ways. That such protectionnist positions could be theoretically grounded is shown in the remainder of this section.

In order to investigate the issue concerning regional covergence or divergence, we start with an inconditional autarky equilibrium λ^+ that exhibits partial agglomeration, given by (41), and ask the following question: How does the spatial structure of the economy change when a decrease in transport costs allows firms of the larger region H to enter the peripheral market F? Let us start with an illustrative example:

Figure 2 plots the two indirect utility differentials $\Delta V^\star_{uni}$ and $\Delta V^\star_{aut}$ on the interval $[1/2, 1]$ and illustrates a possible situation in the case of unilateral trade. Because we are in the conditional case, the pattern of trade (autarky or unilateral) depends on the spatial distribution λ. Using condition (17), the expression of the indirect utility differential changes at the critical value

$$\lambda_{sup} := 1 - \frac{2\phi(a - b\tau)}{\tau c L} = 1 - \frac{2\phi\epsilon(\alpha - \tau)}{\tau\gamma L}, \qquad (50)$$

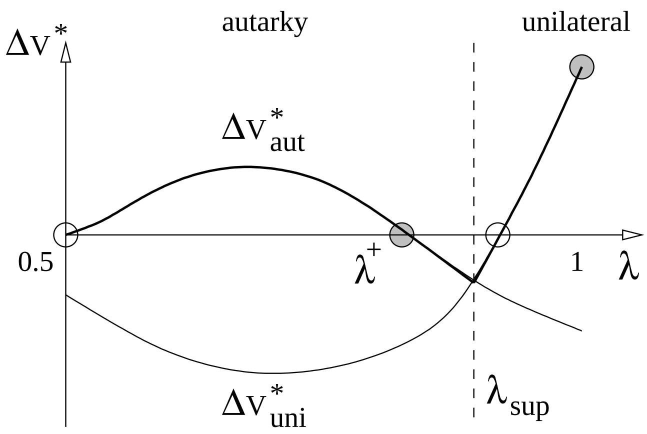

Figure 2: Indirect utility differential in the conditional case

which is graphically represented by the dashed vertical line in Figure 2. From Proposition 3.1, we know the indirect utility differential remains a continuous function of λ. The *effective indirect utility differential* that takes into account the structural change in the pattern of trade is represented in Figure 2 by the heavy line, which turns out to be the upper envelop of $\Delta V^{\star}_{uni}$ and $\Delta V^{\star}_{aut}$. As one can also see from Figure 2, there are *two stable equilibria associated with two different patterns of trade* in our example. The first one corresponds to complete agglomeration under unilateral trade at $\lambda = 1$ and the second one to partial agglomeration under autarky at $\lambda = \lambda^{+} > 0.5$ (as given by expression (41)). Figure 2 further shows that there are two unstable equilibria, corresponding also to two different patterns of trade. Hence, *there are seven equilibria in our model if we consider the general case in which* $\lambda \in [0, 1]$. This shows that if we take into account the structure of trade, the number of equilibria increases and changes with that structure. Finally, Figure 2 shows that the indirect utility differential is a non-differentiable function of λ. The kink is located at $\lambda = \lambda_{sup}$, which is strictly greater than the interior equilibrium λ^{+} in our example.

Expression (50) clearly shows that λ_{sup} is an increasing function of τ. Hence, the kink λ_{sup} 'moves to the left' as τ decreases. [6] What happens when τ decreases

[6] As one can see from (50), λ_{sup} decreases with decreasing transport costs, increasing product differentiation, increasing scale economies and increasing consumer preference for the differentiated product. Hence, the passage to trade does not necessarily rely on a sole

sufficiently so that $\lambda_{sup} = \lambda^{+}$, i.e. when the industry of the larger region H is able to profitably enter the market of the smaller region F? Our main result, which is in line with the 'kaldorian absorption scenario', is summarized in the next proposition.

Proposition 4.3 (ABSORPTION OF THE SMALLER REGION'S INDUSTRY)

Assume that $\lambda > 1/2$ so that H is the large region. Consider a stable initial autarky equilibrium with partial agglomeration. Suppose that the level of transport costs τ decreases so that $\tilde{\tau} \leq \tau \leq \tau_F(\lambda)$, which allows the firms of the larger region H to profitably enter the market of the smaller region F. Then the initial equilibrium becomes unstable and we observe more agglomeration in the larger region.

PROOF. We have to show that the derivative of the indirect utility differential $\Delta V^{\star}_{uni}$ in the case of unilateral trade, given by (38), is always positive when evaluated at $\lambda = \lambda_{sup}$. Hence, when τ decreases so that unilateral trade becomes profitable for firms of the larger region H, the stability of the initial partially agglomerated equilibrium is broken and more agglomeration arises in region H. Due to its length, part of the proof is relegated to appendices B and C. In Appendix B, we establish the analytical expression of the derivative of the indirect utility differential in the unilateral trade case at $\lambda = \lambda_{sup}$. In Appendix C, we show that this derivative is always positive when the condition

$$A \leq A_b + \frac{2\phi\epsilon}{\gamma}\left(1 - \frac{\alpha}{\tau}\right) + \frac{5}{4}L \qquad (51)$$

holds. Rewrite (51) as $A \leq A_b + f(\tau)$, where

$$f(\tau) = \frac{2\phi\epsilon}{\gamma}\left(1 - \frac{\alpha}{\tau}\right) + \frac{5}{4}L.$$

decrease in the value of trade costs. For a given and fixed value of transport costs, unilateral trade can emerge as a consequence of technological evolutions in the productive structures: larger production scale and stronger product differentiation can trigger agglomeration and a passage to trade, even if trade costs stay constant and high. As far as we know, this aspect has not been treated in the literature until now. More work is called for here.

Clearly, f is an increasing function of τ. Because $A \leq A_b$ must hold (recall that we assume we have an interior equilibrium in autarky) and since f is increasing with respect to τ, condition (51) always holds if $f(\tilde{\tau}) \geq 0$. Using the definition of $\tilde{\tau}$, given by (16), we have

$$f(\tilde{\tau}) = \frac{2\phi\epsilon}{\gamma}\left(1 - \frac{\alpha}{\tilde{\tau}}\right) + \frac{5}{4}L = \frac{2\phi\epsilon}{\gamma}\left(1 - \frac{4\epsilon + \gamma N}{4\epsilon}\right) + \frac{5}{4}L = \frac{3}{4}L \geq 0.$$

Hence

$$A \leq A_b \leq A_b + f(\tau), \quad \forall \tau \in [\tilde{\tau}, \alpha],$$

so that condition (51) always holds. □

As shown by Proposition 4.3, (part of the) firms of region F relocate to the larger region H as soon as market F opens to imports from H. Let us restate this fundamental result in the following theorem.

Theorem 4.4 (NO REGIONAL CONVERGENCE THROUGH UNILATERAL TRADE)

Consider a stable initial autarky equilibrium with either full or partial agglomeration. Suppose that the value of transport costs τ decreases so that the firms of the larger region can profitably enter the market of the smaller one. Then agglomeration in the larger region is non-decreasing so that we never observe a process of dispersion. Hence, in the presence of mobile production factors, regional convergence can never result from the unilateral opening to trade of one region alone.

PROOF. Being a simple reformulation of Propositions 4.2 and 4.3, the proof is omitted. □

Let us close this section with an important final remark. As one can see from expression (41), the interior equilibrium λ^+ is independent of the value of transport costs τ. Hence, λ^+ does not change as τ decreases below τ_a, *as long as there is no structural change in the model* (i.e. as long as the pattern of trade remains stable). This implies

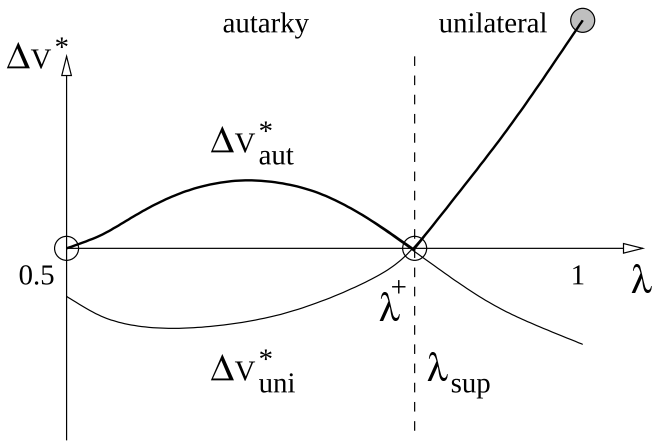

Figure 3: Indirect utility differential as autarky breaks

that for all $1/2 \leq \lambda \leq \lambda^+$, the relevant indirect utility differential is that of autarky given by $\Delta V^\star_{aut}$.

Figure 3 illustrates the passage from autarky to unilateral trade in the case of a partially agglomerated initial equilibrium. While the interior equilibrium λ^+ is stable under autarky (refer to Figure 2) it becomes unstable once there is unilateral trade, i.e. as soon as λ_{sup} hits the value λ^+ of the interior equilibrium. The new stable equilibrium in our example is one of complete agglomeration *in which the larger region H has absorbed the whole industrial sector of the smaller region F*.

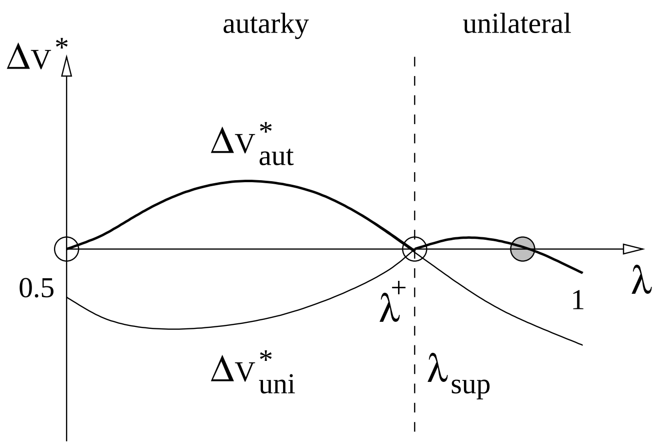

Figure 4: Increasing partial agglomeration with unilateral trade ?

This example raises the question of whether once the interior equilibrium gets unstable there is always complete agglomeration in the larger region. Because the general analytical expression of the derivative of $\Delta V^{\star}_{uni}$ is very complicated, we have no formal proof. Especially, we have not been able to determine whether $\Delta V^{\star}_{uni}$ is a monotonous function for $\lambda \geq \lambda_{sup}$ or not. Numerical simulations suggest that the indirect utility differential is indeed monotonically increasing beyond λ_{sup}. Hence, increasing agglomeration with partially agglomerated equilibria, as depicted in Figure 4, seems not to be a possible issue. More work is called for here.

5. Utility under different core-periphery structures

As shown by Ottaviano et al. [19] and by Behrens [1], there is not only one but there are *three types of core-periphery structure, depending on the pattern of trade between the two regions*. We can have full agglomeration of all firms in region H under either autarky, unilateral or bilateral trade. Therefore, there are *three qualitatively different core-periphery structures in our model*. Each core-periphery structure is susceptible to yield a different level of welfare in the periphery, depending on the actual structure of trade in the economy. We can hence investigate whether there is a core-periphery structure that yields a higher level of utility in the periphery than the other core-periphery structures do. One might be tempted to think that utility in the periphery gradually rises as we first pass from autarky to unilateral trade and then from unilateral to bilateral trade. That things are not that simple and that the degree of openess to trade is not monotonously correlated with the level of welfare in the periphery will be explained in this section.

As shown in Section 4, the indirect utilities in the peripheral region F under a core-periphery structure are given by (42) and (43) respectively. In order to evaluate the utilities in region F under all three basic patterns of trade, we have to establish the expression of the indirect utility in region F for bilateral trade. Using the results established by Ottaviano et al. [19], one can show that the indirect utility in the bilateral core-periphery structure is given by

$$V^{\star}_{F\,bil}\Big|_{\lambda=1} = \frac{1}{4\epsilon\phi(2\epsilon+\gamma N)^2}\Big[2L\epsilon(\epsilon+\gamma N)(\alpha-\tau)^2 + \frac{A}{2}(2\alpha\epsilon+\tau\gamma N)^2 + \left(\frac{A}{2}+L\right)(2\alpha\epsilon-\tau(2\epsilon+\gamma N))^2\Big]. \quad (52)$$

Using (42), (43) and (52), one can show that the three indirect utilities are linked by simple relations, which are given by

$$V^{\star}_{F\,uni}\Big|_{\lambda=1} = V^{\star}_{F\,aut}\Big|_{\lambda=1} \times \left(\frac{2\alpha\epsilon+\tau\gamma N}{2\alpha\epsilon+\alpha\gamma N}\right)^2 + \frac{N(\alpha-\tau)^2(\epsilon+\gamma N)}{2[2\epsilon+\gamma N]^2} \quad (53)$$

and

$$V^{\star}_{F\,bil}\Big|_{\lambda=1} = V^{\star}_{F\,uni}\Big|_{\lambda=1} + \frac{\big(2\alpha\epsilon-\tau(2\epsilon+\gamma N)\big)^2}{4\epsilon\phi(2\epsilon+\gamma N)^2}\left(\frac{A}{2}+L\right). \quad (54)$$

respectively. As one can see from (54), the passage from unilateral trade to bilateral trade is always beneficial to the periphery because the second term is non-negative. If a firm was to deviate to the periphery, it would be able to offer its workers a higher level of utility under a bilateral trade pattern than under a unilateral one. This result is easy to understand: competition in region F is not fiercer under bilateral trade than it is under unilateral trade, but bilateral trade allows the periphery to access the market and products of the core. The impact of the passage from autarky to unilateral trade is less straightforward to analyze. As one can see from (53), the utility in the periphery can either increase or decrease. This is due to the fact that

$$\frac{2\alpha\epsilon+\tau\gamma N}{2\alpha\epsilon+\alpha\gamma N} \le 1,$$

because $\tau \le \alpha$ must hold in order for unilateral trade to be feasible for at least some values of λ. Hence, it seems that nothing precise can be said a priori about the evolution of the indirect utility in the peripheral region under the core-periphery structure when passing from autarky to unilateral trade. Nevertheless, one can check that

$$\frac{\partial V^{\star}_{F\,uni}}{\partial\tau}\Big|_{\lambda=1} \ge 0 \quad \Leftrightarrow \quad A \le \frac{4\phi\epsilon(\alpha-\tau)(\epsilon+\gamma N)}{\gamma(2\alpha\epsilon+\tau\gamma N)}. \quad (55)$$

Therefore, the slope of $V^{\star}_{F\,uni}\big|_{\lambda=1}$ is always positive at $\tau = \tau_a = \alpha$, which shows that the passage from autarky to unilateral trade (which implies a decrease in τ) deteriorates the conditions in the periphery.

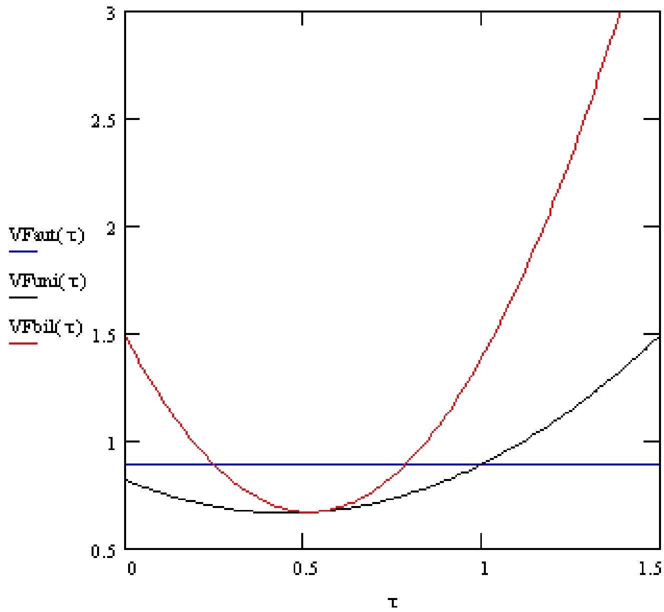

Figure 5: Evolution of $V^{\star}_{F}$ under a core-periphery structure with high ϵ

One can check that the functions (42), (43) and (52) describe the indirect utility in the periphery for all values of τ and that the global function is continuous with respect to τ. Figures 5 and 6 plot $V^{\star}_{F}\big|_{\lambda=1}$ for the different structures of trade. As one can see from Figure 5, the indirect utility first decreases, then usually starts to increase again with the passage to bilateral trade. It stays below the autarky level for the unilateral trade configuration and eventually exceeds the autarky level after transport costs decrease sufficiently so that bilateral trade becomes possible Under which conditions is trade preferable to autarky for at least some values of τ? Using (53) and (54), we have

$$V^{\star}_{F\,bil}\Big|_{\substack{\lambda=1\\ \tau=0}} = V^{\star}_{F\,aut}\Big|_{\substack{\lambda=1\\ \tau=0}} \times \left(\frac{2\epsilon}{2\epsilon+\gamma N}\right)^2 + \frac{N\alpha^2(\epsilon+\gamma N)}{2[2\epsilon+\gamma N]^2} + \frac{\alpha^2\epsilon}{\phi(2\epsilon+\gamma N)^2}\left(\frac{A}{2}+L\right), \quad (56)$$

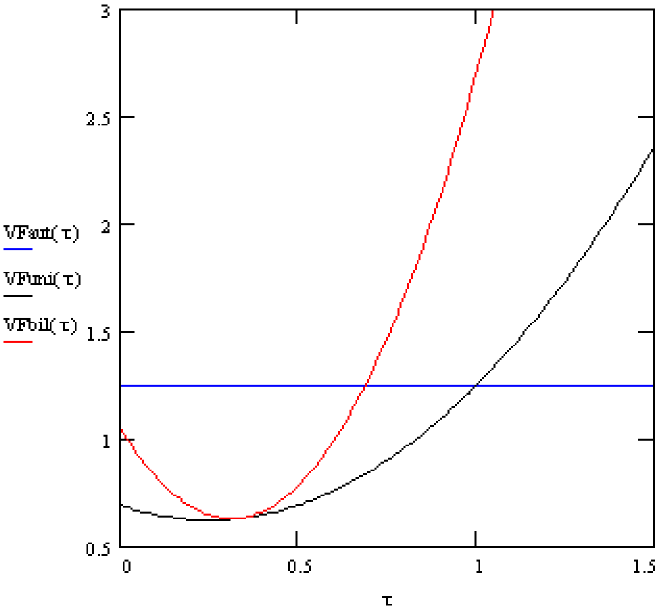

Figure 6: Evolution of $V_F^\star$ under a core-periphery structure with low ϵ

which shows that the condition

$$V_{F\,bil}^\star \Big|_{\substack{\lambda=1\\ \tau=0}} \geq V_{F\,aut}^\star \Big|_{\substack{\lambda=1\\ \tau=0}}$$

is equivalent to

$$\frac{N}{2}(\epsilon + \gamma N) + \epsilon\left(\frac{A}{2\phi} + N\right) \geq \frac{A\gamma N}{2\phi}\left(1 + \frac{\gamma N}{4\epsilon}\right). \tag{57}$$

As one can see from condition (57), the opening to bilateral trade is more likely to increase the utility in the periphery when compared to autarky if products are highly differentiated (larger values of ϵ and lower values of γ) and if scale economies are significant (larger values of ϕ). One might legitimately ask whether unilateral trade always yields the lowest possible utility in the periphery or not. The main result is summarized in the following proposition.

Proposition 5.1 (MINIMUM UTILITY IN THE PERIPHERY UNDER A CORE-PERIPHERY STRUCTURE)

Suppose that $\lambda = 1$ is a spatial equilibrium for all values of τ. Then the *minimum of the indirect utility* $V_F^\star \big|_{\lambda=1}$ *is reached in the range of unilateral trade if*

$$\tau^\circ := \frac{\alpha\big[\phi(\epsilon+\gamma N)-\gamma A\big]}{2\phi(\epsilon+\gamma N)+\gamma^2 AN} \in \,]\tau_{trade}\,,\ \tau_a[. \tag{58}$$

PROOF. Expression (58) is an interior extremum of $V^\star_{F\,uni}$. The second order conditions reveal that this point is a local minimum. Condition (54) shows that $V^\star_{F\,bil}$ lies above $V^\star_{F\,uni}$; hence there is no lower value for $\tau < \tau^\circ$. Condition (55) shows that $V^\star_{F\,uni}$ is increasing for $\tau > \tau^\circ$. Because $V^\star_{F\,aut}$ is constant and lies strictly above $V^\star_{F\,uni}(\tau^\circ)$, we conclude that the minimum is achieved at $\tau = \tau^\circ$. □

Proposition 5.1 explicits conditions under which *the worst possible situation for the periphery is achieved when trade is unilateral*. This is reminiscent of the situation in many of todays developing countries, in which the large metropolitan core regions provide all goods and services and in which peripheral regions are unable to provide goods or services to the core in return. Hence, *unilateral trade, by lowering the attractivity of the periphery, is a strong force working against economic development and regional convergence. Locations that allow for unilateral access to peripheral markets attract economic activities, which in turn enhances their access capacity and diminishes that of the periphery*.

Note finally that if Proposition 5.1 does not hold, the minimum utility in the peripheral region under a core-periphery structure is just reached at the passage from unilateral to bilateral trade (as is the case in figures 5 and 6). [7] This shows that *autarky never corresponds to the worst situation for the periphery, at least under a core-periphery structure*.

[7] Numerical simulations reveal that this case is the most frequently observed one. It occurs whenever the derivative of the indirect utility differential, evaluated at $\tau{=}\tau_{trade}$ is positive under unilateral and negative under bilateral trade.

6. Conclusions

We have shown in this paper that, despite a priori symmetric transport costs between regions, asymmetries in regional access can endogenously appear. This endogenous asymmetry arises naturally if firms are unevenly distributed in the space-economy, because this uneven distribution is synonymous with different degrees of local price competition in segmented markets. Hence, it is tempting to conclude that *transport costs are never symmetric between regions, at least as long as those transport costs remain higher than a certain threshold* (given by τ_{trade} in our model). This aspect is of fundamental importance and is reminiscent of basic questions addressed in the ever growing literature on hubs and endogenous network formation. As argued by Mori and Nishikimi [15], *"interregional trade patterns and the structure of the transport network are determined endogenously as a result of the interaction between industrial location behavior and increasing returns in transportation, in particular economies of transport density"*. As shown in this paper, there is no need for economies of transport density in order for market access between regions to be eventually asymmetric; price competition among monopolistically competitive firms producing imperfect substitutes can be sufficient in order to create endogenous asymmetries. Of course, economies of transport density do play a role in creating even more asymmetry in market access. Hence, we may safely conclude that *both price competition and economies of transport density work together to create significant distortions in market access in todays differentiated industries*. Unfortunately, we are not aware of any empirical work trying to investigate the degree of asymmetry in market access and the impact of this market access on the spatial distribution of economic activities (see, however, Head and Mayer [10] for some interesting developments). More work is called for here.

We have also shown in this paper that asymmetries in trade patterns give rise to strong agglomeration forces, because they introduce a significant distortion into firms' market access capacities. In case firms in region H can access market F but not the other way round, region H offers a significant locational advantage: the overall market-size accessible from region H is larger than that accessible from region F, while price competition is not fiercer. This in turn deteriorates the operating conditions in region F and leads to relocation of (a part of) the industry of region F to region H. As shown by Theorem 4.4, *the unilateral opening to trade of one region leads to the absorption of its industry by the other region*. Hence, there is no reason to assume that unilateral trade will lead to regional convergence.

Finally, we have highlighted the fact that it is difficult to speak of *the* core-periphery structure of the economy, because there are three core-periphery structures corresponding to three basic patterns of trade. As shown in Section 5, the welfare results for those three configurations are very different. Contrary to what one might expect, the periphery is worst off in case of unilateral trade, because in this situation the terms of trade are the most detrimental. Hence, the passage from autarky to unilateral trade, which can be a forced stage as transport costs decrease, deteriorates significantly the attractivity of the periphery. This result is reminiscent of ancient colonial trade policies, which imposed unilateral patterns of trade between the mother country and the colonies and hence significantly slowed down their economic development. Only when transport costs become sufficiently low and barriers to trade sufficiently negligible will bilateral trade emerge and lead to increasing utility in the periphery.

Appendix A

PROOF. We have to show that the condition for a completely agglomerated equilibrium

$$3\alpha^2\phi N\epsilon + \alpha^2\phi\gamma N^2 + A\alpha^2\epsilon - A\frac{[2\alpha\epsilon + \gamma\tau N]^2}{4\epsilon} \geq (\alpha - \tau)^2\left[\epsilon(\phi N - A) + \phi\gamma N^2\right]$$

always holds for all $\tilde{\tau} \leq \tau \leq \alpha$ if we start from complete agglomeration in autarky (i.e. for $0 \leq A \leq A_s$). This is equivalent to showing that the function

$$\begin{aligned} f(A,\tau) := {} & 3\alpha^2\phi N\epsilon + \alpha^2\phi\gamma N^2 + A\alpha^2\epsilon \\ & - A\frac{[2\alpha\epsilon + \gamma\tau N]^2}{4\epsilon} - (\alpha - \tau)^2\left[\epsilon(\phi N - A) + \phi\gamma N^2\right] \end{aligned} \quad (59)$$

is non-negative on the set

$$\Omega = \left\{(A,\tau) \in \mathbb{R}^2, \tilde{\tau} \leq \tau \leq \alpha,\ \ 0 \leq A \leq A_s\right\}, \quad (60)$$

where

$$A_s := \frac{12\epsilon^2\phi^2 + 4\gamma\epsilon L\phi}{\gamma[4\epsilon\phi + \gamma L]} \quad \text{and} \quad \tilde{\tau} := \frac{4\alpha\epsilon}{4\epsilon + \gamma N}. \quad (61)$$

First, it is straightforward to show that

$$f(A_s, \alpha) = 0, \quad (62)$$

so that the zero-level curve of the function f passes through the upper-right corner of the rectangle Ω. What we show now is that the function f is non-negative on the right border of Ω (given by $A = A_s$ and $\tilde{\tau} \leq \tau \leq \alpha$) and that it is decreasing with respect to A for any fixed $\tau \in [\tilde{\tau}, \alpha]$. Therefore, by continuity, f cannot take negative values on Ω, which proves our claim.

Let us begin by showing that f is decreasing with respect to A. The partial derivative of f with respect to A is given by

$$\frac{\partial f}{\partial A}(A,\tau) = \frac{1}{4\epsilon}\left[4\alpha^2\epsilon^2 - (2\alpha\epsilon + \tau\gamma N)^2 + 4\epsilon^2(\alpha - \tau)^2\right]$$

and we have to show that this partial derivative is negative for all values of $\tilde{\tau} \le \tau \le \alpha$. Define

$$\xi := \frac{\tau}{\alpha} \quad \text{and} \quad k := \frac{\gamma N}{2\epsilon}, \tag{63}$$

so that the condition $\partial f/\partial A \le 0$ can be expressed as

$$4\alpha^2\epsilon^2 g(\xi) \le 0, \tag{64}$$

where

$$g(\xi) = 1 - (1 + k\xi)^2 + (1 - \xi)^2. \tag{65}$$

Since $(A, \tau) \in \Omega$, we know that

$$\xi \in [\bar{\xi}, 1] \quad \text{where} \quad \bar{\xi} := \frac{\tilde{\tau}}{\alpha} = \frac{4\epsilon}{4\epsilon + \gamma N} = \frac{2}{2+k}. \tag{66}$$

Consequently

$$g'(\xi) = -2k(1 + k\xi) - 2(1 - \xi) \le 0. \tag{67}$$

Since g is decreasing with respect to ξ, g is always negative on Ω if it is negative at $\bar{\xi}$. Straightforward calculus shows that the condition

$$g(\bar{\xi}) \le 0 \quad \Leftrightarrow \quad 7k^2 + 8k \ge 0$$

automatically holds since $k \ge 0$. Therefore since, by (67), g is decreasing with respect to ξ, $g \le 0$ on Ω and hence the partial derivative of f is negative with respect to A on Ω.

In order to establish the non-negativity of f on Ω, we can show that f is non-negative on the right side of the rectangle Ω. Since, as shown previously, f is decreasing with respect to A, the result follows.

First, consider the partial derivative of f with respect to τ. Using (63), this derivative can be written as

$$\frac{\partial f}{\partial \tau}(A,\tau) = \alpha\Big[-A(1+k\xi)\gamma N + 2(1-\xi)\big[\epsilon(\phi N - A) + \phi\gamma N^2\big]\Big]. \qquad (68)$$

Clearly, (68) is negative for $\tau = \alpha$ i.e. for $\xi = 1$. One can further see that for a given value of A, this derivative is at most equal to zero a single time. Therefore, since $f(A_s,\alpha) = 0$, f is non-negative on the right border of the rectangle Ω if $f(A_s,\tilde{\tau}) \geq 0$. That this holds indeed is shown now.

First, note that (59) can be rewritten as

$$f(A,\tau) = \alpha^2\epsilon\left(3\phi N + \frac{\phi\gamma N^2}{\epsilon} + A - A\left(1+\frac{\tau\gamma N}{2\alpha\epsilon}\right)^2 - \left(1-\frac{\tau}{\alpha}\right)^2\left[\phi N - A + \frac{\phi\gamma N^2}{\epsilon}\right]\right)$$

which, using the relations (63) yields

$$f(A,\xi) = \alpha^2\epsilon\big(L(3+2k) + A[1-(1+k\xi)^2+(1-\xi)^2] - (1-\xi)^2 L(1+2k)\big). \quad (69)$$

The function f, given by (69), has to be evaluated at $(A_s,\bar{\xi})$. Using (61) and (63), A_s can be expressed as

$$A_s = L\frac{(3+2k)}{k(2+k)}. \qquad (70)$$

Using (69) and (70), the condition $f(A_s,\bar{\xi}) \geq 0$ is equivalent to

$$L(3+2k) + A_s[1-(1+k\bar{\xi})^2 + (1-\bar{\xi})^2] - (1-\bar{\xi})^2 L(1+2k) \geq 0,$$

which, after some calculus, reduces to

$$k(10k^2 + 26k + 15) \geq 0. \qquad (71)$$

Clearly, (71) always holds since k is non-negative. Therefore

$$f(A_s,\tilde{\tau}) = \frac{\alpha^2\epsilon k L(10k^2+26k+15)}{(2+k)^3},$$

which is clearly non-negative.

Since f is continuous (with respect to A and τ), decreasing with respect to A and non-negative on the right side of the rectangle Ω, we conclude that f cannot take negative values on Ω, which proves our claim.

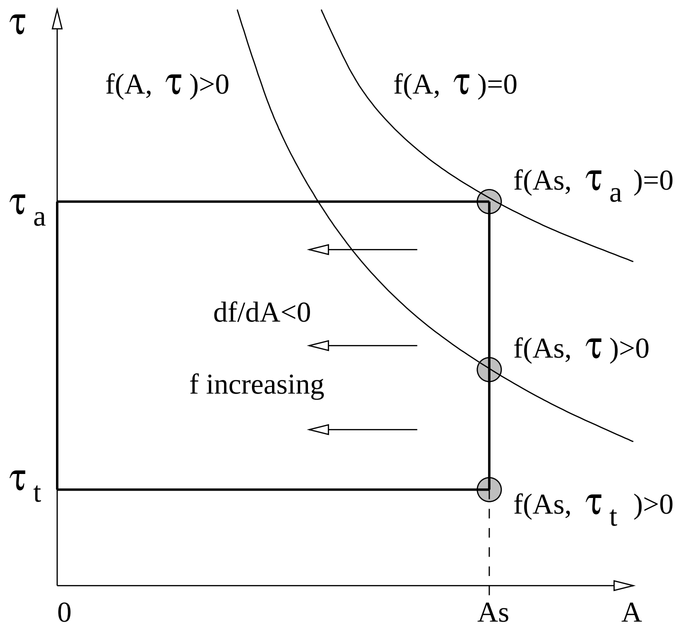

Figure 7: Positivity of f on Ω

Figure 7 illustrates the technique of proof used in this appendix.

Appendix B

PROOF. We show in this appendix that the derivative of $\Delta V^{\star}_{uni}$ is always positive at $\lambda = \lambda_{sup}$ if we are in the case of an initial partially agglomerated equilibrium. This establishes that the opening of the smaller region to unilateral trade renders the equilibrium unstable and leads to more agglomeration in the larger region.

We evaluate

$$\frac{\partial(\Delta V^{\star}_{uni})}{\partial\lambda}(\lambda) = \frac{\partial(\Delta V^{\star}_{Huni})}{\partial\lambda}(\lambda) - \frac{\partial(\Delta V^{\star}_{Funi})}{\partial\lambda}(\lambda) \tag{72}$$

at $\lambda = \lambda_{sup}$, where λ_{sup} is given by (50). Let us start with the derivative of $\Delta V^{\star}_{Huni}$ at the critical value λ_{sup}. First note that $\Delta V^{\star}_{Huni}$, as given by (33), can be expressed as

$$\Delta V^{\star}_{Huni}(\lambda) = \frac{1}{2}u(\lambda)^2 v(\lambda) + w(\lambda)\frac{z(\lambda)}{c_1}, \tag{73}$$

where

$$u(\lambda) := \frac{\alpha}{2\epsilon + \lambda\gamma N}, \quad v(\lambda) := 3\lambda\epsilon N + \lambda^2\gamma N^2 + \frac{A\epsilon}{\phi} \tag{74}$$

and

$$w(\lambda) := \frac{A}{2} + (1-\lambda)L, \quad z(\lambda) := [2\epsilon(\alpha - \tau) - (1-\lambda)\tau\gamma N]^2. \tag{75}$$

Finally,

$$c_1 := 4\phi\epsilon[2\epsilon + \gamma N]^2 \tag{76}$$

is a constant. Since

$$2\epsilon(\alpha - \tau) - (1 - \lambda_{sup})\tau\gamma N = 0, \tag{77}$$

we have $z(\lambda_{sup}) = z'(\lambda_{sup}) = 0$. The derivative of $\Delta V^{\star}_{Huni}(\lambda)$ evaluated at λ_{sup} is then given by

$$\frac{\partial(\Delta V^{\star}_{Huni})}{\partial\lambda}(\lambda_{sup}) = u(\lambda_{sup})u'(\lambda_{sup})v(\lambda_{sup}) + \frac{1}{2}u^2(\lambda_{sup})v'(\lambda_{sup}) + \frac{1}{c_1}\big[w'(\lambda_{sup})z(\lambda_{sup}) + w(\lambda_{sup})z'(\lambda_{sup})\big]. \quad (78)$$

Plugging (74), (75) and (76) into (78) yields, after some simplifications,

$$\frac{\partial(\Delta V^{\star}_{Huni})}{\partial\lambda}(\lambda_{sup}) = \frac{\alpha^2 N\epsilon}{(2\epsilon + \lambda_{sup}\gamma N)^3}\left[3\epsilon + \frac{1}{2}\lambda_{sup}\gamma N - \gamma\frac{A}{\phi}\right].$$

Using (50) and some more simplifications finally yields

$$\frac{\partial(\Delta V^{\star}_{Huni})}{\partial\lambda}(\lambda_{sup}) = \frac{\alpha^2 N\epsilon}{(4\epsilon + \gamma N - 2\epsilon\alpha\tau^{-1})^3}\left[4\epsilon + \frac{\gamma}{\phi}\left(\frac{1}{2}L - A\right) - \frac{\epsilon\alpha}{\tau}\right] \quad (79)$$

which is the derivative of $\Delta V^{\star}_{Huni}$ evaluated at the critical value.

Let us turn next to $\Delta V^{\star}_{Funi}$ as given by (34). Its derivative is best evaluated using the relation

$$\frac{\partial(\Delta V^{\star}_{Funi})}{\partial\lambda}(\lambda_{sup}) = \frac{\partial A_1}{\partial\lambda}(\lambda_{sup}) + \frac{\partial A_2}{\partial\lambda}(\lambda_{sup}) + \frac{\partial A_3}{\partial\lambda}(\lambda_{sup})\cdot \quad (80)$$

Let us begin by evaluating the derivative of A_1 at λ_{sup}. Using (35), we can write

$$A_1(\lambda) = \lambda N\frac{u(\lambda)}{c_1}\left[c_2 - \tau v(\lambda) - \frac{1}{2}w(\lambda)\frac{u(\lambda)}{\epsilon}\right],$$

where we have *redefined* u, v and w as

$$u(\lambda) = 2\epsilon(\alpha-\tau) - \tau\gamma(1-\lambda)N, \;\; v(\lambda) = 2\epsilon + \gamma(\lambda+1)N, \;\; w(\lambda) = \epsilon + \gamma\lambda N \quad (81)$$

and where

$$c_2 = 2\alpha(\epsilon + \gamma N) \quad (82)$$

and c_1, given by (76), are constants. By (77) we know that $u(\lambda_{sup}) = 0$ so that

$$\frac{\partial A_1}{\partial \lambda}(\lambda_{sup}) = \lambda_{sup} N \frac{u'(\lambda_{sup})}{c_1}\big[c_2 - \tau v(\lambda_{sup})\big].$$

Using

$$v(\lambda_{sup}) = 4\epsilon + 2\gamma N - \frac{2\epsilon\alpha}{\tau}$$

we get

$$\frac{\partial A_1}{\partial \lambda}(\lambda_{sup}) = \left(1 - \frac{2\phi\epsilon(\alpha-\tau)}{\tau\gamma L}\right) N \frac{\tau\gamma N}{4\epsilon[2\epsilon+\gamma N]^2}\Big[2\alpha(\epsilon+\gamma N) - 4\epsilon\tau - 2\tau\gamma N + 2\epsilon\alpha\Big]$$

which, after some simplifications, can be expressed as

$$\frac{\partial A_1}{\partial \lambda}(\lambda_{sup}) = \frac{N(\alpha-\tau)}{2\epsilon[2\epsilon+\gamma N]}\big[N\tau\gamma - 2\epsilon(\alpha-\tau)\big]. \tag{83}$$

Next, we use (36) in order to evaluate the derivative of A_2. The expression A_2 can be written as

$$A_2(\lambda) = (1-\lambda) N \frac{u(\lambda)}{c_1}\left[c_2 - \frac{1}{2\epsilon}u(\lambda)v(\lambda)\right]$$

where we have *redefined* u and v as

$$u(\lambda) = 2\alpha\epsilon + \gamma\lambda\tau N, \quad v(\lambda) = 3\epsilon + \gamma(1-\lambda)N \tag{84}$$

and c_2 as

$$c_2 = 2\alpha[2\epsilon + \gamma N]. \tag{85}$$

Note that c_1 is still given by (76). Hence, the derivative of A_2 is given by

$$\frac{\partial A_2}{\partial \lambda}(\lambda_{sup}) = \big[-Nu(\lambda_{sup}) + (1-\lambda_{sup})u'(\lambda_{sup})N\big]\left(c_2 - \frac{1}{2\epsilon}u(\lambda_{sup})v(\lambda_{sup})\right)$$
$$+ (1-\lambda_{sup}) N \frac{u(\lambda_{sup})}{c_1}\left(-\frac{1}{2\epsilon}\big[u'(\lambda_{sup})v(\lambda_{sup}) + u(\lambda_{sup})v'(\lambda_{sup})\big]\right).$$

Some longer calculus yields

$$\frac{\partial A_2}{\partial \lambda}(\lambda_{sup}) = \frac{\alpha - 0.5\tau}{4\epsilon[2\epsilon + \gamma N]}\Big(-N\tau(2\epsilon + \gamma N) + 2\epsilon(\alpha - \tau)N\Big)$$
$$- \frac{1}{4\epsilon[2\epsilon + \gamma N]}(\alpha - \tau)\Big[\tau N\Big(\epsilon + \frac{2\epsilon\alpha}{\tau}\Big) - \tau N(2\epsilon + \gamma N)\Big],$$

which can be expressed as

$$\frac{\partial A_2}{\partial \lambda}(\lambda_{sup}) = \frac{N}{4\epsilon[2\epsilon + \gamma N]}\Big[\Big(\alpha - \frac{\tau}{2}\Big)\big[2\epsilon(\alpha - \tau) - \tau(2\epsilon + \gamma N)\big]$$
$$- (\alpha - \tau)\big[\epsilon(2\alpha - \tau) - \tau\gamma N\big]\Big]. \quad (86)$$

Let us finally evaluate the derivative of A_3. Note that A_3, given by expression (37), can be written as

$$A_3(\lambda) = \frac{u(\lambda)}{c_1}\big[v(\lambda)u(\lambda) - \lambda(1 - \lambda)c_2 w(\lambda)\big]$$

where we have *redefined* u, v and w as

$$u(\lambda) = 2\alpha\epsilon + \gamma\lambda\tau N, \quad v(\lambda) = \frac{A}{2\phi} + (1 - \lambda)N, \quad w(\lambda) = 2\epsilon(\alpha - \tau) - \tau\gamma(1 - \lambda)N \quad (87)$$

and c_2 as

$$c_2 = \frac{\gamma N^2}{\epsilon}. \quad (88)$$

Since $w(\lambda_{sup}) = 0$ by (77), the derivative of A_3 is given by

$$\frac{\partial A_3}{\partial \lambda}(\lambda_{sup}) = \frac{u'(\lambda_{sup})}{c_1}v(\lambda_{sup})u(\lambda_{sup}) + \frac{u(\lambda_{sup})}{c_1}\big[v'(\lambda_{sup})u(\lambda_{sup})$$
$$+ u'(\lambda_{sup})v(\lambda_{sup}) - \lambda_{sup}(1 - \lambda_{sup})c_2 w'(\lambda_{sup})\big].$$

Some longer calculus and simplifications finally yield

$$\frac{\partial A_2}{\partial \lambda}(\lambda_{sup}) = \frac{N\tau}{4\epsilon[2\epsilon + \gamma N]}\Big[\frac{1}{2}\gamma\tau\Big(\frac{A}{\phi} - N\Big) - (\alpha - \tau)\Big(N\gamma - \frac{2\epsilon\alpha}{\tau}\Big) - \epsilon\tau\Big]. \quad (89)$$

Adding expressions (83), (86) and (89), we finally get the expression of the derivative of the indirect utility of region F evaluated at λ_{sup}, given by

$$\frac{\partial(\Delta V^{\star}_{Funi})}{\partial\lambda}(\lambda_{sup}) = \frac{N\tau}{4\epsilon[2\epsilon+\gamma N]}\left[2(\alpha-\tau)\epsilon+\tau\left(\frac{\gamma}{\phi}\left(A-\frac{3}{2}L\right)-3\epsilon\right)\right]. \quad (90)$$

It is the sign of expression (90) that determines whether partial agglomeration remains stable as the smaller region is opened to trade. As we show in Appendix C, the sign of (90) is always positive when we start from initial partial agglomeration, so that the equilibrium always becomes unstable. □

Appendix C

PROOF. We derive in this appendix a sufficient condition under which the derivative of the indirect utility differential $\Delta V^{\star}_{uni}$ is always positive at $\lambda = \lambda_{sup}$ if we start from an autarkic situation in which there are stable interior equilibria.

Since by definition

$$\frac{\partial(\Delta V^{\star}_{uni})}{\partial \lambda}(\lambda_{sup}) = \frac{\partial(\Delta V^{\star}_{Huni})}{\partial \lambda}(\lambda_{sup}) - \frac{\partial(\Delta V^{\star}_{Funi})}{\partial \lambda}(\lambda_{sup}), \tag{91}$$

it is sufficient to show that the first term is positive and the second negative. Let us start with the first term and let us show that this term is always positive. From (79) in Appendix B, we know that

$$\frac{\partial(\Delta V^{\star}_{Huni})}{\partial \lambda}(\lambda_{sup}) = \frac{\alpha^2 N \epsilon}{(4\epsilon + \gamma N - 2\epsilon\alpha\tau^{-1})^3}\left[4\epsilon + \frac{\gamma}{\phi}\left(\frac{L}{2} - A\right) - \frac{\epsilon\alpha}{\tau}\right].$$

First, we need to show that

$$4\epsilon + \gamma N - \frac{2\epsilon\alpha}{\tau} \geq 0,$$

which can be rewritten as

$$\tau \geq \frac{2\alpha\epsilon}{4\epsilon + \gamma N} \tag{92}$$

Since we know from (16) that

$$\tilde{\tau} := \frac{4\alpha\epsilon}{4\epsilon + \gamma N} > \frac{2\alpha\epsilon}{4\epsilon + \gamma N},$$

and since we are in the unilateral case (hence $\tilde{\tau} \leq \tau \leq \tau_a$), it is obvious that condition (92) holds. Second, we show that

$$4\epsilon + \frac{\gamma}{\phi}\left(\frac{1}{2}L - A\right) - \frac{\epsilon\alpha}{\tau} \geq 0. \tag{93}$$

Rewrite (93) as

$$4\epsilon + \frac{1}{2}\gamma N - A\frac{\gamma}{\phi} \geq \frac{\epsilon\alpha}{\tau}\cdot$$

Using

$$A_b := \frac{12\epsilon\phi + \gamma L}{4\gamma}$$

we have

$$4\epsilon + \frac{1}{2}\gamma N - A\frac{\gamma}{\phi} = \frac{\gamma}{\phi}\left[\frac{\epsilon\phi}{\gamma} + (A_b - A) + \frac{L}{4}\right] \geq 0 \tag{94}$$

since $A \leq A_b$ (else there could be no stable interior equilibria under autarky, which would violate our initial assumption). Condition (93) is equivalent to

$$\tau \geq \frac{\alpha\epsilon}{4\epsilon + \frac{1}{2}\gamma N - \gamma\frac{A}{\phi}} = \frac{\alpha\epsilon}{4\epsilon + \frac{\gamma}{\phi}(\frac{L}{2} - A)}\cdot \tag{95}$$

Since $\tau \geq \tilde{\tau}$, condition (95) holds if

$$\tilde{\tau} \geq \frac{\alpha\epsilon}{4\epsilon + \frac{\gamma}{\phi}(\frac{L}{2} - A)}\cdot$$

One can show that this last condition is equivalent to $A \leq A_b$, which holds by assumption of existence of stable interior equilibria.. Hence, since (92) and (93) hold, we conclude that

$$\frac{\partial(\Delta V^{\star}_{Huni})}{\partial\lambda}(\lambda_{sup}) \geq 0$$

for all values of the parameters such that there are partially agglomerated equilibria under autarky.

Let us turn next to $[\partial(\Delta V^{\star}_{Funi})/\partial\lambda](\lambda_{sup})$ and let us show that the sign of this term is ambiguous. From (90) we know that

$$\frac{\partial(\Delta V^{\star}_{Funi})}{\partial\lambda}(\lambda_{sup}) = \frac{N\tau}{4\epsilon[2\epsilon + \gamma N]}\left[2(\alpha - \tau)\epsilon + \tau\left(\frac{\gamma}{\phi}\left(A - \frac{3}{2}L\right) - 3\epsilon\right)\right].$$

In order for this term to be negative, the condition

$$2(\alpha-\tau)\epsilon+\tau\left[\frac{\gamma}{\phi}\left(A-\frac{3}{2}L\right)-3\epsilon\right]\leq 0 \tag{96}$$

must hold. Using the definition of A_b, this condition can be expressed as

$$2\alpha\epsilon+\tau\frac{\gamma}{\phi}\left[(A-A_b)-\frac{2\phi\epsilon}{\gamma}-\frac{5}{4}L\right]\leq 0. \tag{97}$$

As one can see from (97), its sign is a priori undetermined. A sufficient condition for absorption of the smaller region F by the larger region H, i.e.

$$\frac{\partial(\Delta V^{\star}_{Funi})}{\partial\lambda}(\lambda_{sup})\leq 0$$

is finally given by

$$A\leq A_b+\frac{2\phi\epsilon}{\gamma}\left(1-\frac{\alpha}{\tau}\right)+\frac{5}{4}L.$$

□

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