

About the reinterpretation of the Ghosh model as a price model

JEL classification. C67, D46, D57.

KEYWORDS. Input-Output, Demand, Supply, Ghosh

AUTHOR. Prof. Louis de MESNARD

AFFILIATION. LATEC (UMR CNRS 5118), Faculty of Economics, University of Burgundy

ADDRESS.

LATEC

2 Bd Gabriel, B.P. 26611,

F-21066 Dijon Cedex,

FRANCE

Tel: (33) 3 80 39 35 22

Fax: (33) 3 80 39 35 22

E-mail : *louis.de-mesnard@u-bourgogne.fr*

ABSTRACT. The Ghosh model assumes that, in an input-output framework, each commodity is sold to each sector in fixed proportions. This model is strongly criticized because it seems implausible in the traditional input-output field. To answer to these critics, Dietzenbacher stresses that it can be reinterpreted as a price model: the Leontief price model is equivalent to the Ghosh model when this one is interpreted as a price model. This paper shows that the interpretation of the Ghosh model as a price model cannot be accepted because Dietzenbacher makes a strong assumption, dichotomy, while the Ghosh model does not determine prices.

I. Introduction

The Ghosh model assumes that, in an input-output framework, each commodity is sold to each sector in fixed proportions. This model is strongly criticized because in the traditional input-output field it seems implausible for many ¹. To answer to those critics, Dietzenbacher (1997) stresses that it can be reinterpreted as a price model. Considering the demand-driven (Leontief) and supply-driven (Ghosh) models, Dietzenbacher demonstrates that the Leontief price model -- the Leontief model solved by columns, i.e., the dual of the traditional Leontief model solved by rows -- is equivalent to the Ghosh model when this one is interpreted as a price model. This is seductive because it could vindicate strongly the Ghosh model, even if this new interpretation removes its quality of quantity model. However, I will show that the interpretation of the Ghosh model as a price model cannot be accepted, by returning first to the foundations of the Leontief and Ghosh models.

II. The foundations of the linear production models

A. *The hypothesis*

One consider n sectors, producing n commodities, and a set of final consumers. Each sector produces one and only one commodity and each commodity is produced by one and only one sector ². The physical quantity bought by sector j to sector i when j produces the commodity j

¹ See a discussion in (Miller, 1989) (Oosterhaven, 1988, 1989, 1996) (Rose and Allison, 1989).

² To stay in the Dietzenbacher's framework, here I focus only of the more simple linear

is denoted \bar{x}_{ij} . By hypothesis $\bar{x}_{ij} \geq 0$. The model is closed: for each commodity i , the total that is sold is equal to the total that is produced: it is denoted \bar{x}_i , with $\bar{x}_i = \sum_j \bar{x}_{ij} + \bar{f}_i$, where $\bar{f}_i \geq 0$ denotes the final demand. Note that if homogeneity of rows of matrix $\bar{\mathbf{X}}$ is granted, sums by columns cannot be done. Commodities i have a price p_i . Initially, the value of each commodity i is equal to $\bar{x}_i p_i$ and each sector j has in hand x_j of money. The model is monetary closed: the agents have the same quantity of money before and after the exchange, i.e., the model is at equilibrium sector by sector, that is $\bar{x}_i p_i = x_i$ for all i . So, after exchange, the value of each product i is disposed of completely:

$$(1) \quad \sum_j x_{ij} + f_i = x_i \Leftrightarrow \sum_j \bar{x}_{ij} + \bar{f}_i = \bar{x}_i \text{ for all } i$$

where $f_i = \bar{f}_i p_i$ is the final demand in value, and each sector j spends completely the money that he has in hand:

$$(2) \quad \sum_i x_{ij} + v_j = x_j \Leftrightarrow \sum_i \bar{x}_{ij} p_i + \bar{v}_j w = \bar{x}_j p_j \text{ for all } j$$

where $v_j = \bar{v}_j w$ is the added-value measured in value, while \bar{v}_j is the amount of labor used by sector j and w is the wage rate (the profits are assimilated to the remuneration of the manager). Then, two main hypotheses of behavior can be done: either the demand drives the model (Leontief) either the supply does it (Ghosh).

models, where one product is produced by one and only one sector, and conversely, leaving aside the more complicated cases, as the "von Neumann model".

B. The demand-driven model

When the demand-driven model is considered, one assumes that each sector buys each commodity in fixed proportions, but there are two possibilities: coefficients can be defined in physical terms or in value terms.

If coefficients are defined in physical terms, it is assumed that $\bar{a}_{ij} = \frac{\bar{x}_{ij}}{\bar{x}_j}$ for all i and j are stable

³. In matrix terms, this writes: $\bar{\mathbf{A}} = \bar{\mathbf{X}} \langle \bar{\mathbf{x}} \rangle^{-1}$. The economy must be at equilibrium by row and by columns. By rows (I call this the *primal*), the accounting identity (1) becomes $\sum_j \bar{a}_{ij} x_j + \bar{f}_i = \bar{x}_i$, that is:

$$(3) \quad \bar{\mathbf{A}} \bar{\mathbf{x}} + \bar{\mathbf{f}} = \bar{\mathbf{x}} \Leftrightarrow (\mathbf{I} - \bar{\mathbf{A}}) \bar{\mathbf{x}} = \bar{\mathbf{f}}$$

This Cramer system has a non trivial solution only if the determinant $|\mathbf{I} - \bar{\mathbf{A}}|$ is not equal to zero; this solution is $\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} \bar{\mathbf{f}}$. By columns (I call this the *dual*), the accounting identity (2) becomes $\sum_i \bar{a}_{ij} \bar{x}_j p_i + \bar{v}_j w = \bar{x}_j p_j$ or $\sum_i \bar{a}_{ij} p_i + \bar{l}_j w = p_j$, where $\bar{l}_j = \frac{\bar{v}_j}{\bar{x}_j}$ are the input coefficients of labor in quantity and w is the wage rate. This can be noted in matrix terms:

$$(4) \quad \mathbf{p}' \bar{\mathbf{A}} + w \bar{\mathbf{l}}' = \mathbf{p}'$$

³ Note that nothing prevents the coefficients to be higher than 1, their magnitude depends on the chosen scale but the determinant is independent to the scale, and the result also, after the appropriate conversion of scale.

When coefficients are defined in value, are assumed to be stable the coefficients:

$a_{ij} = \frac{x_{ij}}{x_j} = \frac{\bar{x}_{ij} p_i}{\bar{x}_j p_j} = \bar{a}_{ij} \frac{p_i}{p_j}$. By rows, (1) becomes: $\sum_j a_{ij} x_j + f_i = x_i$ for all i , that is:

$$(5) \quad \mathbf{A} \mathbf{x} + \mathbf{f} = \mathbf{x} \Leftrightarrow (\mathbf{I} - \mathbf{A}) \mathbf{x} = \mathbf{f}$$

This will have a solution $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f}$ if $|\mathbf{I} - \mathbf{A}| \neq 0$. By columns, (2) becomes $\sum_i a_{ij} + l_j = 1$

where $l_j = \frac{v_j}{x_j}$, that is:

$$(6) \quad \mathbf{s}' \mathbf{A} + \mathbf{l}' = \mathbf{s}' \Leftrightarrow \mathbf{s}' (\mathbf{I} - \mathbf{A}) = \mathbf{l}'$$

where $\mathbf{s}' = \left(1 \dots 1 \right)$. This is not a Cramer system of equation to be solved in \mathbf{s} , but only an accounting identity that must be respected: if $|\mathbf{I} - \mathbf{A}| = 0$ then $\mathbf{s}' = 0$, what is impossible, so (6) requires $|\mathbf{I} - \mathbf{A}| \neq 0$. Hence the demand-driven model in value (5) always has a non-trivial solution. And as $|\mathbf{I} - \mathbf{A}| = |\mathbf{I} - \hat{\mathbf{p}} \bar{\mathbf{A}} \hat{\mathbf{p}}^{-1}| = \hat{\mathbf{p}} |\mathbf{I} - \bar{\mathbf{A}}| \hat{\mathbf{p}}^{-1}$, if $|\mathbf{I} - \mathbf{A}| = 0$ then $|\mathbf{I} - \bar{\mathbf{A}}| = 0$ unless prices are null. Then the demand-driven model in quantity (3) will have a non-trivial solution always unless prices are null. Moreover, if $\mathbf{f} > 0$ and as $\mathbf{f} = \mathbf{x} - \mathbf{A} \mathbf{x}$, then $\mathbf{x} > \mathbf{A} \mathbf{x}$: matrix \mathbf{A} is always productive⁴.

C. The supply-driven model

Each commodity is assumed to be sold in fixed proportions to each sector. Again there are two possibilities. In physical terms, $\bar{b}_{ij} = \frac{\bar{x}_{ij}}{\bar{x}_i}$ is assumed to be stable for all i, j ; in matrix this

writes $\bar{\mathbf{B}} = \langle \bar{\mathbf{x}} \rangle^{-1} \bar{\mathbf{X}}$. By rows, (1) becomes $\sum_j \bar{b}_{ij} + \bar{d}_i = 1$ for all i where $\bar{d}_i = \frac{\bar{f}_i}{\bar{x}_i}$, that is:

⁴ The demonstration of the condition of productivity $\mathbf{x} > \mathbf{A} \mathbf{x}$ can be found in (Gale, 1989, p. 296-...).

$$(7) \quad \bar{\mathbf{B}} \mathbf{s} + \bar{\mathbf{d}} = \mathbf{s} \Leftrightarrow (\mathbf{I} - \bar{\mathbf{B}}) \mathbf{s} = \bar{\mathbf{d}}$$

Again, this is an identity, not a system of equations to be solved, what imposes $|\mathbf{I} - \bar{\mathbf{B}}| \neq 0$ to be true. By columns, (2) is transformed into $\sum_i \bar{b}_{ij} \bar{x}_i + \bar{v}_j = \bar{x}_j$ for all i , that is:

$$(8) \quad \bar{\mathbf{x}}' \bar{\mathbf{B}} + \bar{\mathbf{v}}' = \bar{\mathbf{x}}' \Leftrightarrow \bar{\mathbf{x}}' (\mathbf{I} - \bar{\mathbf{B}}) = \bar{\mathbf{v}}'$$

what has always a solution $\bar{\mathbf{x}}' = \bar{\mathbf{v}}' (\mathbf{I} - \bar{\mathbf{B}})^{-1}$ since $|\mathbf{I} - \bar{\mathbf{B}}| \neq 0$. In money terms, the stable

coefficients are defined as follows: $b_{ij} = \frac{x_{ij}}{x_i} = \frac{\bar{x}_{ij} p_i}{\bar{x}_i p_i} = \frac{\bar{x}_{ij}}{\bar{x}_i} = \bar{b}_{ij}$ for all i, j . As they are equal to

the coefficients in quantities, the model is exactly the same, either by rows, either by columns.

Note that as $\mathbf{A} = \hat{\mathbf{x}} \mathbf{B} \hat{\mathbf{x}}^{-1}$, one has $\mathbf{I} - \mathbf{A} = \mathbf{I} - \hat{\mathbf{x}} \mathbf{B} \hat{\mathbf{x}}^{-1} = \hat{\mathbf{x}} (\hat{\mathbf{x}}^{-1} \hat{\mathbf{x}} - \mathbf{B}) \hat{\mathbf{x}}^{-1} = \hat{\mathbf{x}} (\mathbf{I} - \mathbf{B}) \hat{\mathbf{x}}^{-1}$ and

then $|\mathbf{I} - \mathbf{A}| = |\mathbf{I} - \mathbf{B}| \neq 0$, either in quantities or in prices.

III. Discussion

Let's summarize. The primal (conventionally computed by rows) of the quantity Leontief model solves in outputs (measured in quantity) while its dual (conventionally computed by columns) solves in prices. The primal of the value Leontief model solves in outputs (measured in value) while the dual is an identity. The primal of all Ghosh models (in quantity as well as in price) is an identity while the dual solves in outputs (in quantity or in value) and is the dual of the primal of the Leontief value model (the same value added generates the same output). So, prices are undetermined in all Ghosh models and in the value Leontief model, while they are

not in the quantity Leontief model compute by columns ⁵. To summarize, *stricto sensu*, you have true prices only in the dual of the Leontief quantity model.

Now, let's discuss Dietzenbacher's argument. He uses the formula (Dietzenbacher, 1997, p. 633, Eq. (9)):

$$(9) \quad \pi' = \pi' \mathbf{A}_0 + \mathbf{v}' \mathbf{L}_0 \Leftrightarrow \pi' = \mathbf{v}' \mathbf{W}_0 (\mathbf{I} - \mathbf{A}_0)^{-1}$$

to compute the price ratios π from the wage ratios $\mathbf{v}' = \hat{\mathbf{v}}_1 \hat{\mathbf{v}}_0^{-1}$ (assuming the dichotomy between prices and quantities, the variation of wages in money will generate only variations in prices), where $\mathbf{L}_0 = \hat{\mathbf{v}}_0 \hat{\mathbf{x}}_0^{-1}$ and where the subscript 0 refers to the data before wage raise and the subscript 1 to the data after wage raise. Dietzenbacher retrieves the fact that the table generated by the variation of the added value with the Ghosh value model is the same that the table generated by the same variation of the added value with the dual of the Leontief value model: this is true because, in value, one model is the dual of the other. However, formula (9) is presented as a price ratio equation ⁶, but this is not true because it only corresponds to the equation of the dual of the Leontief value model (formula 6).

To be convinced of this, just replace \mathbf{v} by \mathbf{s} , (wages are fixed) in formula (9): you obtain formula (6) where π is replaced by \mathbf{s} (prices are fixed). This is why the result found by Dietzenbacher is equal to a ratio between outputs (in value), that is $\pi' = \mathbf{x}'_1 \hat{\mathbf{x}}_0$: introducing a

⁵ Note that the supply-driven model in quantity is not the dual of the demand-driven model in quantity, while they are in their value version.

⁶ "... the new price ratios (π') for the products are such that they satisfy the accounting equations." (Dietzenbacher, 1997, p.633).

change in the value added generates a change in the output, but it is only by a supplementary assumption that Dietzenbacher is able to say that it is a change in prices.

This supplementary hypothesis is dichotomy. True dichotomy comes naturally in the quantity Leontief model: solving the primal gives outputs in quantity but prices are fixed, while solving the dual provides prices but quantities are fixed. But in the other models since prices are never determined, dichotomy could be only a strong assumption. In other words, in the value Leontief model, you see that outputs (in value) vary and you say: *as quantities are fixed by hypothesis, this is a variation of prices...* Obviously, the reverse hypothesis could have been chosen: *prices are fixed by hypothesis and variations of quantities are found*. As well as, dichotomy could have been abandoned for a mixed hypothesis: one percent of variation of value outputs is decomposed into a variation of λ percent of quantities and of $1 - \lambda$ percent of prices. All would have been possible...

Note that the same reasoning holds in duality for formula:

$$(10) \quad \boldsymbol{\theta} = \mathbf{B} \boldsymbol{\theta} + \mathbf{D}_0 \boldsymbol{\varphi} \Leftrightarrow \boldsymbol{\theta} = (\mathbf{I} - \mathbf{B})^{-1} \mathbf{D}_0 \boldsymbol{\varphi}$$

(Dietzenbacher, 1997, p. 639, Eq. (12)) where $\mathbf{D}_0 = \hat{\mathbf{x}}_0^{-1} \hat{\mathbf{f}}_0$, $\boldsymbol{\varphi} = \hat{\mathbf{f}}_0^{-1} \mathbf{f}_2$ and the subscript 2 refers to a new value; it is found that $\boldsymbol{\theta} = \hat{\mathbf{x}}_0^{-1} \mathbf{x}_2$: $\boldsymbol{\theta}$ is again a ratio of outputs. Formula (10) corresponds to the primal of the Ghosh model, what could be verified by replacing $\boldsymbol{\varphi}$ by \mathbf{s} (final demands are fixed): this gives formula (7) where $\boldsymbol{\theta}$ is replaced by \mathbf{s} (outputs are fixed).

IV. Conclusion

Only the dual of the traditional Leontief quantity model (that is, solved by columns) can be considered as a price model, while the primal is the traditional Leontief model that computes

the output of sectors. Prices remain always undetermined in the Leontief value model and all Ghosh models (in quantity and in value). As the Dietzenbacher's demonstration is based on the Leontief value model and Ghosh value model, prices cannot be determined, even under the form of prices indexes, except is a supplementary assumption about dichotomy is made: these price indexes introduced by Dietzenbacher depend on an hypothesis, the fixity of quantities in relation to prices in the value models. In other words, dichotomy is here a strong assumption, while dichotomy comes naturally without any supplementary hypotheses when the Leontief quantity model is solved by rows and generates outputs (in the primal) or when it is solved by columns and generates prices (in the dual). So, as is, the Ghosh model cannot be reinterpreted as a price model.

Have we to abandon the Ghosh model? Probably no, because of many reasons. First, technical coefficients are not much stable than allocation coefficients over time (Bon 1986, 2000), (Mesnard, 1997), so the Ghosh hypothesis does not seem less reliable than the Leontief hypothesis. Second, even if the Ghosh quantity model (supply push) seems implausible in the traditional input-output field, one must consider that it is more accepted outside, for example in business economics ⁷.

⁷ Many firms compute allocation coefficients to know what is the usual share of their products that is sold to each of its main customers (these main customers are often firms themselves, but also administrations). When these shares divert, they are alerted: if the share of one customer increases, this could be satisfactory; but perhaps it is the sign that the price is too low for him; and conversely if the share decreases. So, it could be asserted that the firm "pushes" its supply toward the customer. So, in business economics, one can say that this

V. Bibliographical references

BON, Ranko 1986. "Comparative Stability Analysis of Demand-Side and Supply-side Input-Output Models," *International Journal of Forecasting* 2, 231-235.

_____ 2000b. *Economic structure and maturity. Collected Papers in input-output modelling and applications*, Ashgate, Adelshot.

DIETZENBACHER, Erik. 1997. "In vindication of the Ghosh model: a reinterpretation as a price model", *Journal of Regional Science* 37, 629-651.

GALE, David. 1989. *The theory of linear economic models.*, Chicago Press, Chicago (originally published in 1960).

de MESNARD, Louis 1997. "A biproportional filter to compare technical and allocation coefficient variations", *Journal of Regional Science*, 37, 4, pp. 541-564.

MILLER, Ronald E., 1989. "Stability of Supply Coefficients and Consistency of Supply-Driven and Demand-Driven Input-Output Models: a Comment," *Environment and Planning A* 21, 1113-1120.

OOSTERHAVEN, Jan 1988. "On the Plausibility of the Supply-Driven Input-Output Model," *Journal of Regional Science* 28, 203-217.

_____ 1989. "The Supply-Driven Input-Output Model: A New Interpretation but Still Implausible," *Journal of Regional Science* 29, 459-465.

model could be fully functional as a quantity model, as much as the Leontief quantity model.

_____ 1996. "Leontief Versus Ghoshian Price And Quantity Models", *Southern Economic Journal* 62-3, pp. 750-759.

ROSE, Adam and Tim ALLISON 1989. "On the Plausibility of the Supply-Driven Input-Output Model: Empirical Evidence on Joint Stability," *Journal of Regional Science* 29, 451-458.