Qualitative methods of structural analysis:
Layer-based methods are informationally trivial

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ABSTRACT. Some methods of qualitative structural analysis, as MFA, are based on the analysis of layers (flow matrices generated at each iteration when the equilibrium of an input-output model is computed). MFA mixes the analysis of the pure structure of production (the technical coefficients) and of the final demand. I have demonstrated that all column-coefficient matrices (or row-coefficient matrices) computed from each layer are the same in MFA: the information brought by one layer is identical to those of another layer. For a given structure of production, the only element of variability over layers is caused by the flows that final demand generates.If the new definition of layers proposed by the creators of MFA is adopted, the method becomes similar to a quantitative method of structural analysis.

I. Introduction

Qualitative methods of structural analysis are useful to analyze the structures of production and exchange, for example in space, as the structure of exchange between countries, regions or cities but also for non-space systems as structure of production. In this last application, when structures of production are considered in an input-output framework, these methods are often called *Qualitative Input-Output Analysis* or *QIOA* [1].

Among qualitative methods of structural analysis, some are based on "layers", as Schnabl's MFA or *Minimal Flow Analysis* (Holub, Schnabl and Tappeiner, 1985), (Holub and Schnabl, 1985), (Schnabl, 1992, 1994, 1995), (Weber and Schnabl, 1998): for that, they can be considered as intermediate between qualitative and quantitative methods. What are layers? Layers are intermediary flow matrices generated when the equilibrium of an input-output model is computed. They are analyzed by the mean topologic tools (after being transformed into boolean matrices). This idea seems to be seductive. However, the information carried by layers is in question: if each layer carries a specific information then it is interesting to study layers, but if all layers carry the same information the interest of the method is challenged. After recalling what exactly are layer-based methods, this paper examines the problem of the information carried by layers.

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[1] Bon (1989) has presented the basic principles of QIOA, but its paper is also a good introduction to qualitative methods of structural analysis, in general.

II. Recall

The general framework is the Leontief model (Leontief, 1986): $\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{y}$, where $\mathbf{x}$ is the output vector, $\mathbf{y}$ is the final demand vector, and $\mathbf{A}$ is the matrix of fixed technical coefficients calculated by $\mathbf{A} = \mathbf{Z}\,\hat{\mathbf{x}}^{-1}$ where $\mathbf{Z}$ is the matrix of transaction flows given by the national accounting system. The solution of the model is $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{y}$, with $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$. Denote $\mathbf{H}$ any input-output matrix, that can be $\mathbf{Z}$ or $\mathbf{A}$ or $\mathbf{B}$ (the allocation coefficient matrix), and h_{ij} the terms of this matrix. In qualitative terms, a sector j is influencing sector i if $h_{ij} \geq \phi$, where ϕ is the value of the filter. In other terms, a boolean matrix $\mathbf{W}^{(1)}(\phi)$ is deduced from matrix $\mathbf{H}$: $w_{ij}^{(1)}(\phi) = 1 \Leftrightarrow h_{ij} \geq \phi$.

In ordinary boolean methods, the matrix $\mathbf{W}^{(2)}(\phi)$ is computed as a boolean product, $\mathbf{W}^{(2)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(1)}(\phi)$, $*$ denoting the boolean product. $w_{lj}^{(2)}(\phi) = 1$ if and only if there exists at least one sector i such that there is a direct path between i and l, i.e., $w_{li}^{(1)}(\phi) = 1$ and a direct path between i and j, i.e., $w_{ij}^{(1)}(\phi) = 1$. Again, $\mathbf{W}^{(3)}(\phi)$ is calculated from $\mathbf{W}^{(2)}(\phi)$ following the same rule, $\mathbf{W}^{(3)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(2)}(\phi)$, etc. This is generalized by:

(1) $$\mathbf{W}^{(k)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(k-1)}(\phi)$$

Then, a dependency matrix is computed: $\mathbf{D} = \sum_k \mathbf{W}^{(k)}$.

In MFA, one starts from $\mathbf{Z} = \mathbf{A}\,\langle \mathbf{x} \rangle$. As,

(2) $$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{y} = \left(\sum_{k=0}^{\infty} \mathbf{A}^k \right) \mathbf{y}$$

one can write:

(3) $$\mathbf{Z} = \mathbf{A}\,\langle \mathbf{y} \rangle + \mathbf{A}\,\langle \mathbf{A}\,\mathbf{y} \rangle + \mathbf{A}\,\langle \mathbf{A}^2\,\mathbf{y} \rangle + \ldots + \mathbf{A}\,\langle \mathbf{A}^k\,\mathbf{y} \rangle + \ldots$$

The *layers*, i.e., the matrices $\mathbf{Z}_0 = \mathbf{A}\,\langle\mathbf{y}\rangle$, $\mathbf{Z}_1 = \mathbf{A}\,\langle\mathbf{A}\,\mathbf{y}\rangle$, $\mathbf{Z}_2 = \mathbf{A}\,\langle\mathbf{A}^2\,\mathbf{y}\rangle$, ..., $\mathbf{Z}_k = \mathbf{A}\,\langle\mathbf{A}^k\,\mathbf{y}\rangle$, can be interpreted as the successive flow matrices generated by an initial demand vector $\mathbf{y}$ at steps 1, 2, 3, ..., *k*, respectively. The matrix $\mathbf{W}_k$, built from each layer $\mathbf{Z}_k$, indicates if there is a link between vertices in $\mathbf{Z}_k$. By analogy with formula (1), matrices $\mathbf{W}_k$ are combined by the recursive formula (Schnabl, 1994, p. 52, eq. 2):

$$\mathbf{W}^{(k)} = \mathbf{W}_{k-1} * \mathbf{W}^{(k-1)} \tag{4}$$

with $\mathbf{W}^{(0)} = \mathbf{I}$. The result of (2) is the following at the first step: $\mathbf{W}^{(1)} = \mathbf{W}_0 * \mathbf{W}^{(0)} = \mathbf{W}_0$, i.e. $w_{lj}^{(1)} = 1$ and only if there exists at least one sector *i* such that there is a direct path with a length equal to 1 between *j* and *i*, i.e. $(w_0)_{ij} = 1$. In the second step, $\mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \mathbf{W}_1 * \mathbf{W}_0$, i.e., $w_{lj}^{(2)} = 1$ if and only if:

- there exists at least one sector *i* such that there is a direct link between *j* and *i*, $(w_0)_{ij} = 1$ in the first layer $\mathbf{Z}_0$ (in the matrix of flows after an impulsion of final demand),
- and *i* is in direct relation with *l* i.e. $(w_1)_{li} = 1$ in the second layer $\mathbf{Z}_1$.

At the step *k*, one has: $\mathbf{W}^{(k)} = \mathbf{W}_{k-1} * \mathbf{W}_{k-2} * ... * \mathbf{W}_1$. A dependency matrix is also computed by boolean summation of the $\mathbf{W}^{(k)}$.

III. Informational problem with layer-based methods

<u>Remark</u>. The same output $\mathbf{x}_k$ can be generated either by $\mathbf{Z}_{k-1}\,\mathbf{s}$ ($\mathbf{s}$ is the sum vector) or by $\mathbf{A}^k\,\mathbf{y}$ because one have:

$$\mathbf{x}_k = \mathbf{A}^k\,\mathbf{y} = \mathbf{A}^k\,\langle\mathbf{y}\rangle\,\mathbf{s} = \mathbf{Z}_{k-1}\,\mathbf{s} = \mathbf{A}\,\langle\mathbf{x}_{k-1}\rangle\,\mathbf{s} = \mathbf{A}\,\mathbf{x}_{k-1}$$

so $\mathbf{A}^k$ corresponds to the same iteration than $\mathbf{Z}_{k-1}$, but as $\mathbf{A}^k\,\langle\mathbf{y}\rangle \neq \mathbf{Z}_{k-1}$, analyzing the power matrix $\mathbf{A}^k$ is not the same as analyzing the layer $\mathbf{Z}_{k-1}$. ■

When they are considered in relative terms, **all layers carry the same information**. To prove this, first one must define column coefficients. Column coefficients are not exactly technical coefficients, they are similar but do not count the value-added of each sector; $\hat{\mathbf{A}}_k$ is such that:

(5) $$\hat{a}_{ij}^{k} = \frac{(z_{ij})_k}{(z_{\bullet j})_k}$$

where $(z_{ij})_k$ is a term of $\mathbf{Z}_k$ and $(z_{\bullet j})_k = \sum_i (z_{ij})_k$. That is in matrix terms:

(6) $$\hat{\mathbf{A}}_k = \mathbf{Z}_k \langle \mathbf{s}' \mathbf{Z}_k \rangle^{-1}$$

<u>Property 1</u>. All matrices $\hat{\mathbf{A}}_k$ of column coefficients, deduced from layers $\mathbf{Z}_k$ at each step k, are identical: $\hat{\mathbf{A}}_k = \hat{\mathbf{A}}_0$, for all k. ■

So, all layers carry the same information because all boolean matrices $\hat{\mathbf{W}}_k$ found from matrices $\hat{\mathbf{A}}_k$ are the same. The information carried out by matrices $\mathbf{Z}_k$ is **trivial** in terms of column coefficients.

<u>Proof</u>.

$\hat{\mathbf{A}}_k = \mathbf{Z}_k \langle \mathbf{s}' \mathbf{Z}_k \rangle^{-1} = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \langle \mathbf{s}' \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \rangle^{-1}$

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle [\langle \mathbf{s}' \mathbf{A} \rangle \langle \mathbf{A}^k \mathbf{y} \rangle]^{-1}$ (because if $\mathbf{b}$ and $\mathbf{c}$ are two vectors, $\langle \mathbf{b}' \langle \mathbf{c} \rangle \rangle = \langle \mathbf{b} \rangle \langle \mathbf{c} \rangle$)

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \langle \mathbf{A}^k \mathbf{y} \rangle^{-1} \langle \mathbf{s}' \mathbf{A} \rangle^{-1}$

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{s}' \mathbf{A} \rangle^{-1}$, for all k, so $\hat{\mathbf{A}}_k$ is a constant. ■

MFA is not a method of pure structural analysis because it analyzes flows that depend on the production structure and of final demand. At this step, one argument appears: MFA is concerned with the structure of flows including the structure of final demand, while the above demonstration concerns the structure of production only. In other words, to save MFA, one could say: the above developements demonstrate only that the pure structure of production is stable from one layer to the other, but, as MFA studies flows by mixing the structure of production and final demand, the stability over layers of the pure production structure is not a problem. And MFA could continue to work fine. This argument is not acceptable on a Cartesian view point.

- First, MFA cannot study any other thing than the pure production structure and the effect of final demand on flows that pass by this production structure.

- Second, in the Leontief model, technical coefficients are assumed to be stable while only final demand is variable: the Hawkins-Simon theorem shows that changes in the structure of production are independent to changes in final demand (Hawkins and Simon, 1949).

- Third, MFA studies flows in layers, that depend on the production structure and on final demand by the formula $\mathbf{Z}_k = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle$.

- Fourth, I have demonstrated that the pure production structure -- measured by the column coefficients -- is stable in MFA.

To summurize, final demand, and flows in layers change, but the pure production structure remains stable from one layer to the other. Then, logically, for a given structure of production **A**, the only element of variability over layers is caused by final demand and the flows that it generates at each layers.

<u>Example</u>.

Consider $\mathbf{A} = \begin{bmatrix} 0.6 & 0.15 & 0.05 \\ 0.2 & 0.5 & 0.25 \\ 0.1 & 0.3 & 0.55 \end{bmatrix}$

1) $\mathbf{y} = \begin{pmatrix} 500 \\ 1000 \\ 2000 \end{pmatrix}$, $\Phi = 300$, $\phi = 0.3$

- $\mathbf{x}_0 = \begin{pmatrix} 500 \\ 1000 \\ 2000 \end{pmatrix}$

- $\mathbf{Z}_0 = \begin{bmatrix} 300 & 150 & 100 \\ 100 & 500 & 500 \\ 50 & 300 & 1100 \end{bmatrix}$, $\mathbf{x}_1 = \begin{pmatrix} 550 \\ 1100 \\ 1450 \end{pmatrix}$

  $\mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(1)} = \mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

  but,

  $\hat{\mathbf{A}}_0 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix}$

  and,

  $\mathbf{A}^1 = \begin{bmatrix} 0.6 & 0.15 & 0.05 \\ 0.2 & 0.5 & 0.25 \\ 0.1 & 0.3 & 0.55 \end{bmatrix}$, $\widetilde{\mathbf{W}}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

- $\mathbf{Z}_1 = \begin{bmatrix} 330 & 165 & 72,5 \\ 110 & 550 & 362,5 \\ 55 & 330 & 797,5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 567.5 \\ 1022.5 \\ 1182.5 \end{pmatrix}$

  $\mathbf{W}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\hat{\mathbf{A}}_1 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix} = \hat{\mathbf{A}}_0$$

$$\mathbf{A}^2 = \begin{bmatrix} 0.395 & 0.180 & 0.095 \\ 0.245 & 0.355 & 0.273 \\ 0.175 & 0.330 & 0.383 \end{bmatrix}, \ \tilde{\mathbf{W}}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

- $$\mathbf{Z}_2 = \begin{bmatrix} 340.500 & 153.375 & 59.125 \\ 113.500 & 511.250 & 295.625 \\ 56.750 & 306.750 & 650.375 \end{bmatrix}, \ \mathbf{x}_3 = \begin{pmatrix} 553.000 \\ 920.375 \\ 1013.875 \end{pmatrix}$$

$$\mathbf{W}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(3)} = \mathbf{W}_2 * \mathbf{W}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\hat{\mathbf{A}}_2 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix} = \hat{\mathbf{A}}_1 = \hat{\mathbf{A}}_0$$

$$\mathbf{A}^3 = \begin{bmatrix} 0.283 & 0.178 & 0.117 \\ 0.245 & 0.296 & 0.251 \\ 0.209 & 0.306 & 0.302 \end{bmatrix}, \tilde{\mathbf{W}}^{(3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

- ...

Remark. Since the first column in $\mathbf{Z}_1$ is greater than the first column in $\mathbf{Z}_0$, the influence of sector 1 seems to be increasing from step 2 to step 3, but it is only an artefact of MFA because in terms of column coefficients, nothing at all has changed from one step to the other:

$$\hat{\mathbf{A}}_k = \begin{bmatrix} 0.171 & 0.266 & 0.462 \\ 0.398 & 0.200 & 0.308 \\ 0.432 & 0.533 & 0.231 \end{bmatrix} \text{ for all } k. \blacksquare$$

2) Now, consider exactly the same example but with another vector of demand:

$$\mathbf{y} = \begin{pmatrix} 0 \\ 1000 \\ 1700 \end{pmatrix}, \Phi = 300, \phi = 0.3$$

- $$\mathbf{x}_0 = \begin{pmatrix} 1000 \\ 1000 \\ 1000 \end{pmatrix}$$

- $$\mathbf{Z}_0 = \begin{bmatrix} 600 & 150 & 50 \\ 200 & 500 & 250 \\ 100 & 300 & 550 \end{bmatrix}, \mathbf{x}_1 = \begin{pmatrix} 800 \\ 950 \\ 950 \end{pmatrix}$$

$$\mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(1)} = \mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

but the pure production structure is the same as above, that is:

$$\hat{\mathbf{A}}_0 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix}, \mathbf{A}^1 = \begin{bmatrix} 0.6 & 0.15 & 0.05 \\ 0.2 & 0.5 & 0.25 \\ 0.1 & 0.3 & 0.55 \end{bmatrix} \text{ and } \tilde{\mathbf{W}}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

- $$\mathbf{Z}_1 = \begin{bmatrix} 480 & 142.5 & 47.5 \\ 160 & 475 & 237.5 \\ 80 & 285 & 522.5 \end{bmatrix}, \mathbf{x}_2 = \begin{pmatrix} 670 \\ 872.5 \\ 887.5 \end{pmatrix}$$

$$\mathbf{W}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Rightarrow \mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and the following are unchanged by respect to the previous case:

$$\hat{\mathbf{A}}_1 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix} = \hat{\mathbf{A}}_0$$

$$\mathbf{A}^2 = \begin{bmatrix} 0.395 & 0.180 & 0.095 \\ 0.245 & 0.355 & 0.273 \\ 0.175 & 0.330 & 0.383 \end{bmatrix}, \tilde{\mathbf{W}}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

- $\mathbf{Z}_2 = \begin{bmatrix} 402 & 130.875 & 44.375 \\ 134 & 436.250 & 221.875 \\ 67 & 261.750 & 488.125 \end{bmatrix}, \mathbf{x}_3 = \begin{pmatrix} 577.250 \\ 792.125 \\ 816.875 \end{pmatrix}$

  $\mathbf{W}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Rightarrow \mathbf{W}^{(3)} = \mathbf{W}_2 * \mathbf{W}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

  again the following are unchanged:

  $\hat{\mathbf{A}}_2 = \begin{bmatrix} 0.666 & 0.158 & 0.588 \\ 0.222 & 0.526 & 0.294 \\ 0.111 & 0.316 & 0.647 \end{bmatrix} = \hat{\mathbf{A}}_1 = \hat{\mathbf{A}}_0$

  $\mathbf{A}^3 = \begin{bmatrix} 0.283 & 0.178 & 0.117 \\ 0.245 & 0.296 & 0.251 \\ 0.209 & 0.306 & 0.302 \end{bmatrix}, \tilde{\mathbf{W}}^{(3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

- ...

As it can be seen, the second computation with another final demand vector, but the same matrix **A**, gives different results than the first, the layers are not the same, the boolean matrices of MFA are not the same, while the pure structure is unchanged. It seems that only the influence of final demand is caught by MFA. What happens with an unchanged final demand vector but another matrix **A**?

3) Consider the same final demand vector than in case 1) but another matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 0.6 & 0.15 & 0.05 \\ 0.2 & 0.5 & 0.25 \\ 0.1 & 0 & 0.55 \end{bmatrix}, \mathbf{y} = \begin{pmatrix} 500 \\ 1000 \\ 2000 \end{pmatrix}, \Phi = 300, \phi = 0.3$$

- $\mathbf{x}_0 = \begin{pmatrix} 500 \\ 1000 \\ 2000 \end{pmatrix}$

- $$\mathbf{Z}_0 = \begin{bmatrix} 300 & 150 & 100 \\ 100 & 500 & 500 \\ 50 & 0 & 1100 \end{bmatrix}, \mathbf{x}_1 = \begin{pmatrix} 550 \\ 1000 \\ 1450 \end{pmatrix}$$

  $$\mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(1)} = \mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

  $$\hat{\mathbf{A}}_0 = \begin{bmatrix} 0.666 & 0.231 & 0.059 \\ 0.222 & 0.769 & 0.294 \\ 0.111 & 0 & 0.647 \end{bmatrix}$$

- $$\mathbf{Z}_1 = \begin{bmatrix} 330 & 165 & 57.5 \\ 110 & 550 & 287.5 \\ 55 & 0 & 632.5 \end{bmatrix}, \mathbf{x}_2 = \begin{pmatrix} 552.5 \\ 947.5 \\ 687.5 \end{pmatrix}$$

  $$\mathbf{W}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- $$\mathbf{Z}_2 = \begin{bmatrix} 331.500 & 142.125 & 34.375 \\ 110.500 & 473.750 & 171.875 \\ 55.250 & 0.000 & 378.125 \end{bmatrix}, \mathbf{x}_3 = \begin{pmatrix} 508.000 \\ 756.125 \\ 433.175 \end{pmatrix}$$

  $$\mathbf{W}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(3)} = \mathbf{W}_2 * \mathbf{W}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- Etc. As it can be seen, with another matrix **A** the results are not the same than in the first case, but, again, from one layer to the other, the column coefficients remain unchanged.

Let's summarize. Another matrix **A**, another result; another final demand vector, another result; but for a given production structure and a given final demand vector, the k^{th} layer brings no new information compared to the $(k+1)^{th}$ layer. ■

IV. Epilogue

In their recent paper, Weber and Schnabl (1998) claim to apply MFA to the analysis of the energy sector and its role in the German economy. Intermediary flows of energy are studied by premultiplying equations (2) by $\mathbf{E}^{x}$ and replacing $\mathbf{y}$ by $\langle \mathbf{y} \rangle$:

$$\mathbf{X}_{\mathrm{E}} = \mathbf{E}^{x}\,(\mathbf{I} - \mathbf{A})^{-1}\,\langle \mathbf{y} \rangle$$

$$\Leftrightarrow\ \mathbf{X}_{\mathrm{E}} = \mathbf{E}^{x}\,\langle \mathbf{y} \rangle + \mathbf{E}^{x}\,\mathbf{A}\,\langle \mathbf{y} \rangle + \mathbf{E}^{x}\,\mathbf{A}^{2}\,\langle \mathbf{y} \rangle + \ldots + \mathbf{E}^{x}\,\mathbf{A}^{k}\,\langle \mathbf{y} \rangle + \ldots$$

where "the columns of the subsystem matrix $\mathbf{X}_{\mathrm{E}}$ indicate the composition of the cumulated energy requirements of product group j by sector i of (final) energy consumption", $\mathbf{E}^{x}$ is the energy input coefficient matrix: $\mathbf{E}^{x} = \mathbf{S}^{x}\,\langle \mathbf{x} \rangle^{-1}$, and $\mathbf{S}^{x}$ is the matrix of energy input (Weber and Schnabl, 1998, pp. 3398-339 and 350). Apart this, MFA is claimed to be applied in a similar way.

However, if this has the taste of MFA, this is not MFA! The terms $\mathbf{E}^{x}\,\mathbf{A}^{k}\,\langle \mathbf{y} \rangle$ are not layers in the sense of MFA as defined in (Schnabl, 1992, 1994, 1995). As said, a layer $\mathbf{Z}_{k} = \mathbf{A}\,\langle \mathbf{A}^{k}\,\mathbf{y} \rangle$ is the flow matrix that appears at the iteration $k+1$, or if k is seen as a time index, it is the matrix of intermediary flows exchanged at the date k: $\mathbf{A}^{k}\,\mathbf{y}$ is the output generated at the date k, and the product $\mathbf{A}\,\langle \mathbf{A}^{k}\,\mathbf{y} \rangle$ is the quantity of intermediate commodities demanded to each sector in order to produce this output.

A term of $\mathbf{A}^{k}\,\langle \mathbf{y} \rangle$ is not a flow generated at date k. A term $a_{ij}^{(k)}$ of matrix $\mathbf{A}^{k}$ is the result of the summation of all the successive coefficients found on paths of a length equal to k. For example, the term $a_{ij}^{(2)}$ of $\mathbf{A}^{2}$ is equal to $\sum_{l=1}^{n} a_{il}\,a_{lj}$: n paths of length 2 between j and i are of type $j \rightarrow l \rightarrow i$. The term $a_{ij}^{(2)}\,y_{j}$ of the matrix $\mathbf{A}^{k}\,\langle \mathbf{y} \rangle$ is the intermediate demand addressed to sector i in commodity i to respond to the final demand of commodity j when an indirect relation of length k is considered. It is not the flow exchanged at step k!

Weber and Schnabl have tried to transform MFA to respond to the objections developed in this paper [2]. Anyway, transformed in this new way, the new layers become very close to a simple power matrix as $\mathbf{A}^k$: this concept is analyzed by quantitative methods when the strongest paths of all possible lengths are searched in the structure; see, for example, (Lantner, 1974).

Remark. Note that $\mathbf{A}^k \neq \mathbf{A}^l$ for all $k \neq l$ generally: the series of power matrices is not informationally trivial. Consider $\widetilde{\mathbf{W}}^{(k)}$ defined as the boolean matrix found from $\mathbf{A}^k$ by the relation:

$$a_{ij}^{(k)} \geq \phi \Leftrightarrow \widetilde{w}_{ij}^{(k)} = 1$$

where $a_{ij}^{(k)}$ is a term of the power matrix $\mathbf{A}^k$. Generally, $\mathbf{W}^{(k)} \neq \widetilde{\mathbf{W}}^{(k)}$.

When one considers the Leontief model, only the matrix $\mathbf{A}$ plays a role from one iteration to the following. One can write: $\mathbf{Z}_{k+1} = \mathbf{A} \langle \mathbf{Z}_k \, \mathbf{s} \rangle$, because $\mathbf{x}_{k+1} = \mathbf{Z}_k \, \mathbf{s} = \mathbf{A}^{k+1} \, \mathbf{y}$, and so the only change between $\mathbf{Z}_{k+1}$ and $\mathbf{Z}_k$ is brought by $\mathbf{A}$, but it is also the case for $\mathbf{A}^{k+1}$ and $\mathbf{A}^k$, $\mathbf{A}^{k+1} = \mathbf{A} \, \mathbf{A}^k$ [3]. So, the lack of information between any two iterations of layers, in relative terms, does not rely on this fact. ■

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[2] It's not a temporal paradox: they were exposed to them at the *Twelfth International Conference on Input-Output Techniques*, New-York City

[3] Similar reasoning comes from matrix **B**.

V. Conclusion

Layer-based methods as MFA belong to the general category of qualitative methods of structural analysis -- or qualitative input-output analysis -- even if they are a hybrid between qualitative and quantitative methods. MFA mixes the analysis of the pure structure of production (the technical coefficients) and of the final demand. I have demonstrated that all column-coefficient matrices (or row-coefficient matrices) computed from each layer are the same in MFA: it is illusory to believe that the information brought by one layer is different to those of another layer and MFA's approach is not informationally discerning. In other terms, for a given structure of production, the only element of variability over layers is caused by the flows that final demand generates at each layers. Finally, MFA is not a method of strcutural analysis. If the new definition of layers proposed by the creators of MFA is adopted, the method becomes similar to a quantitative method of structural analysis.

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