Interpretation of the RAS method: absorption and fabrication effects are incorrect

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ABSTRACT. The r and s vectors of the RAS method of updating matrices are presented often as corresponding to an absorption effect and a fabrication effect. Here, it is proved that these vectors are unidentified, so their interpretation in terms of fabrication and absorption effect is incorrect.

I. Introduction

Stone and Brown (1962), followed by others, as Paelinck and Waelbroeck (1963), give an interpretation of the left and right multipliers, that are found when a RAS method is used to update a matrix, in terms of *absorption effect* and *fabrication effect* [1]. However, I will show that a technical difficulty prevents them to be relevant of such interpretation.

II. Fabrication and absorption effects

Consider two matrices of technical coefficients, for two different dates t_1 and t_2 $\mathbf{A}^{t_1}$ and $\mathbf{A}^{t_2}$. To remove the price effect from year t_1 to year t_2, one computes the matrix denoted $\mathbf{A}^{t_1/t_2}$, that is the matrix $\mathbf{A}^{t_1}$ at the prices of t_2: $\mathbf{A}^{t_1/t_2} = \hat{\mathbf{p}}\ \mathbf{A}^{t_1}\ \hat{\mathbf{p}}^{-1}$, where $\mathbf{p}$ is the price vector of the date t_2 with the date t_1 as price base. Then the projection of $\mathbf{A}^{t_1/t_2}$, the matrix of the year t_1 at the prices of the year t_2, on the year t_2 is the transformation of $\mathbf{A}^{t_1/t_2}$ such a this matrix get the same margins than $\mathbf{A}^{t_2}$; this is given by: $RAS(\mathbf{A}^{t_1/t_2},\ \mathbf{A}^{t_2}) = \mathbf{R}\ \mathbf{A}^{t_1/t_2}\ \mathbf{S}$, where $RAS(\ \ )$ is the operator well known as the "RAS method". In this case the diagonal matrix $\mathbf{R}$ is interpreted by the above authors as an absorption (or also substitution) effect: factors r_i affect a row of $\mathbf{A}^{t_1/t_2}$ and reflect the modification of the outlet of a commodity; the diagonal matrix $\mathbf{S}$ as a fabrication (or also transformation) effect: factors s_j affect a column of $\mathbf{A}^{t_1/t_2}$ and reflect the modification in the so-called "degree of fabrication".

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[1] This is exposed also by Snower (1990).

III. Non identification

Even if this interpretation is commonly accepted, as the terms r_i and s_j are **not identified** [2], they **cannot** be interpreted for themselves as a absorption effect and a fabrication effect. If all the coefficients r_i are multiplied by λ then all the coefficients s_j are multiplied by $\frac{1}{\lambda}$ and conversely: multiplying fabrication effects by λ will divide absorption effects by λ, and conversely. This removes all signification to **C** and **D** in terms of fabrication or absorption effects. It is very simple to prove it. By commodity, the demonstration will be based on the algorithm presented by Bachem and Korte (1979):

(1) $$r_i = \frac{a_{i\bullet}^{t_2}}{\sum_{j=1}^{m} s_j \, a_{ij}^{t_1/t_2}} \text{ for all } i, \text{ and } s_j = \frac{a_{\bullet j}^{t_2}}{\sum_{i=1}^{n} r_i \, a_{ij}^{t_1/t_2}} \text{ for all } j$$

Bachem and Korte (1979) have proved that their algorithm is equivalent to the RAS algorithm. This type of algorithm has an iterative numerical solution (k is an index of iteration), for example:

(2) $$r_i(k+1) = \frac{a_{i\bullet}^{t_2}}{\sum_{j=1}^{m} s_j(k) \, a_{ij}^{t_1/t_2}} \text{ for all } i, \text{ and } s_j(k+1) = \frac{a_{\bullet j}^{t_2}}{\sum_{i=1}^{n} r_i(k+1) \, a_{ij}^{t_1/t_2}} \text{ for all } j$$

After an initialization, for example by $s_j(0) = 1$, for all j, this leads to an equilibrium [3]:

(3) $$r_i^* = \frac{a_{i\bullet}^{t_2}}{\sum_{j=1}^{m} s_j^* \, a_{ij}^{t_1/t_2}} \text{ for all } i, \text{ and } s_j^* = \frac{a_{\bullet j}^{t_2}}{\sum_{i=1}^{n} r_i^* \, a_{ij}^{t_1/t_2}} \text{ for all } j$$

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[2] I use this term by reference with econometrics, even if there nothing stochastic here.

[3] It is not the aim of this paper to discuss about the properties of this equilibrium, and more generally, of biproportional algorithms; for further informations, see (Bacharach, 1970),

Now, assume that all $s_j(0)$ are replaced by $\tilde{s}_j(0) = \lambda\, s_j(0)$. Then, the $r_i(1)$ will become replaced by $\tilde{r}_i(1)$:

(4) $$\tilde{r}_i(1) = \frac{a_{i\bullet}^{t_2}}{\sum_{j=1}^{m} \tilde{s}_j(0)\, a_{ij}^{t_1/t_2}} = \frac{a_{i\bullet}^{t_2}}{\sum_{j=1}^{m} \lambda\, s_j(0)\, a_{ij}^{t_1/t_2}} = \frac{1}{\lambda}\, r_i(1), \text{ for all } i$$

and

(5) $$\tilde{s}_j(1) = \frac{a_{\bullet j}^{t_2}}{\sum_{i=1}^{n} \tilde{r}_i(1)\, a_{ij}^{t_1/t_2}} = \frac{a_{\bullet j}^{t_2}}{\sum_{i=1}^{n} \frac{1}{\lambda}\, r_i(1)\, a_{ij}^{t_1/t_2}} = \lambda\, s_j(1), \text{ for all } j$$

By recurrence, r_i^* is replaced by $\tilde{r}_i^* = \frac{1}{\lambda}\, r_i^*$ and s_j^* by $\tilde{s}_j^* = \lambda\, s_j^*$. Similar results are found by reverting the role of the *r* and *s*.

IV. Conclusion

Factors **R** and **S** have no signification at all by themselves, only the products $r_i\, s_j$, for all $(i,\ j)$, are identified. Even if the RAS method and its generalization in terms of biproportional methods can be credited of some technical qualities [4] and even if RAS -- and biproportion -- can be seen as a simple generalization of "naïve" approaches [5], its economic interpretation remains difficult.

(Bachem and Korte, 1979), (Balinski and Demange, 1989), (Mesnard, 1994).

[4] As the non negativity of the projected matrix, or as the characteristic of the projected matrix to be the closer to the initial matrix in terms of information theory or entropy.

[5] As Leontief technical coefficients or Ghosh allocation coefficients when they are used to measure change in a linear system of production (Mesnard, 1997).

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