On boolean topological methods of structural analysis

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<u>ABSTRACT</u>. The properties of Boolean methods of structural analysis are used to analyze the intern structure of linear or non linear models. Here they are studied on the particular example of qualitative methods of input-output analysis. First, it is shown that these methods generate informational problems like biases when working in money terms instead of percentages, losses of information, increasing of computation time, and so on. Second, considering three ways to do structural analysis, analysis from the inverse matrix, from the direct matrix and from layers (intermediate flow matrices), these methods induce topological problems; the adjacency of the adjacency cannot be defined from the inverse matrix; the binary relation of influence may be non transitive from the direct matrix, with poorer results than with quantitative methods; for layers - based methods, the information carried out by layers is trivial.

I. Introduction

Structural analysis is useful to understand the functioning of a multivariable multiequation model [1]. One way to perform it is Boolean Topological Structural Analysis or Qualitative Structural Analysis [2]. For example to detect causalities between variables, one can replace the value of the coefficient that links one variable *i* with another variable *j*, i.e. the value of the oriented arc linking these two variables, by a boolean coefficient, 1 or 0, depending of the fact that the value exceeds or not a given value called a filter. This method seems interesting to have a global view of the structure because it allows to exhibit the "skeleton" of the structure. However, there are some disadvantages to proceed like that.

In this paper, one will present the problems caused by boolean methods of structural analysis, confining on the particular and simple case of linear models for more clarity, specially the input-output (I-O) model, but the results will be transposable to other types of computable models. So the model that one will consider is the most simple in I-O analysis (Leontief, 1986): $\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{y}$, where **x** is output vector, **y** is the final demand vector and **A** is the matrix of fixed technical coefficients calculated by $\mathbf{A} = \mathbf{Z}\,\hat{\mathbf{x}}^{-1}$ where **Z** is the matrix of flow given by the national accounting system, and the solution of the model is $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{y}$, with $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$. This model will be largely sufficient to explain the problems, the most part of them being topological.

1 One a the first references seems to be (Solow, 1952).

2 See (Rossier, 1980), (Royer and Ritschard, 1984), (Ancot, 1985), (Gilli, Ritschard and Royer, 1985), (Dagum, 1985), (Maybee, 1985).

II. Remind: qualitative methods versus quantitative methods

There are two great types of structural analysis methods which can be divided each into two sub-types.

A. Working on one matrix only

One work on the direct matrix. It is possible to work in absolute terms, i.e. in monetary units, in dollars for example, on the flow matrix **Z**. Alternately, it is possible to work on relative terms, either on the technical coefficient matrix **A** or on the allocation coefficient matrix **B**. Technical coefficients are absolute elasticities: a_{ij} describes the variation $\frac{\Delta x_i}{\Delta x_j}$ of the output of sector i caused by a variation of 1 dollar of the output of sector j . The analysis can be also conducted on the matrix of allocation coefficients, $b_{ij} = \frac{z_{ij}}{x_i}$ or $\mathbf{B} = \langle \mathbf{x} \rangle^{-1}\ \mathbf{Z}$, in order to capture in percentage the effect of each sector over each sector: they are relative elasticities describing the variation $\frac{\Delta x_i / xi}{\Delta x_j / x_j}$ of the sector i output in percentage caused by the variation of the sector j output; this approach eliminates completely the size effect (for the case of demand-driven models). Also, instead of **A** (or **B**), it is possible to work on normalized (or Markovian) matrices, of which column or row margins are unitary, to eliminate the effect of added value or the effect of final demand.

Also, one can work on the inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$ or even $(\mathbf{I} - \mathbf{B})^{-1}$.

Qualitative methods build a binary relation of influence: the adjacency of a sector is defined as the sectors that are influenced by, or that are influencing, a sector. Denote **H** an input-output matrix (**Z**, or **A**, or **B**) and h_{ij} the terms of this matrix. In qualitative terms, a sector j is influencing a sector i if $h_{ij} \geq \phi$, where ϕ is the value of the filter: $j\ R\ i \Leftrightarrow h_{ij} \geq \phi$. In other terms, a boolean matrix $\mathbf{W}^{(1)}(\phi)$ is deduced from the matrix **H**: $w_{ij}^{(1)}(\phi) = 1 \Leftrightarrow h_{ij} \geq \phi$. When a sector *i* is linked to another sector *j* by a "1" in $\mathbf{W}^{(1)}(\phi)$, it is declared to be in the adjacency of this sector *j* [3]. In some works, $\phi = 0$, for example, (Bon, 1989), in other works, $\phi > 0$. This does not characterize qualitative methods because this process remains descriptive and, as said in (Mesnard, 1995), it could be so interesting to use a visual representation of the structure. Searching the adjacency of the adjacency is characterizing qualitative methods: one search the vertices *l* that are influenced by a vertex *i* what is influenced itself by a vertex *j* or the sectors *l* that are influenced by *j* through *i*, or the sectors *l* that are influenced by *i*, *i* being influenced by *j*.

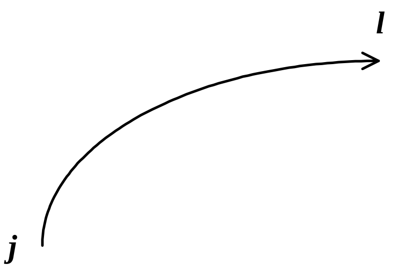
To do this, the matrix $\mathbf{W}^{(2)}(\phi)$ is computed as a boolean product, $\mathbf{W}^{(2)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(1)}(\phi)$, where $*$ is the following operation: $w_{lj}^{(2)}(\phi) = 1$ if and only if there exists at least one sector *i* such that there is a direct path between *i* and *l*, i.e. $w_{li}^{(1)}(\phi) = 1$ and a direct path between *i* and *j*, i.e. $w_{ij}^{(1)}(\phi) = 1$. Again, $\mathbf{W}^{(3)}(\phi)$ is calculated from $\mathbf{W}^{(2)}(\phi)$ following the same rule, $\mathbf{W}^{(3)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(2)}(\phi)$, etc., up to

(1) $$\mathbf{W}^{(k)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(k-1)}(\phi)$$

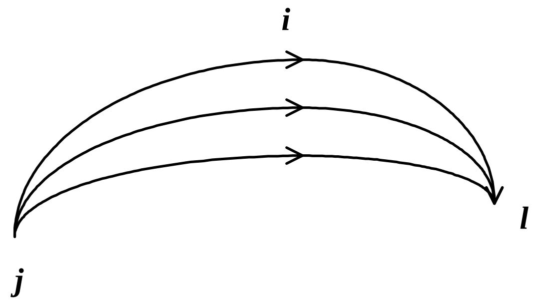
where one have $w_{lj}^{(k)}(\phi) = 1$ if and only if there exists at least one sector *i* such that there is a direct link between *j* and *i*, i.e. $w_{ij}^{(1)}(\phi) = 1$ and *l* is in the adjacency of the adjacency of the adjacency ... of the adjacency ($k-1$ times) of *i*, i.e. $w_{li}^{(k-1)}(\phi) = 1$. This recursive formula can

[3] In I-O analysis, boolean methods of structural analysis are called improperly *qualitative methods of input-output analysis*, that is *QIOA*. They are found, with some variations that sometimes may lessen critics strongly, in most works like Mougeot, Duru, Auray (1977), Kleine and Meyer (1981), Auray and Duru (1984), Auray, Duru, Mougeot and Seffert (1985), Holub and Schnabl (1985), Bon (1989), Schnabel (1992, 1994, 1995), Leicht, Kwong Wong and Wyatt (1993), Aroche-Reyes (1996), Cassetti (1995), Gillen and Guccione (1996), etc.

be solved by writing $\mathbf{W}^{(k)}(\phi) = \mathbf{W}^{(1)}(\phi) * \mathbf{W}^{(1)}(\phi) * ... * \mathbf{W}^{(1)}(\phi)$, k times. It means that traditional boolean methods use topology [4].



Graph for $\mathbf{W}^{(1)}$



Graph for $\mathbf{W}^{(2)} = \mathbf{W}^{(1)} * \mathbf{W}^{(1)}$

(arrows are oriented in the direction of influence)

Then, a dependency matrix is computed: $\mathbf{D} = \sum_k \mathbf{W}^{(k)}$.

B. Working on a sequence of matrices

There are two great manners for working on sequence of matrices.

1. Working on the sequence of *layers* $\{\mathbf{Z}_0, \mathbf{Z}_1, \mathbf{Z}_2, ..., \mathbf{Z}_k, ...\}$

Starting from $\mathbf{Z} = \mathbf{A}\,\langle \mathbf{x} \rangle$, where $\mathbf{Z}$ is the flow matrix, as $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{y} = \left(\sum_{k=0}^{\infty} \mathbf{A}^k \right) \mathbf{y}$, one can write:

$$\mathbf{Z} = \mathbf{A}\,\langle \mathbf{y} \rangle + \mathbf{A}\,\langle \mathbf{A}\,\mathbf{y} \rangle + \mathbf{A}\,\langle \mathbf{A}^2\,\mathbf{y} \rangle + ... + \mathbf{A}\,\langle \mathbf{A}^k\,\mathbf{y} \rangle + ...$$

What Schnabl (1992, 1994, 1995) calls *layers*, i.e. the matrices $\mathbf{Z}_0 = \mathbf{A}\,\langle \mathbf{y} \rangle$, $\mathbf{Z}_1 = \mathbf{A}\,\langle \mathbf{A}\,\mathbf{y} \rangle$, $\mathbf{Z}_2 = \mathbf{A}\,\langle \mathbf{A}^2\,\mathbf{y} \rangle$, ..., $\mathbf{Z}_k = \mathbf{A}\,\langle \mathbf{A}^k\,\mathbf{y} \rangle$, can be interpreted as the successive flow matrices generated by an initial demand vector $\mathbf{y}$ at steps 1, 2, 3, ..., k, respectively: so, layers based methods work in absolute terms.

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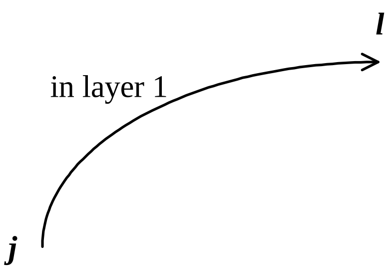
4 Bon, which is a supporter of qualitative methods uses the term (Bon, 1989, p. 223).

At this point, Schnabl's method (1992, 1994, 1995), MFA or *Minimal Flow Analysis*, consists into constructing a boolean matrix $\mathbf{W}_k$ from each layer $\mathbf{Z}_k$ to indicate if there is a link between vertices in $\mathbf{Z}_k$. Then matrices $\mathbf{W}_k$ are combined by the recursive formula:

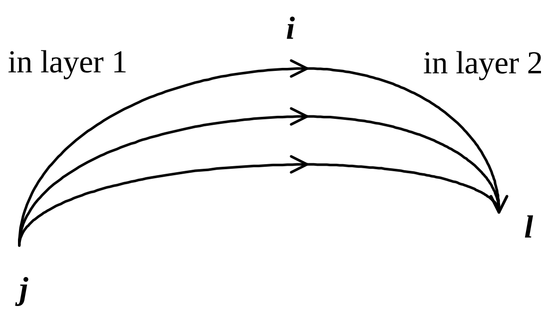
$$\mathbf{W}^{(k)} = \mathbf{W}_{k-1}\ \mathbf{W}^{(k-1)} \qquad (2)$$

with $\mathbf{W}^{(0)} = \mathbf{I}$. This formula is set by analogy with the recursive formula (1) used by traditional qualitative input-output analysis [5], but the first term, $\mathbf{W}^{(1)}$, in the right side of equation (1) is fixed and independent to *k*, when the corresponding term, $\mathbf{W}_{k-1}$, in equation (2) is variable and dependent to *k*.

The result of formula (2) is the following at the first step: $\mathbf{W}^{(1)} = \mathbf{W}_0 * \mathbf{W}^{(0)} = \mathbf{W}_0$, i.e. $w_{lj}^{(1)} = 1$ and only if there exists at least one sector *i* such that there is a direct path with a length equal to 1 between *j* and *i*, i.e. $(w_0)_{ij} = 1$. At the second step, $\mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \mathbf{W}_1 * \mathbf{W}_0$, i.e. $w_{lj}^{(2)} = 1$ if and only if there exists at least one sector *i* such that there is a direct link between *j* and *i* $(w_0)_{ij} = 1$ in the first layer $\mathbf{Z}_0$, i.e. in the matrix of flows after an impulsion of final demand, and *i* is in direct relation with *l* i.e. $(w_1)_{li} = 1$ in the second layer $\mathbf{Z}_1$.



Graph for $\mathbf{W}^{(1)} = \mathbf{W}_1 * \mathbf{W}^{(0)} = \mathbf{W}_1$



Graph for $\mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)}$

(arrows are oriented in the direction of influence)

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[5] Cf. (Schnabl, 1994, p. 52, eq. 2).

In other words, $\mathbf{W}^{(k)} = \mathbf{W}_{k-1} * \mathbf{W}_{k-2} * ... * \mathbf{W}_1$. For Schnabl, "$\mathbf{W}^{(k)}$ is reflecting the connections of sectors of the length of k steps".

2. Working on the sequence of power matrices, $\{\mathbf{I},\ \mathbf{A},\ \mathbf{A}^2,\ ...,\ \mathbf{A}^k, ...\}$

Quantitative methods work on the sequence power of matrices. By quantitative methods, one must understand the methods that are staying on valuated oriented graphs and that analyze the structure by calculating the eigenvalues of $\mathbf{A}$, the determinant of $\mathbf{I} - \mathbf{A}$ [6], the multipliers, etc. for the global level. At a more disaggregated level, the sequence of power matrices $\{\mathbf{I},\ \mathbf{A},\ \mathbf{A}^2,\ ...,\ \mathbf{A}^k, ...\}$ will be analyzed: paths can be ordered from those with the stronger influence to those with the lower influence (remembering that the stronger paths are often the shorter), one can search the stronger paths of a given length k (measured in number of arcs, by examining the matrix $\mathbf{A}^k$) [7], or the influence flowing by paths of a length equal or lower than k by examining $\sum_{l=1}^{k} \mathbf{A}^l$, etc. Even if a boolean analysis is required (for example, for very large matrices [8]), one can build a binary relation of influence on the successive power matrices $\mathbf{I},\ \mathbf{A},\ \mathbf{A}^2,\ ...,\ \mathbf{A}^k, ...$ by doing $a_{ij}^{(k)} \geq \phi \Leftrightarrow \tilde{w}_{ij}^{(k)} = 1$, where $a_{ij}^{(k)}$ is a term of the power matrix $\mathbf{A}^k$; in this case, what one may want is to detect paths of **any** length between two sectors j and l, so, it could be sufficient to analyze the boolean summation of the sequence of matrices $\tilde{\mathbf{W}}^{(k)}$.

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6 See Lantner (1972a, 1972b, 1974).

7 At this point, qualitative methods could become boolean in the sense that the same procedure than described above could be applied, but without topological considerations (the adjacency of the adjacency will not be calculated).

8 According to Ranko Bon idea.

Boolean methods are working on simple oriented graphs. In this paper, they will often be compared to quantitative methods for two categories of problems, informational problems and topological problems.

III. Topological problems of one-matrix boolean methods

Topological problems are perhaps the most important because they affect the intern coherence of boolean methods, when preceding problems either can be corrected (by working in relative terms) either affect only the cost and the interest of the method. Here, it is recalled why boolean methods are topologically wrong (Mesnard 1995). There are two main ways to proceed when making use of topological methods: pretopological methods by working on the inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}$ or by working on the direct matrix $\mathbf{A}$ (or even $(\mathbf{I}-\mathbf{B})^{-1}$ and $\mathbf{B}$) and layer-based methods (by working on intermediate matrices in absolute terms).

There are two great types of pretopological methods, those that work on the inverse matrix and those that work on the direct matrix.

A. Boolean analysis on the inverse matrix

Denote m_{ij} as the terms of the inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}$, $\mathbf{A}$ being the direct matrix of technical coefficients. In quantitative terms, it is indicating how the final demand of commodity *j* influences the output of any commodity *i*. In qualitative terms, the above matrix $\mathbf{H}$ becomes the inverse matrix and if $m_{ij} \geq \phi$, then $w_{ij}^{(1)} = 1$.

The problem is that the adjacency of the adjacency has no meaning. As the inverse matrix is yet the limit of an infinite sum of power matrices, $(\mathbf{I}-\mathbf{A})^{-1} = \sum_{t=0}^{\infty} \mathbf{A}^t$, the multiplier m_{ij} captures all the effects between a commodity *j* and another sector *i*. Conversely, if one says that the adjacency of the adjacency has a meaning, one says that there is something after infinite...

In other words, assume that $m_{ij} \geq \phi \Leftrightarrow w_{ij}^{(1)} = 1$ and $m_{jl} \geq \phi \Leftrightarrow w_{jl}^{(1)} = 1$; for qualitative I-O analysis, $w_{il}^{(2)} = 1$ but this has no meaning, even if transitivity is assumed, because the corresponding attitude in quantitative I-O analysis would be to calculate $m_{ij}\, m_{jl}$ or $m_{il}^{(2)} = \sum_j m_{ij}\, m_{jl}$ which is not allowed because the term m_{il} yet indicates **all** the effects of the demand of commodity l on sector i. So, in the inverse matrix, one is able to consider paths with one and only one arc, i.e. $l \rightarrow j$ and $j \rightarrow i$, but not paths with more than one arc like $l \rightarrow j \rightarrow i$: the influence from l over i via j does not exist because one have an influence of l over j initiated by a exogenous impulsion of demand, i.e. $\Delta x_j = m_{jl}\, \Delta\, y_l$ (or $\Delta x_j = \sum_l m_{jl}\, \Delta\, y_l$ to the total) but the influence of l does not continue over i because the influence of j over i comes from *another* exogenous impulsion of demand, i.e. $\Delta x_i = m_{ij}\, \Delta\, y_j$ (or $\Delta xi = \sum_j m_{ij}\, \Delta\, y_j$ to the total). Therefore the interest of boolean analysis on the inverse matrix is very low, or at least not greater than the simple study of the inverse matrix.

B. Boolean analysis on the direct matrix

In the direct matrix, things are quite different because one will be able to calculate the adjacency of the adjacency. The above matrix **H** becomes the direct matrix (here, the technical coefficient matrix **A** is used, but the flow matrix **Z** or the allocation coefficient matrix **B** can also be used to develop the same arguments): $a_{il} \geq \phi \Leftrightarrow w_{il}^{(1)} = 1$.

The relation **W** used by qualitative I-O analysis is transitive if:

$$\left\{ \begin{array}{l} w_{ij}^{(1)} = 1 \\ w_{jl}^{(1)} = 1 \end{array} \right\} \Rightarrow w_{il}^{(1)} = 1 \text{ for all } i, l \text{ and for at least one } j.$$

In many cases, the relation is **not** transitive, as in the following example:

$$\left\{ \begin{array}{l} a_{ij} \geq \phi \Leftrightarrow w_{ij}^{(1)} = 1 \\ a_{jl} \geq \phi \Leftrightarrow w_{jl}^{(1)} = 1 \\ a_{il} < \phi \Leftrightarrow w_{il}^{(1)} = 0 \end{array} \right\}.$$

It is even possible that there is no direct relation between *i* and *l* : $a_{il} < \phi \Leftrightarrow w_{il}^{(1)} = 0$, but only a reverse relation - so, a circuit - between *l* and *i* : $a_{li} > \phi \Leftrightarrow w_{li}^{(1)} = 1$.

Moreover, there is a difficulty of definition. Clearly,

$$\left\{ \begin{array}{l} a_{ij} \geq \phi \Leftrightarrow w_{ij}^{(1)} = 1 \\ a_{jl} \geq \phi \Leftrightarrow w_{jl}^{(1)} = 1 \end{array} \right\} \Rightarrow w_{il}^{(2)} = 1 ,$$

but $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$ are two distinct relations. So transitivity must be defined in a modified relation, the relation **W**, as "$w_{ij} = 1$ if and only if *i* is in the adjacency of *j*, directly **or** indirectly". However even in this case, the relation can be not transitive. Confusion must not be done. For example, in the following expression in $\mathbf{W}^{(1)}$,

$$\left\{ \begin{array}{l} a_{ij} \geq \phi \Leftrightarrow w_{ij}^{(1)} = 1 \\ a_{jl} \geq \phi \Leftrightarrow w_{jl}^{(1)} = 1 \\ a_{li} \geq \phi \Leftrightarrow w_{li}^{(1)} = 1 \end{array} \right\},$$

the two first items, that are implying $w_{il}^{(2)} = 1$ in $\mathbf{W}^{(2)}$, are not incompatible with the third, i.e. $w_{li}^{(1)} = 1$, because $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$ are distinct, even if $w_{li}^{(1)} = 1$ and $w_{il}^{(1)} = 1$ are able to coexist.

The true no-transitivity which is in question here is concerning a contradiction between qualitative and quantitative I-O analysis: in relative terms, one can have $a_{jl}^{(2)} < \phi$ even if $a_{ij} \geq \phi$ and $a_{jl} \geq \phi$, where $a_{jl}^{(2)}$ is a term of the power matrix $\mathbf{A}^2$. Consider again the example where in qualitative terms,

$$\left\{ \begin{array}{l} a_{ij} \geq \phi \Leftrightarrow w_{ij}^{(1)} = 1 \\ a_{jl} \geq \phi \Leftrightarrow w_{jl}^{(1)} = 1 \end{array} \right\} \Rightarrow w_{il}^{(2)} = 1 ,$$

but in quantitative terms one have $a_{il}^{(t+1)} < a_{il}^{(t)}$ generally, so $a_{il}^{(2)} < a_{il}$ generally, where $a_{il}^{(2)} = \sum_j a_{ij}\, a_{jl}$ is the indirect coefficient from l to i and is a term of the matrix $\mathbf{A}^2$, and often $a_{il}^{(2)} < \phi$ which implies that the corresponding term in the boolean matrix is zero: $\tilde{w}_{il}^{(2)} = 0$, by defining the relation $\tilde{\mathbf{W}}^{(k)}$ as $\tilde{w}_{ij}^{(k)} = 1 \Leftrightarrow a_{ij}^{(k)} \geq \phi$ for all i and j, where $a_{ij}^{(k)}$ is a term of $\mathbf{A}^k$. This often holds because for a productive economy, in the open I-O model, $\mathbf{A}^k \underset{k\to\infty}{\longrightarrow} 0$; as an illustration (Mesnard, 1995) one can assume that all coefficients are equal, i.e. $a_{ij} = \dfrac{1}{n+1}$ for all i and j, with n productive sector and one exogenous sector, then $a_{il}^{(2)} = \sum_{j}^{n} a_{ij}\, a_{jl} = \sum_{j=1}^{n} \left(\dfrac{1}{n+1}\right)^2 = \dfrac{n}{(n+1)^2} < a_{il}$. So there is a true contradiction: qualitative I-O analysis may find a relation between l and i, where quantitative I-O analysis may not find; this is called an artefact [9].

IV. Informational problems

A. Lost of information, choice of filters and volume of computation

In qualitative methods, some information is obviously lost because all values under the filter are putted to zero, and all values equal or above the filter are putted to 1. This is not a serious problem if it is only like a rounding that gives the correct result at the end, but here it is not the case. In theory, it is possible to change the value of the filter to explore all the possible set but there is and infinite number of possible values for the filter, so it is not really possible. So, in real applications, a finite number of values of the filter are chosen. Often [10], equidistant values are chosen, but perhaps a different set of values can be chosen (a well-known problem

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[9] One assumes that the value of the filter does not decrease with the parameter k .

[10] Like in MFA (Schnabl, 1992, 1994, 1995).

in econometrics and data analysis, when a quantitative variable must be converted into a qualitative variable), for example by determining dividing the interval between the higher term and the lower term of the matrix into N shares, either in billion of dollars when matrix **H** is expressed in absolute terms, either in percentage when **H** is expressed in relative terms. Another method consist into dividing the set of vectors into N of sensibly equal shares, say 10 parts, and then fix the values of the filter to obtain such parts, so, even if the values of filter are any number, there are between to successive values of the filter a share equal to $\frac{1}{N}$ of the whole set of sectors [11]. Both methods are obviously arbitrary, but it could be acceptable in practice if the number of levels for the filter are sufficiently numerous and if the model is sufficiently "smooth", what is often the case with linear model but what cannot be guaranteed with non linear models. However, the question that raise is: what is the sensibility of the results to the choice of these values? Sometimes, a small variation of the value of the filter may generate a large variation on the boolean matrix: some arcs appear or disappear, what can change strongly the results.

A consequence of all this is that the volume of computation remains larger with boolean methods than with quantitative methods: this is a real paradox, because qualitative methods are normally used when the amount of computation is so large that it can discourage anybody: by replacing real numbers by boolean numbers, all things would normally become more easy and faster to compute.

Boolean mathematics are more easy than ordinary mathematics to compute by hand for small matrices, so, before the computing era, this approach was justified but now, as everybody can

[11] Mougeot, Duru, Auray (1977) practice both methods.

compute easily with its personal computer, it is not acceptable to lost information. In the other hand and paradoxically, for large matrices, the volume of calculation is important.

B. Concealing of successive boolean matrices

Remembering that $\mathbf{W}^{(k)}$ is reflecting a set of couples of vertices, denoted $S^{(k)}$, one have $S^{(k)} \subseteq S^{(k-1)} \subseteq ... \subseteq S^{(1)}$ generally [12], so $\mathbf{D} = \sum_k \mathbf{W}^{(k)} = \mathbf{W}^{(1)} = \mathbf{W}_0$. So, boolean methods lead generally to the same result regarding to the dependency matrices i.e. the first boolean matrix conceals the boolean matrices that follow it.

C. Bias caused by working on absolute coefficients instead on relative coefficients

Often, traditional qualitative I-O analysis, and even more sophisticated methods like Schnabl's MFA, are working on absolute coefficients, i.e. in I-O analysis on the matrix of flows **Z** expressed in dollars. When the filter is expressed in money units and when **Z** is used, small sectors [13] will be removed from the analysis because all their flows are naturally small, and it will remain only the larger sectors. That is why it is preferable to work on **A**, or **B**, with a filter expressed in relative terms, rather than on **Z**, with a filter expressed in absolute terms, allowing more correct comparisons by eliminating the size effect: even a small buying sector can have large technical coefficients or a small selling sector can have large allocation coefficients [14]. This point is very important regarding to the following one.

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[12] This is demonstrated in (Bon, 1989, p. 224).

[13] Like "Glass" or "Leather and Shoes" sector in the French national accounting system.

[14] An alternative method to eliminate the size effect when working on **Z** would be to use

D. Non discerning "layers"

The layers based method MFA is probably better than many other boolean methods. It is apparently more discriminating than traditional boolean methods, because that there are much more zeros in $\mathbf{W}_k$ than in $\mathbf{W}_{k-1}$: as coefficients become smaller from $\mathbf{A}^{k-1}$ to $\mathbf{A}^k$, the process can be stopped in practice for $k \geq 5$, unlike for traditional boolean methods. As $\mathbf{W}_1$ indicates the adjacency for indirect influences, analyzing this matrix is topologically more satisfactory than analyzing matrices of adjacency of adjacency.

However, one will demonstrate that it is more interesting to work on the sequence $\mathbf{I},\ \mathbf{A},\ \mathbf{A}^2,\ ...,\ \mathbf{A}^k$ rather than on layers. First, remember that it is preferable to work in relative terms, when layers based methods work in absolute terms. Second, note that the same output $\mathbf{x}_k$ can be generated either by $\mathbf{Z}_{k-1}\ \mathbf{s}$ ($\mathbf{s}$ is the sum vector) or by $\mathbf{A}^k\ \mathbf{y}$ because one have:

$$\mathbf{x}_k = \mathbf{A}^k\ \mathbf{y} = \mathbf{A}^k\ \langle \mathbf{y} \rangle\ \mathbf{s} = \mathbf{Z}_{k-1}\ \mathbf{s} = \mathbf{A}\ \langle \mathbf{x}_{k-1} \rangle\ \mathbf{s} = \mathbf{A}\ \mathbf{x}_{k-1}$$

so $\mathbf{A}^k$ corresponds to the same iteration than $\mathbf{Z}_{k-1}$, but $\mathbf{A}^k\ \langle \mathbf{y} \rangle \neq \mathbf{Z}_{k-1}$: analyzing the power matrix $\mathbf{A}^k$ is not the same thing than analyzing the layer $\mathbf{Z}_{k-1}$. What is preferable? All layers, evaluated in absolute terms, carry out the same information in relative terms. To prove it, one must define first column coefficients: column coefficients are not exactly technical coefficients, but that are close to them because they are calculated in column without taking into account the added value of each sector:

$\hat{\mathbf{A}}_k$ is such that $\hat{a}_{ij}^k = \dfrac{(z_{ij})_k}{\sum_i (z_{ij})_k}$ where $(t_{ij})_k$ is a term of $\mathbf{Z}_k$, or in matrix terms,

$$\hat{\mathbf{A}}_k = \mathbf{Z}_k\ \langle \mathbf{s}'\ \mathbf{Z}_k \rangle^{-1} \qquad (3)$$

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one value of the filter for each sector, i.e. a vectorial form instead of a scalar form, but it is really more complicated and perhaps less defensible than working on **A** or **B**.

So, the information carried out by matrices $\mathbf{W}_k$ in MFA is trivial: the sequence of matrices $\mathbf{Z}_k$ carries out the same information in terms of column coefficients. This is caused by the fact that all matrices $\hat{\mathbf{A}}_k$ of column coefficients, deduced from layers $\mathbf{Z}_k$ at each step k, are identical: $\hat{\mathbf{A}}_k = \hat{\mathbf{A}}_0$ for all k; so, all layers carry again and again the same information because all boolean matrices $\hat{\mathbf{W}}_k$ found from the $\hat{\mathbf{A}}_k$ are the same (in other terms, a MFA with relative coefficients instead of absolute coefficients gives the same result for all layers). It is easy to prove this property. One have:

$\hat{\mathbf{A}}_k = \mathbf{Z}_k \langle \mathbf{s}' \mathbf{Z}_k \rangle^{-1} = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \langle \mathbf{s}' \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \rangle^{-1}$

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle [\langle \mathbf{s}' \mathbf{A} \rangle \langle \mathbf{A}^k \mathbf{y} \rangle]^{-1}$ (because if $\mathbf{b}$ and $\mathbf{c}$ are two vectors, $\langle \mathbf{b}' \langle \mathbf{c} \rangle \rangle = \langle \mathbf{b} \rangle \langle \mathbf{c} \rangle$)

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{A}^k \mathbf{y} \rangle \langle \mathbf{A}^k \mathbf{y} \rangle^{-1} \langle \mathbf{s}' \mathbf{A} \rangle^{-1}$

$\Leftrightarrow \hat{\mathbf{A}}_k = \mathbf{A} \langle \mathbf{s}' \mathbf{A} \rangle^{-1}$ so $\hat{\mathbf{A}}_k$ is a constant.

Similarly, all matrices of allocation coefficients $\hat{\mathbf{B}}_k$, deduced from layers $\mathbf{Z}_k$, are identical: $\hat{\mathbf{B}}_k = \hat{\mathbf{B}}_0$. First, one have:

$\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{y} = \mathbf{x}\,\mathbf{B}\,\mathbf{x}^{-1}\,\mathbf{x} + \mathbf{y} = \mathbf{x}\,\mathbf{B} + \mathbf{y} \Rightarrow \mathbf{x}\,(\mathbf{I} - \mathbf{B}) = \mathbf{y} \Rightarrow \mathbf{x} = \mathbf{y}\,(\mathbf{I} - \mathbf{B})^{-1}$

$\mathbf{B} = \langle \mathbf{x} \rangle^{-1}\,\mathbf{Z} = \langle \mathbf{Z}\,\mathbf{s} \rangle^{-1}\,\mathbf{Z} \Rightarrow \mathbf{Z} = \langle \mathbf{x} \rangle\,\mathbf{B} = \left\langle \mathbf{y}\,(\mathbf{I} - \mathbf{B})^{-1} \right\rangle \mathbf{B} \Rightarrow \mathbf{Z}_k = \langle \mathbf{y}\,\mathbf{B}^k \rangle\,\mathbf{B}$

So,

$\hat{\mathbf{B}}_k = \langle \mathbf{Z}_k\,\mathbf{s} \rangle^{-1}\,\mathbf{Z}_k = \langle \langle \mathbf{y}\,\mathbf{B}^k \rangle\,\mathbf{B}\,\mathbf{s} \rangle^{-1}\,\langle \mathbf{y}\,\mathbf{B}^k \rangle\,\mathbf{B}$

$\Leftrightarrow \hat{\mathbf{B}}_k == [\langle \mathbf{y}\,\mathbf{B}^k \rangle\,\langle \mathbf{B}\,\mathbf{s} \rangle]^{-1}\,\langle \mathbf{y}\,\mathbf{B}^k \rangle\,\mathbf{B} = \langle \mathbf{B}\,\mathbf{s} \rangle^{-1}\,\langle \mathbf{y}\,\mathbf{B}^k \rangle^{-1}\,\langle \mathbf{y}\,\mathbf{B}^k \rangle\,\mathbf{B}$

(because if $\mathbf{b}$ and $\mathbf{c}$ are two vectors, $\langle \langle \mathbf{c} \rangle\,\mathbf{b} \rangle = \langle \mathbf{c} \rangle\,\langle \mathbf{b} \rangle$

$\Leftrightarrow \hat{\mathbf{B}}_k = \langle \mathbf{B}\,\mathbf{s} \rangle^{-1}\,\mathbf{B}$

Remark. When one consider the Leontief model, only the matrix $\mathbf{A}$ plays a role from one iteration to the following. One can write: $\mathbf{Z}_{k+1} = \mathbf{A}\,\langle \mathbf{Z}_k\,\mathbf{s} \rangle$, because $\mathbf{x}_{k+1} = \mathbf{Z}_k\,\mathbf{s} = \mathbf{A}^{k+1}\,\mathbf{y}$, and so the only change between $\mathbf{Z}_{k+1}$ and $\mathbf{Z}_k$ is brought by $\mathbf{A}$, but it is also the case for $\mathbf{A}^{k+1}$ and

$\mathbf{A}^k$, $\mathbf{A}^{k+1} = \mathbf{A}\,\mathbf{A}^k$ [15]. So, the lack of information between two iterations when layers are considered in relative terms does not lies on this fact.

Finally, in relative terms, the information brought by layers is always the same with both manners to do relative terms, **A** or **B**. Taking into account the fact that working in absolute terms is less preferable than working in relative terms, layers are not discerning.

<u>Example</u>.

$\mathbf{A} = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$ and assume that $\Phi = 200$ and $\phi = 0.2$.

So,

- $\mathbf{x}_0 = \begin{pmatrix} 1000 \\ 1000 \end{pmatrix}$.

- $\mathbf{Z}_0 = \begin{bmatrix} 500 & 200 \\ 100 & 700 \end{bmatrix}$, $\mathbf{x}_1 = \begin{pmatrix} 700 \\ 800 \end{pmatrix}$, $\mathbf{W}_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(1)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^1 = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(1)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

- $\mathbf{Z}_1 = \begin{bmatrix} 350 & 160 \\ 70 & 560 \end{bmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 510 \\ 630 \end{pmatrix}$, $\mathbf{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(2)} = \mathbf{W}_1 * \mathbf{W}^{(1)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^2 = \begin{bmatrix} 0.27 & 0.24 \\ 0.12 & 0.51 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(2)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

- $\mathbf{Z}_2 = \begin{bmatrix} 255 & 126 \\ 51 & 441 \end{bmatrix}$, $\mathbf{x}_3 = \begin{pmatrix} 381 \\ 492 \end{pmatrix}$, $\mathbf{W}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(3)} = \mathbf{W}_2 * \mathbf{W}^{(2)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^3 = \begin{bmatrix} 0.159 & 0.222 \\ 0.111 & 0.381 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(3)} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

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[15] Similar reasoning comes from matrix **B**.

- $\mathbf{Z}_3 = \begin{vmatrix} 190.5 & 98.4 \\ 38.1 & 344.4 \end{vmatrix},$ $\mathbf{x}_4 = \begin{pmatrix} 288.9 \\ 382.5 \end{pmatrix},$

  $\mathbf{W}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(4)} = \mathbf{W}_3 * \mathbf{W}^{(3)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^4 = \begin{bmatrix} 0.1017 & 0.1872 \\ 0.0936 & 0.2889 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(4)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

- $\mathbf{Z}_4 = \begin{vmatrix} 144.45 & 76.50 \\ 28.89 & 267.75 \end{vmatrix},$ $\mathbf{x}_5 = \begin{pmatrix} 220.95 \\ 296.64 \end{pmatrix},$

  $\mathbf{W}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(5)} = \mathbf{W}_4 * \mathbf{W}^{(4)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^5 = \begin{bmatrix} 0.06957 & 0.15138 \\ 0.07569 & 0.22095 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(5)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

- $\mathbf{Z}_5 = \begin{vmatrix} 110.475 & 59.328 \\ 22.095 & 207.648 \end{vmatrix},$ $\mathbf{x}_6 = \begin{pmatrix} 169.803 \\ 229.743 \end{pmatrix},$

  $\mathbf{W}_5 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{W}^{(6)} = \mathbf{W}_5 * \mathbf{W}^{(5)} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

  $\mathbf{A}^6 = \begin{bmatrix} 0.049923 & 0.119880 \\ 0.059940 & 0.169803 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(6)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

- $\mathbf{Z}_6 = \begin{vmatrix} 85.9015 & 46.9486 \\ 16.9803 & 160.8201 \end{vmatrix}, \mathbf{x}_7 = \begin{pmatrix} 130.8501 \\ 177.8004 \end{pmatrix},$

  $\mathbf{W}_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \mathbf{W}^{(7)} = \mathbf{W}_6 * \mathbf{W}^{(6)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

  $\mathbf{A}^7 = \begin{bmatrix} 0.03694955 & 0.09390060 \\ 0.04695030 & 0.01308501 \end{bmatrix}$ and $\widetilde{\mathbf{W}}^{(7)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

- etc...

And the information carried out by each layer in relative terms is:

$$\hat{\mathbf{A}}_0 = \begin{bmatrix} 0.833 & 0.222 \\ 0.167 & 0.777 \end{bmatrix} = \hat{\mathbf{A}}_1 = ... = \hat{\mathbf{A}}_k$$

and for example, by taking $\phi = 0.2$, $\hat{\mathbf{W}}_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \hat{\mathbf{W}}_1 = ... = \hat{\mathbf{W}}_k$.•

V. Conclusion

Four great type of structural analysis methods have been studied: two types of methods based on the analysis of one single matrix, the direct matrix or the inverse matrix, and two types of methods based on the analysis of a sequence of matrices, either the layers (the successive intermediate flow matrices), either the successive power matrices. The first three are *qualitative*, i.e. boolean based methods, the last one is *quantitative*.

Boolean topological and pretopological methods of structural analysis are wrong when they work on the inverse matrix, and fall into the problem of non transitivity when the work on the direct matrix.

One have proved that boolean methods lead to lost information and to a larger volume of computation than quantitative methods. it is more adequate to work on relative terms (in percentage) than in absolute terms (in dollars) to avoid to remove small sectors from the analysis. For layers based methods, when layers are converted into relative terms instead of absolute terms, one layer provide the same information than the following, so these methods are not informationally discerning.

The above developments prove why quantitative analysis is preferable to boolean or qualitative analysis, with tools like eigenvalues, determinant, multipliers, etc.. If it is necessary to have an idea of the intern dynamic of the model [16], in quantitative analysis, one will work on the sequence of power matrices $\mathbf{I}$, $\mathbf{A}$, $\mathbf{A}^2$, ..., $\mathbf{A}^k$, that provide paths of length

[16] See for example (Mesnard, 1992).

equal to 0, 1, 2, ..., *k*, and on its algebraic summation $\sum_{l=1}^{k} \mathbf{A}^l$ that provides the influence flowing by paths of a length equal or lower than *k*.

The transposition of these developments to other fields than input-output is easy, and even for non linear models, because the remarks on non transitivity are very general, and because the remarks about the inverse matrix can be applied to the equilibrium of a non linear model, even if the corresponding matrix cannot be computed simply. The remarks on layers are concerning models where the intern dynamic is important.

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